Computer Graphics and Optimization

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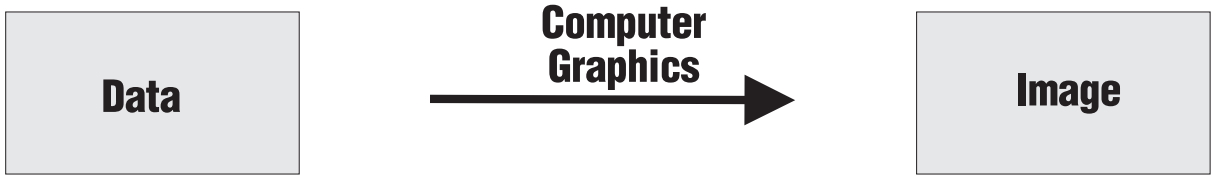
Computer Graphics

- Transforms data into Images



- Relation with Physical universe
  - Visualization $\rightarrow$ Photography
    (2D representation)
  - Vision $\rightarrow$ Human vision
    (3D reconstruction)
- What are the models?
- What are the problems?

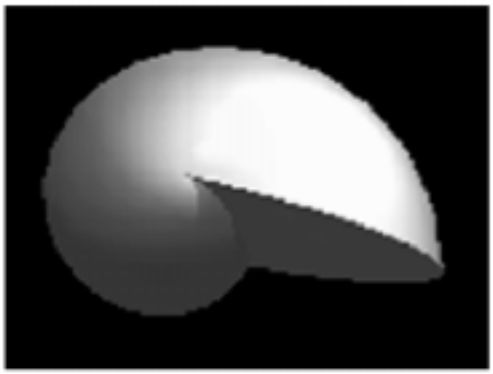
Graphical Objects

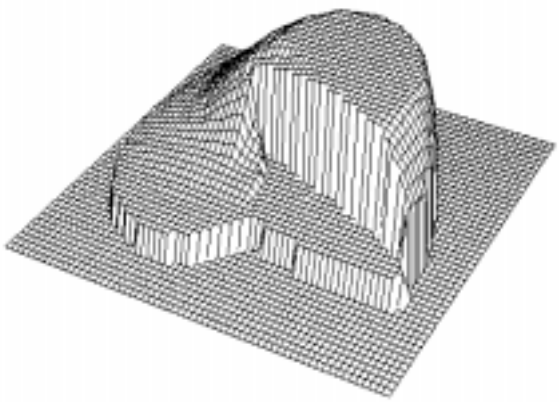
- Data
  - Curves, Surfaces and Solids (Manifolds) (Geometric models)
- Image
  - Color

$$f : U \subset \mathbb{R}^2 \to \mathbb{R}^3$$

  - Grayscale

$$f : U \subset \mathbb{R}^2 \to \mathbb{R}$$

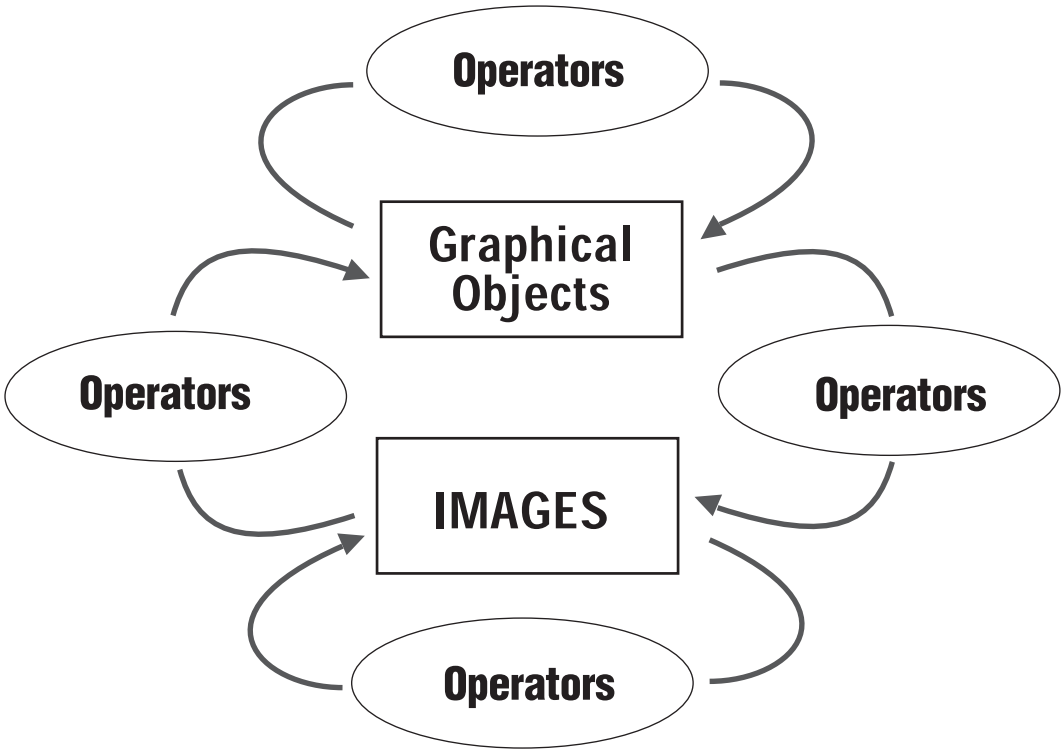




Graphical Objects (GO)

- $\mathcal{O} = (U\,,\, f), \quad f : U \subset \mathbb{R}^n \to \mathbb{R}^m$

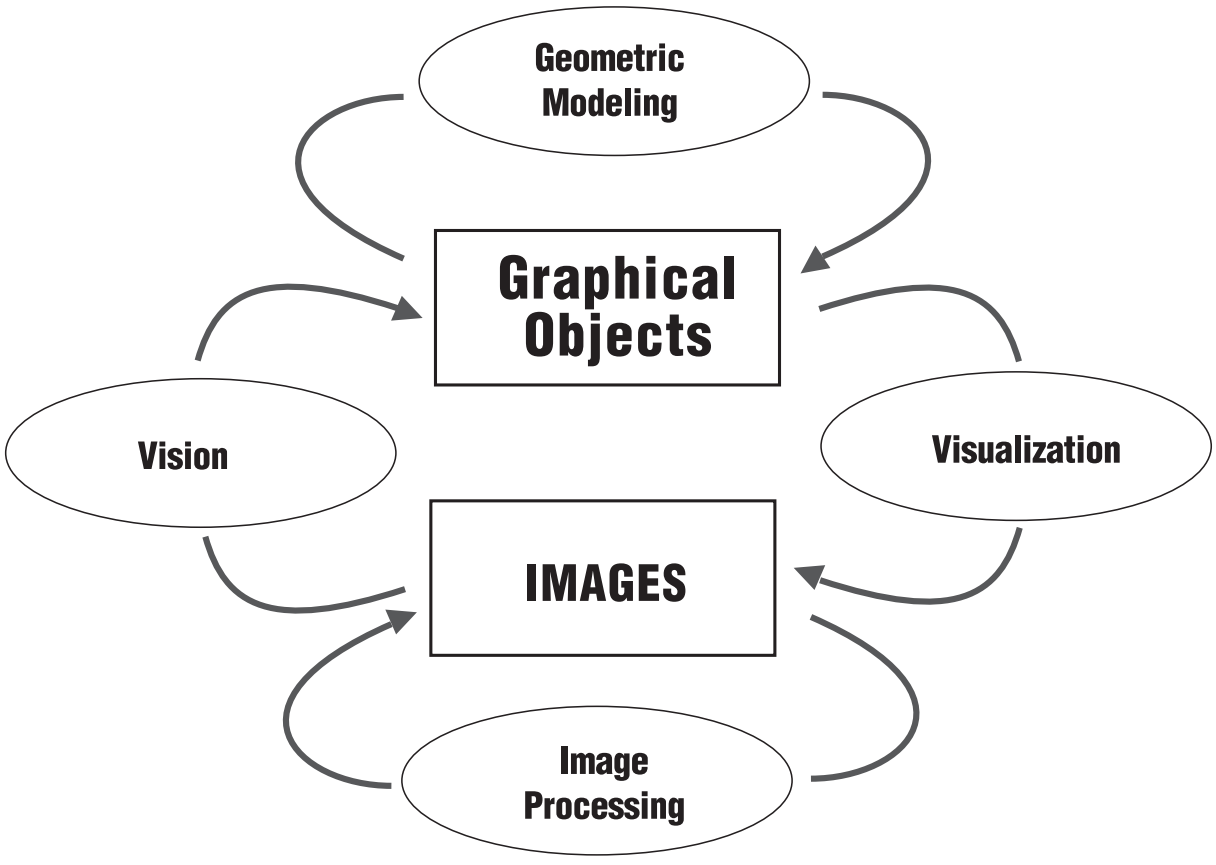
- Geometric Support U,
  Attribute function f,
  Dimension of $\mathcal{O}$



Computer Graphics

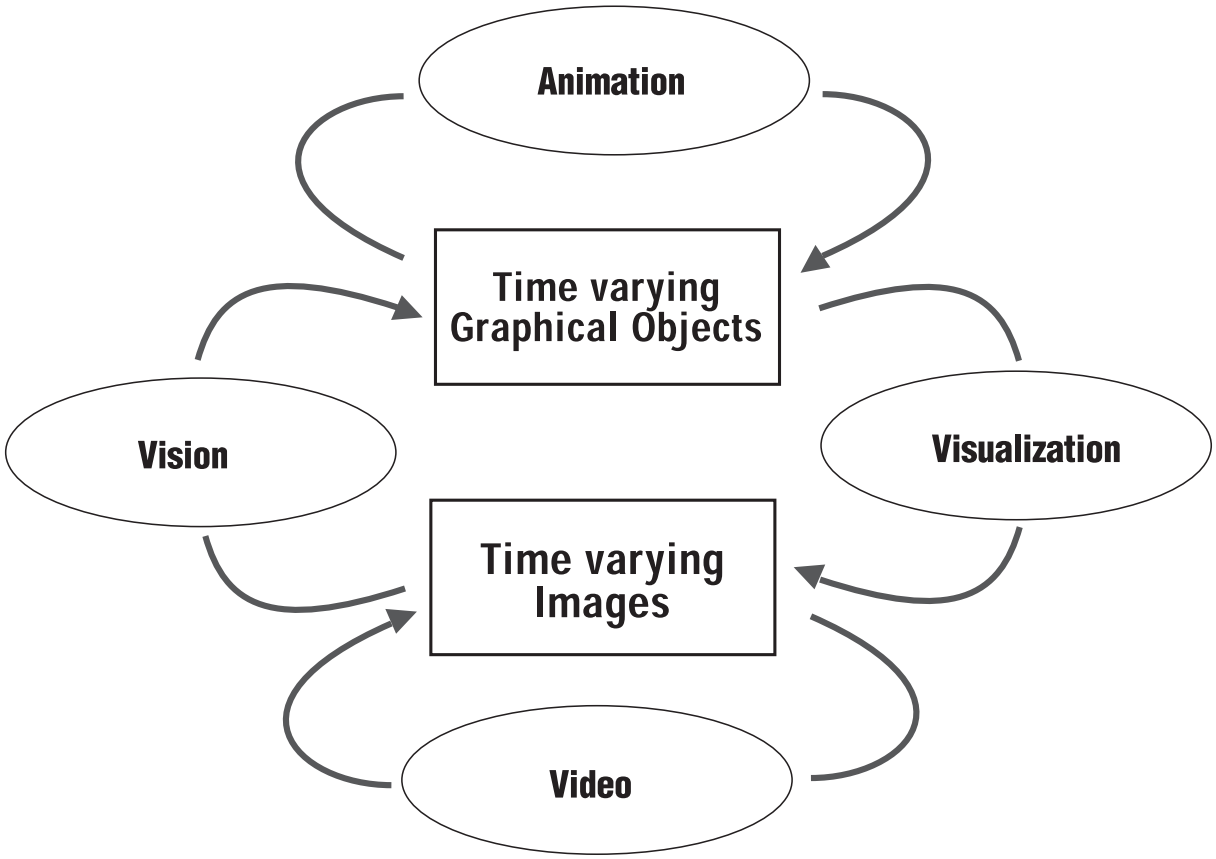
- Processing of graphical objects

- Subjacent areas



An additional Problem: Time

- Time varying graphical objects



- Just another graphical object

$$f : U \times [a, b] \to \mathbb{R}^n$$

Problems

- Spaces of graphical objects
  (Spaces of functions)

- Operators on spaces of graphical objects

$$T\,X = Y$$

- Direct problems

  – Given T and X, find Y

- Inverse problems

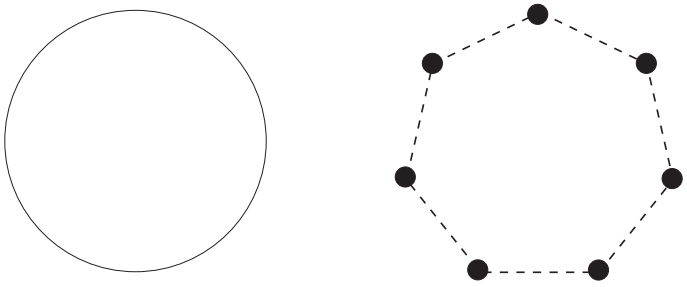
  – Given T and Y, find X

  – Given X and Y, find T

Problems

- Hadamard: well-posed problem
  - Existence of solution
  - Uniqueness of solution
  - Continuous dependence on initial conditions
- Live with ill-posed problems

$$x^2 + y^2 - 1 = 0$$



- "Best" solution

Problems: Visualization

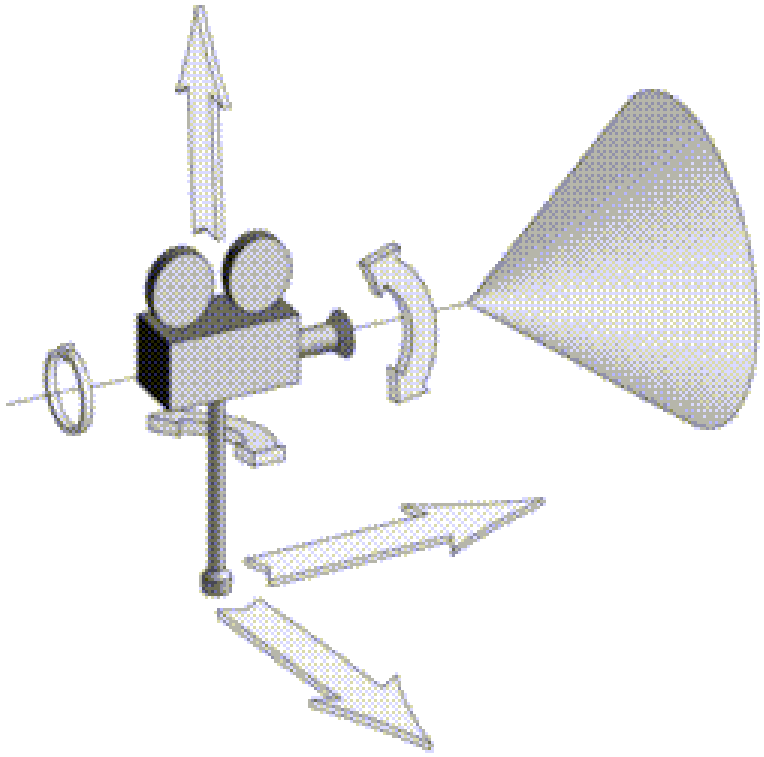
- X is the scene
  - Geometry
  - Attributes
  - Illumination
  - Camera specification (position, focus, etc.)
  - etc.
- T is the *rendering* operator

$$T\,X = Y$$

- Y is the image of the scene
- Direct problem

Problems: Visualization

- Camera specification

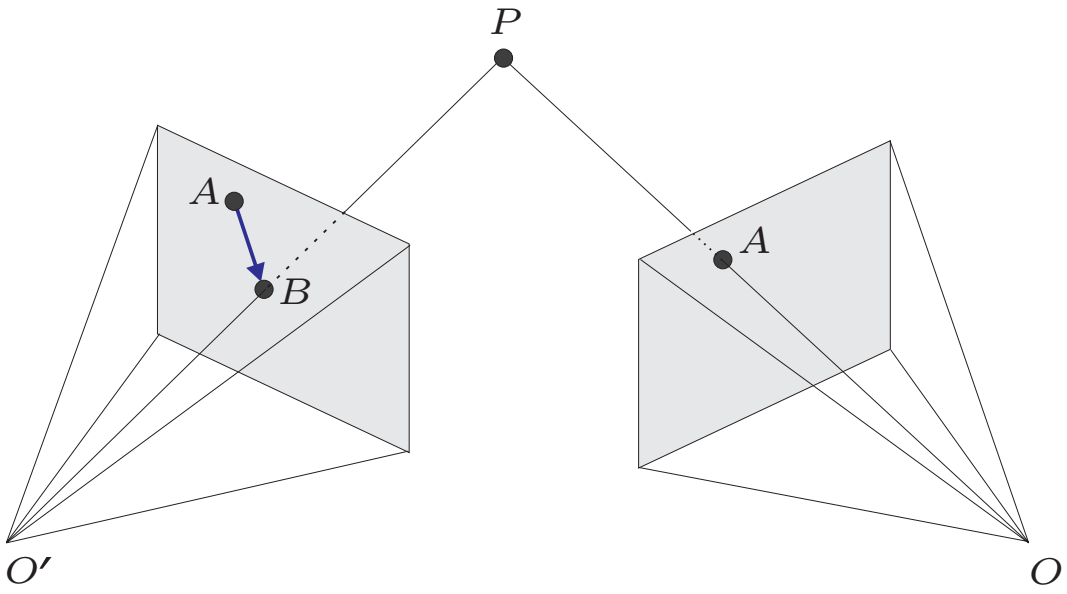


- Direct specification
  Direct problem – Seven parameters

Problems: Visualization

- Camera specification
- Inverse specification

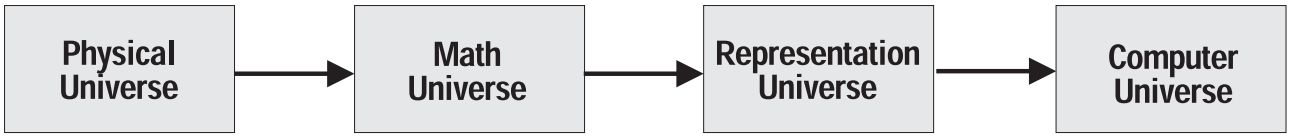
  Ill-posed inverse problem



- Camera calibration
  - Ill-posed inverse problem

Problems

- Representation of graphical objects
- Reconstruction of graphical objects



- Representation operator

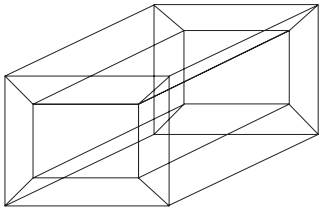
$$R\colon O \to O_d$$

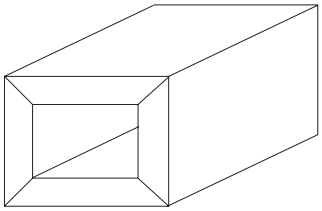
Representation and Reconstruction

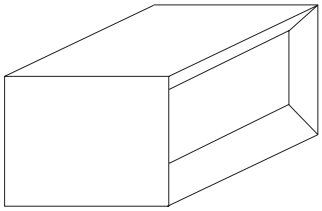
- Why do we need to reconstruct?
  - Representation gives incomplete information
  - Work in the continuous domain to avoid numerical errors
  - Semantics
- Reconstruction
  - Invertibility of the representation operator
  - Exact
  - Non-exact
- Ambiguous Representation
  (Ill posed reconstruction)

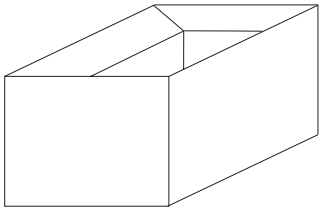
Ambiguous Representation

- Geometric Modeling
  (wireframe representation)



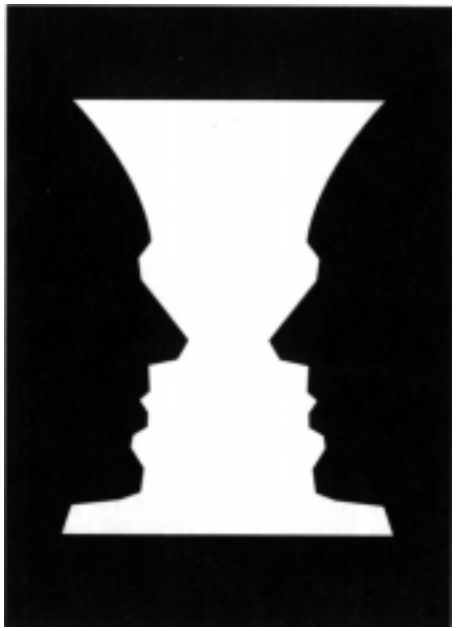






Ambiguous Representation

- Vision



Problems: Modeling

- Representation and reconstruction of curves and surfaces
- Curve reconstruction from points
- Energy minimization approach
  (Variational modeling)
- Internal energy

$$E_{internal} = \lambda E_{tension} + (1 - \lambda) E_{length}$$

$$E_{tension}(\alpha) = \int_{\alpha} k^2(t) dt$$

$$E_{length}(\alpha) = \int_{\alpha} ||\alpha'(t)|| dt$$

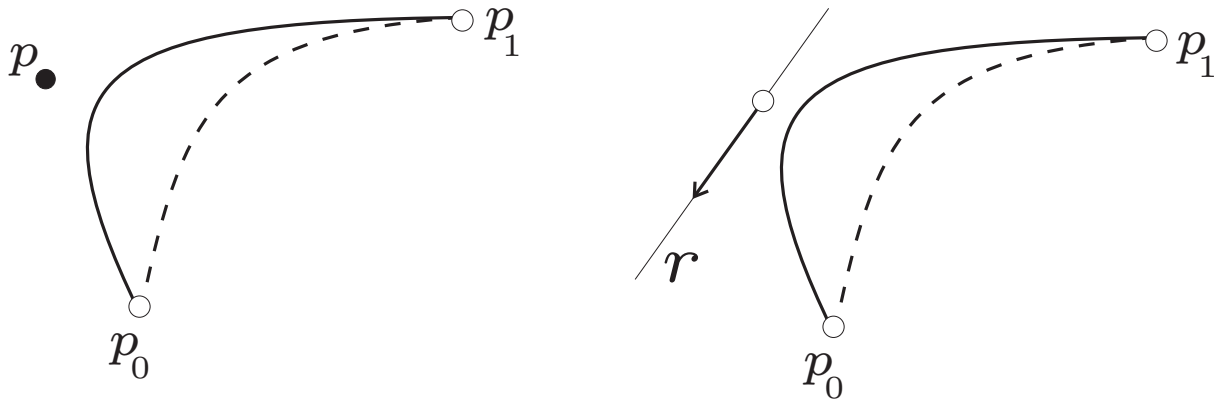
Problems: Modeling

- External energy
  - Attractive or repulsive forces
  - Punctual or directional
- Punctual

$$E_{punctual}(\alpha) = d(\alpha, p)^2 = min_t||\alpha(t) - p||^2$$

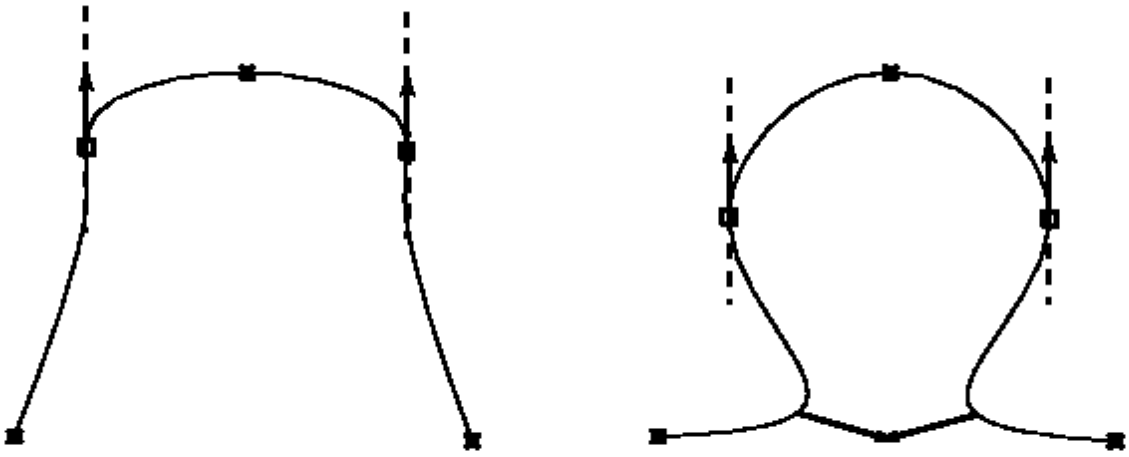
- Directional

$$E_{directional} = min_t||\alpha'(t) \times v||$$



Problems: Modeling

- Examples



Problems: Image boundary

- David Marr Conjecture
  (Reconstruction, ill-posed, problem)

- Boundary description
  - Frequency methods
  - Geometric methods

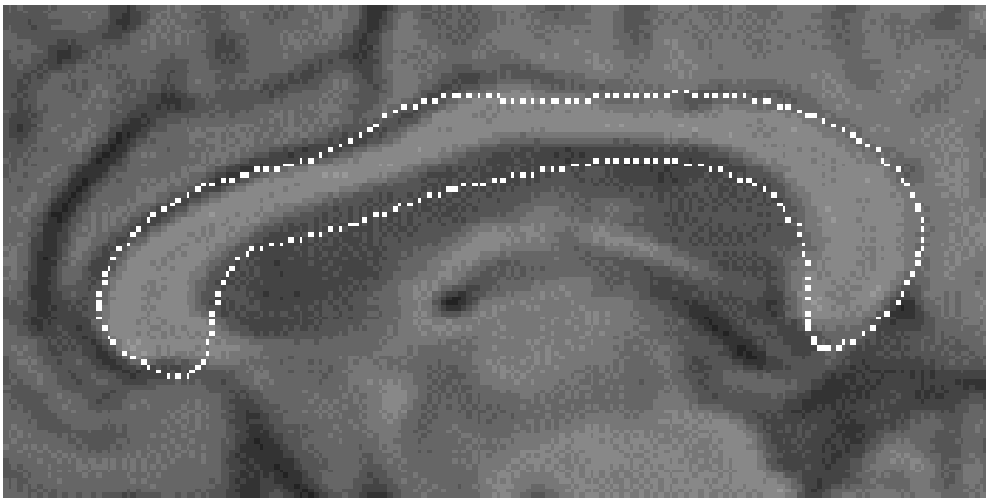
Problems: Image boundary

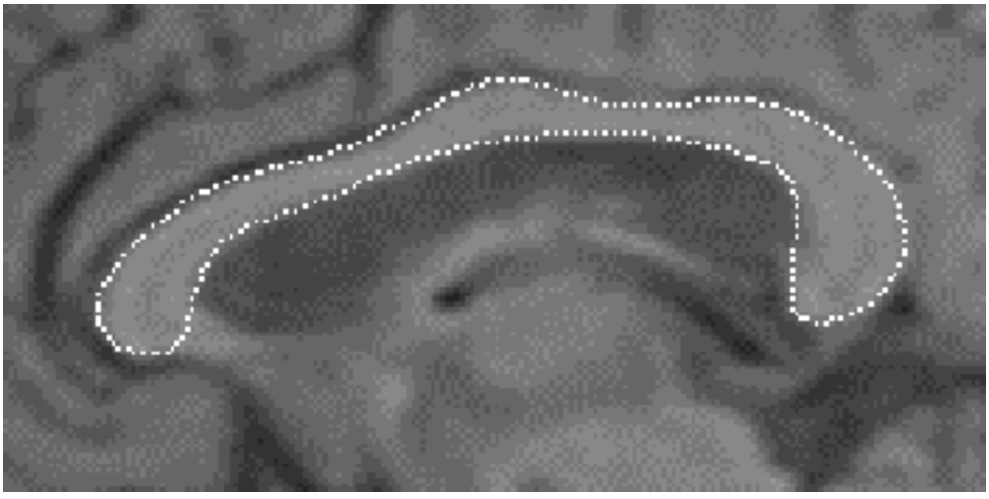
- Frequency computation of boundary

- Marr conjecture and frequency methods
  - Invertibility of boundary operator
  - Different choices of boundary operator

Problems: Image boundary

- Geometric computation of boundary
- Energy minimization approach (snakes)





Optimization

- Choose the best solution among a set of candidates

$$\min_{x \in S} f(x),$$

- f is the objective function

- S is the set of possible solutions

- Types of optimization problems

  - Continuous
    (Variational problems)

  - Discrete

  - Combinatorial

- Different choices to solve a problem

  - Curve minimization problem