IMPA - Instituto de Matemática Pura e Aplicada, Rio de Janeiro, RJ, Brazil

Introduction to Computational Manifolds and Applications

Part 1 - Constructions

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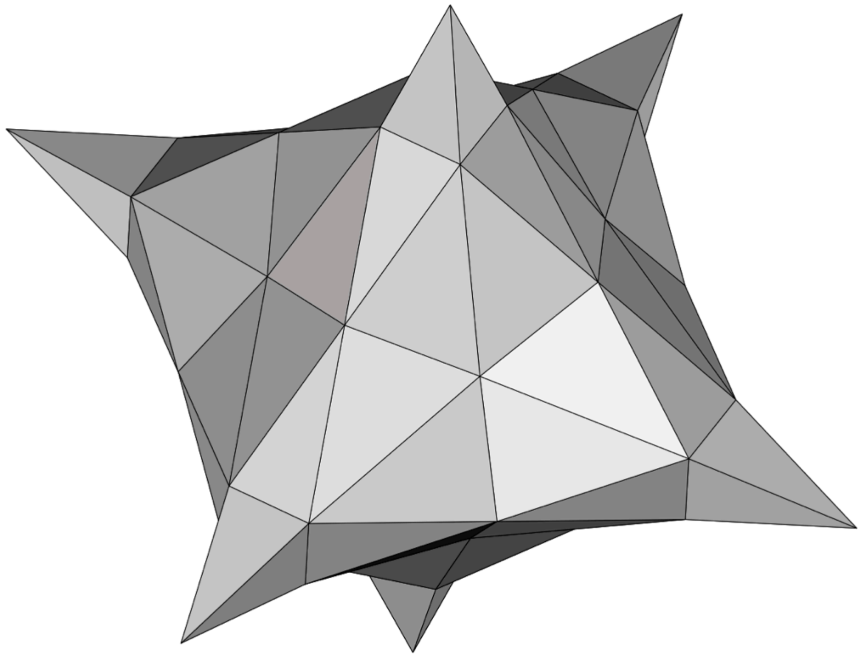
Parametric Pseudo-Manifolds

The User Perspective

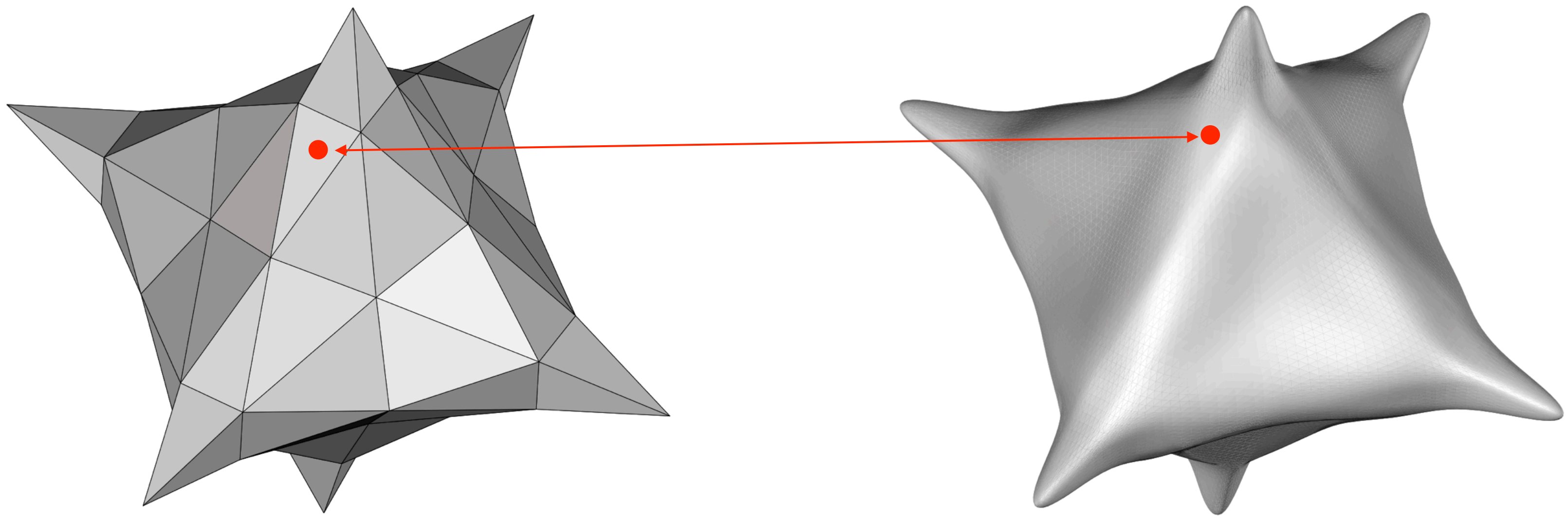
While interacting with a subdivision surface or a B-spline surface for the purposes of editing, rendering, or physical simulation, one uses the "control mesh" as an *interface*.



Parametric Pseudo-Manifolds

The User Perspective

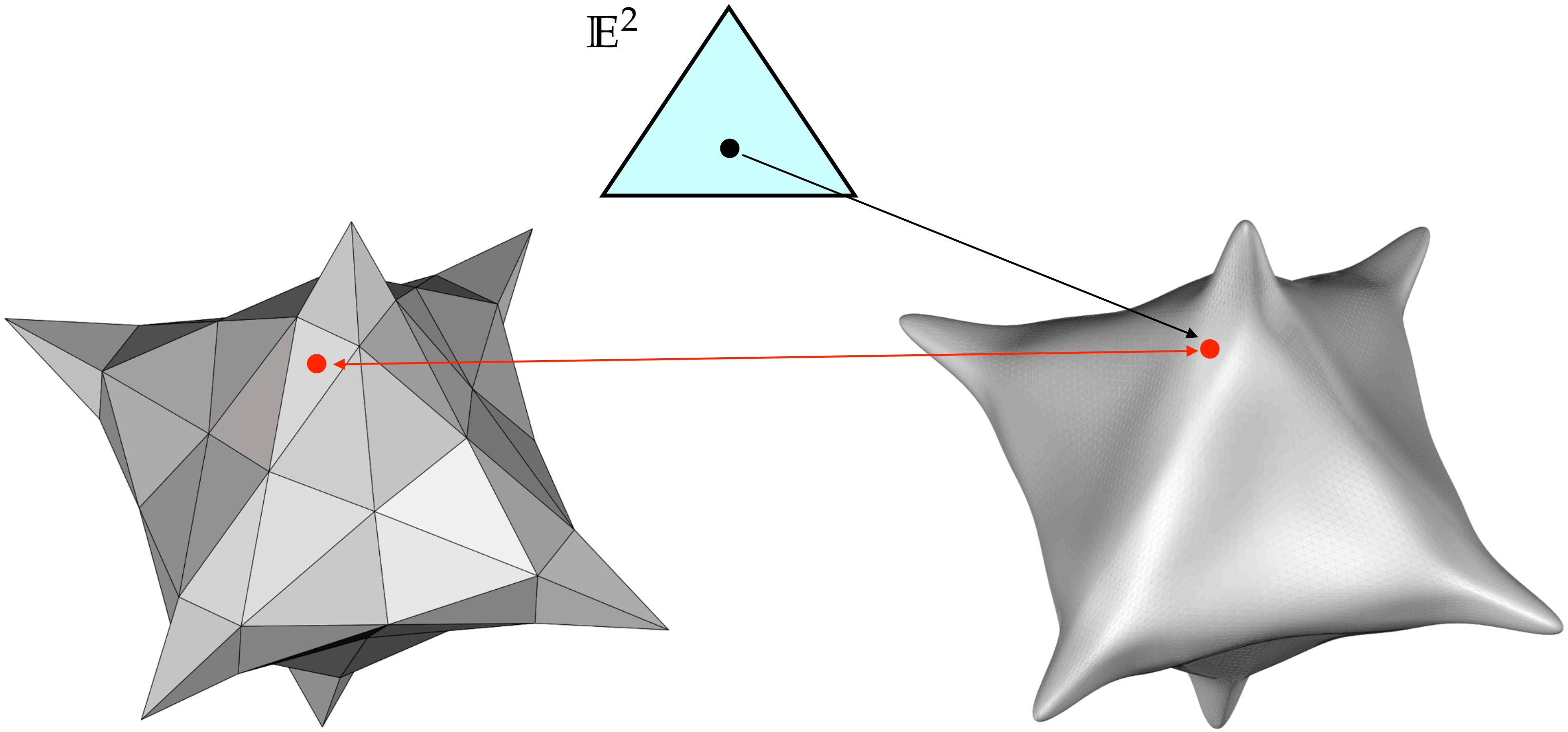
More specifically, the user assumes that there is a one-to-one correspondence between the control mesh and the surface. So, each point in the control mesh corresponds to a point in the surface, and thus the control mesh is viewed as a parameter space.



Parametric Pseudo-Manifolds

The User Perspective

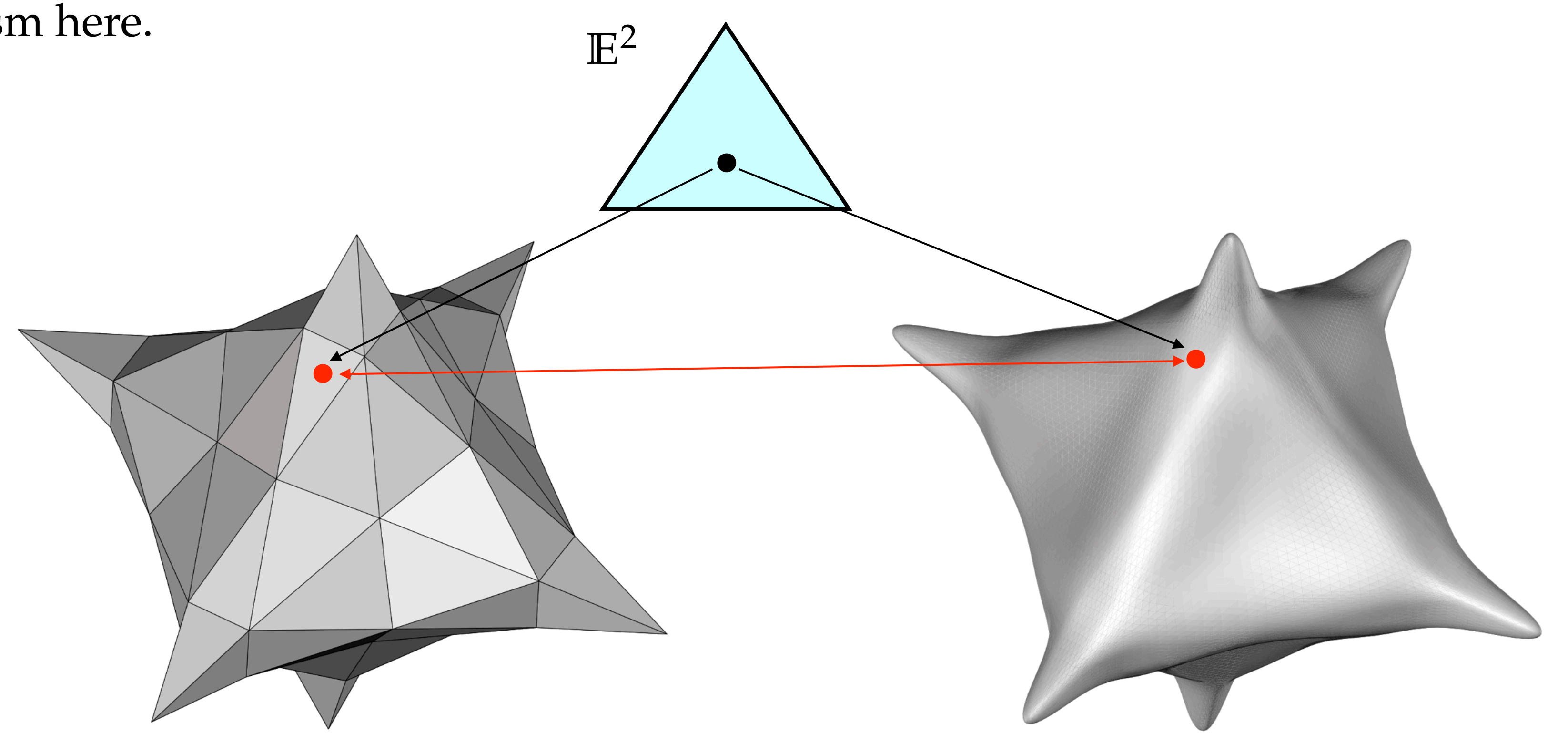
In reality, each point on the surface is the image of a (parameter) point in a triangle in $\mathbb{E}^2$.



Parametric Pseudo-Manifolds

The User Perspective

But, since we can relate the triangle in $\mathbb{E}^2$ with a triangle of the control mesh (via a barycentric mapping), the user "illusion" works fine. We will use a similar mechanism here.

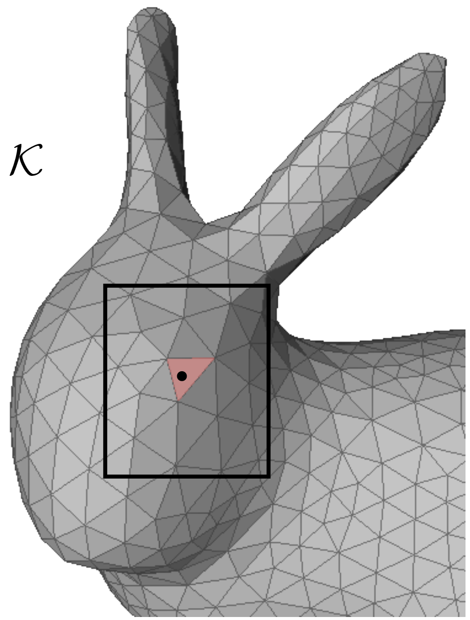


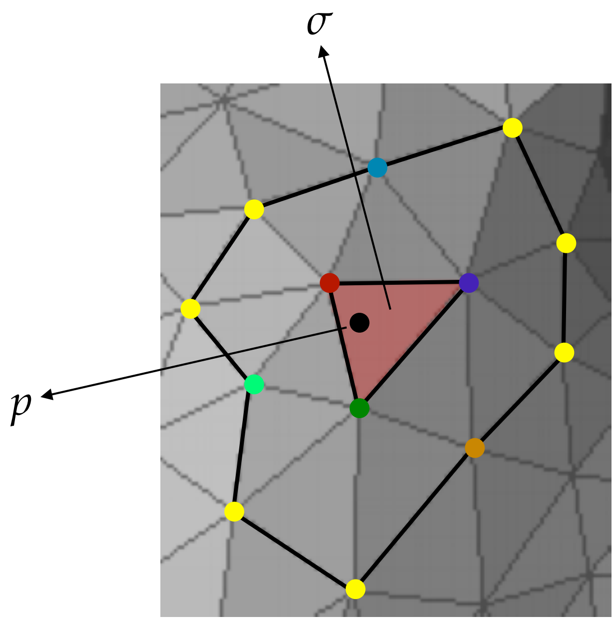
Parametric Pseudo-Manifolds

The User Perspective

Here, the control mesh is the underlying surface, $|\mathcal{K}|$, of the input simplicial surface, $\mathcal{K}$.

Let σ be a triangle in $\mathcal{K}$ and let p be any point in σ.



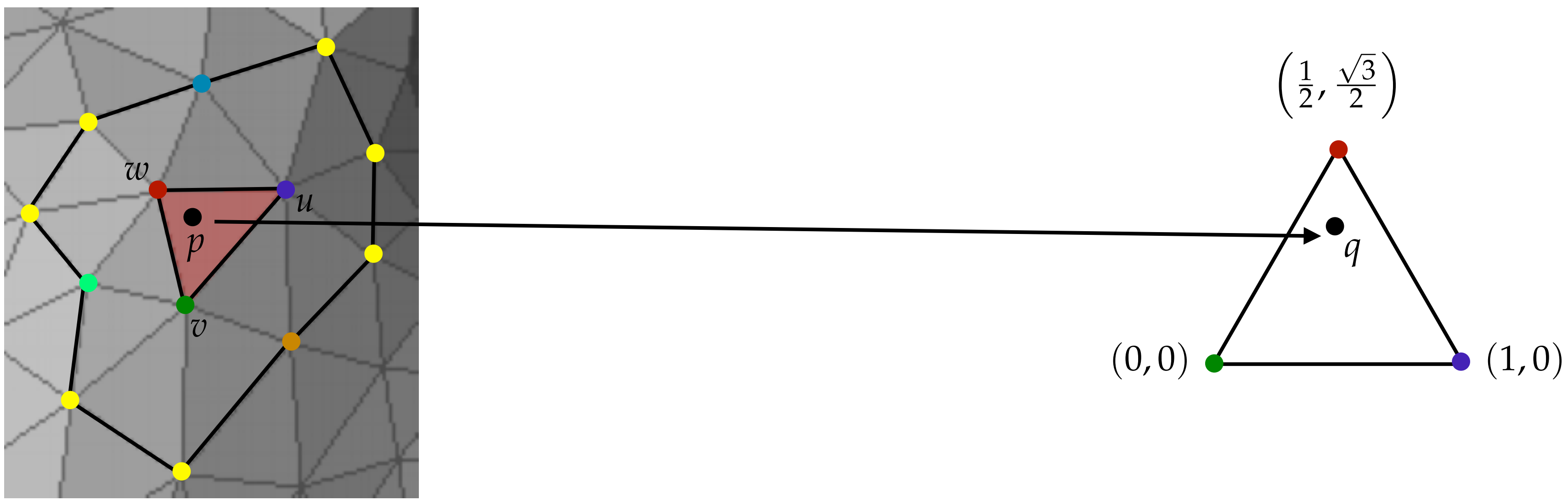


Parametric Pseudo-Manifolds

The User Perspective

Map the point p to a point q belonging to the equilateral triangle, $\triangle \subset \mathbb{E}^2$, with vertices

$$(0,0)\,, \quad (1,0)\,, \quad \text{and} \quad (1/2, \sqrt{3}/2)\,.$$

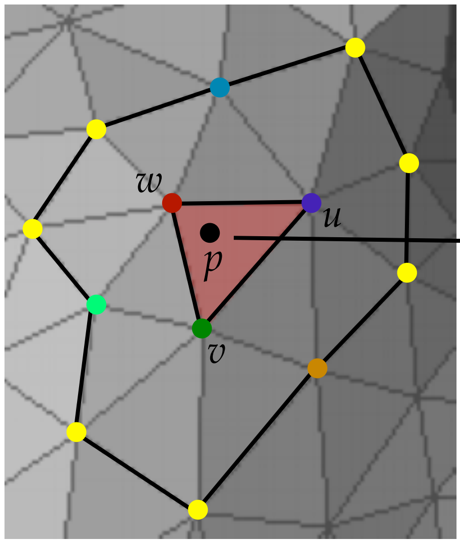


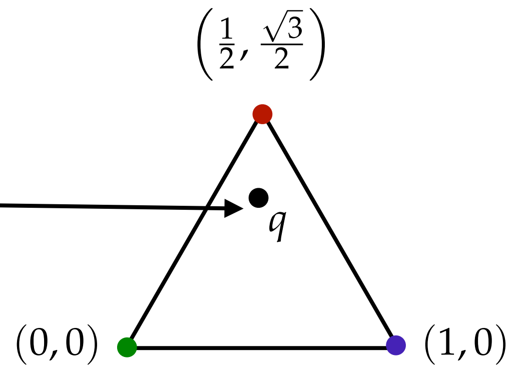
Parametric Pseudo-Manifolds

The User Perspective

We can do that by using the barycentric map that takes the vertices v, u, and w of σ to the vertices $(0,0)$, $(1,0)$, and $(1/2, \sqrt{3}/2)$ of $\triangle$, respectively. So, if $p = \lambda \cdot v + \mu \cdot u + \nu \cdot w$, where $\lambda, \mu, \nu \in \mathbb{R}$, $\lambda, \mu, \nu \geq 0$, and $\lambda + \mu + \nu = 1$, the coordinates of q are given by

$$q = \lambda \cdot (0,0) + \mu \cdot (1,0) + \nu \cdot (1/2, \sqrt{3}/2) \, .$$



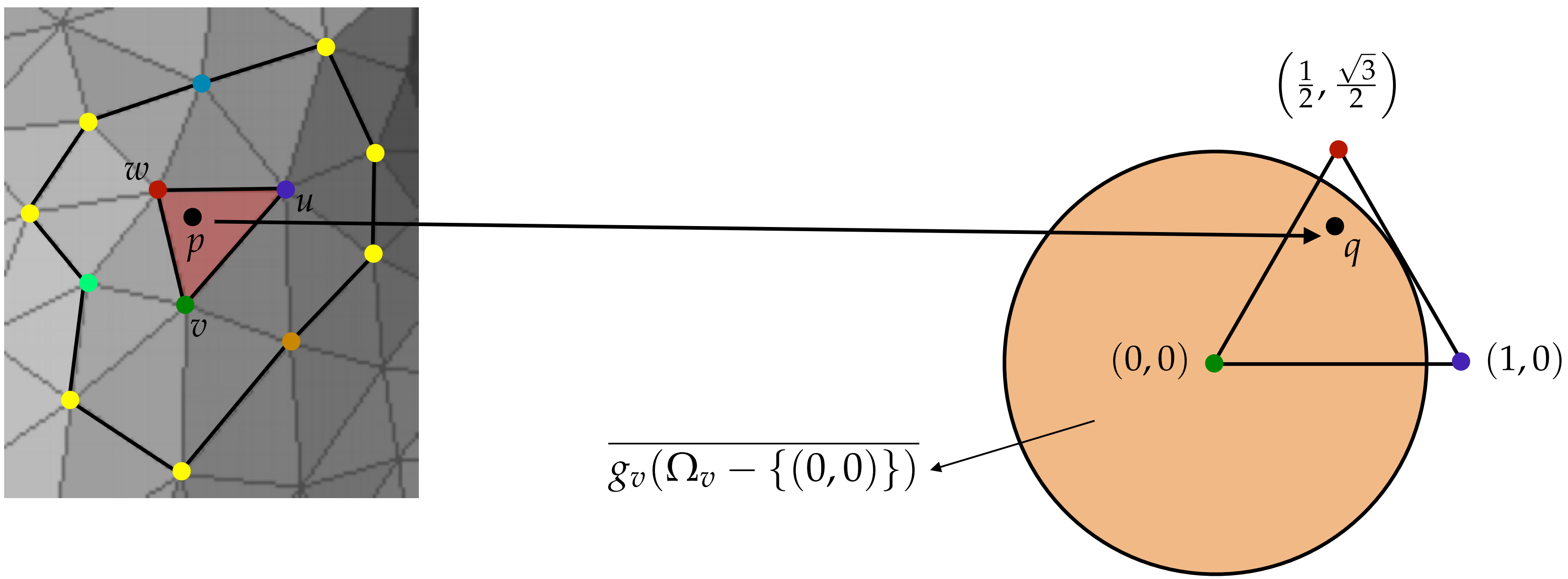


Parametric Pseudo-Manifolds

The User Perspective

Now, we can map q to the p-domain Ω_v using the map $r_{vu}^{-1} \circ g_v^{-1}$ *whenever the distance from q to $(0,0)$ is smaller than* $\cos(\pi/6)$, which is the radius of the circle corresponding to

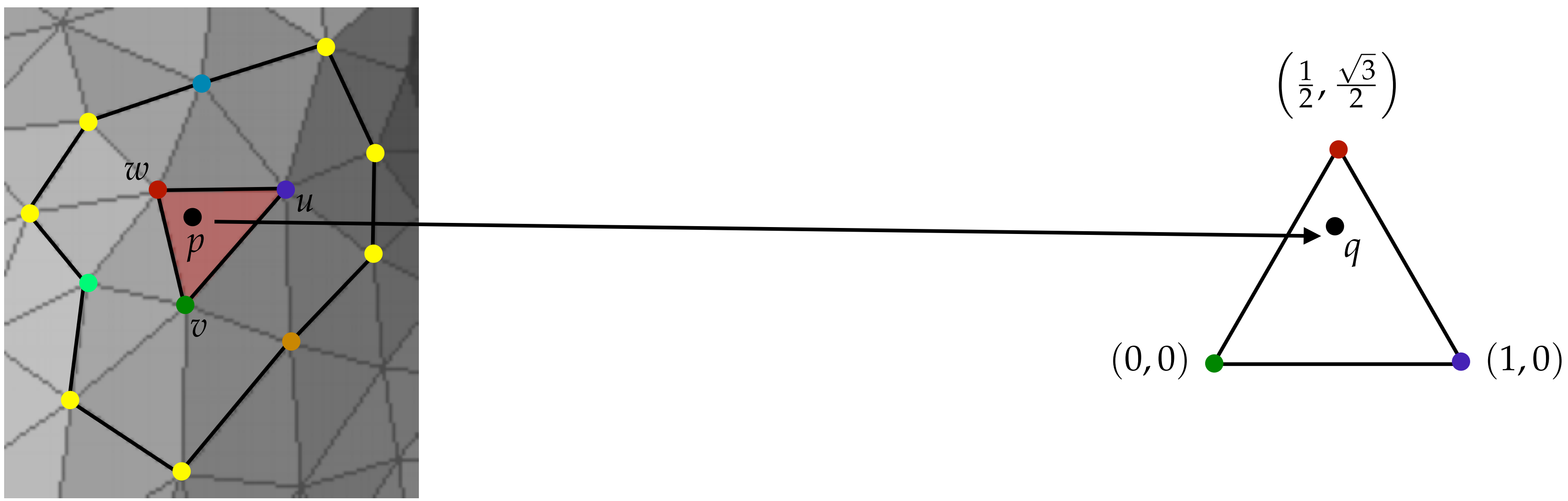
$$\overline{g_v(\Omega_v - \{(0,0)\})}.$$



Parametric Pseudo-Manifolds

The User Perspective

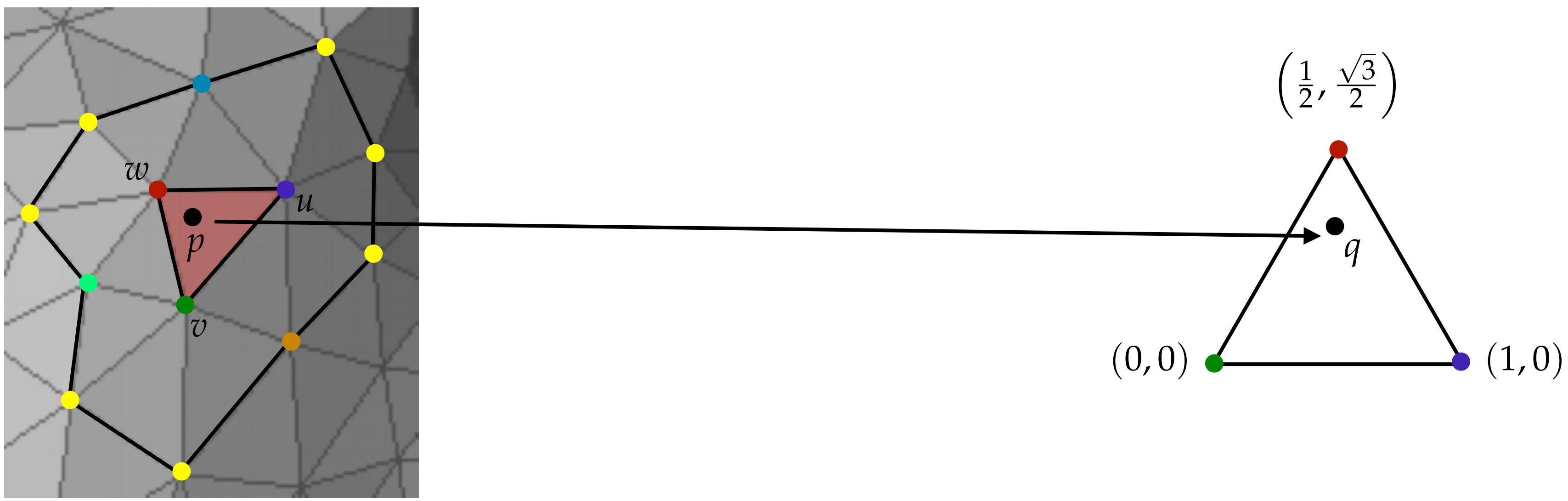
If this is not the case, then we compute the distance from q to $(1,0)$. If this distance is smaller than $\cos(\pi/6)$, we map q to Ω_u. Otherwise, the distance from q to $(1/2, \sqrt{3}/2)$ *has* to be smaller than $\cos(\pi/6)$ — *why is that so*?, and thus we map q to Ω_w.



Parametric Pseudo-Manifolds

The User Perspective

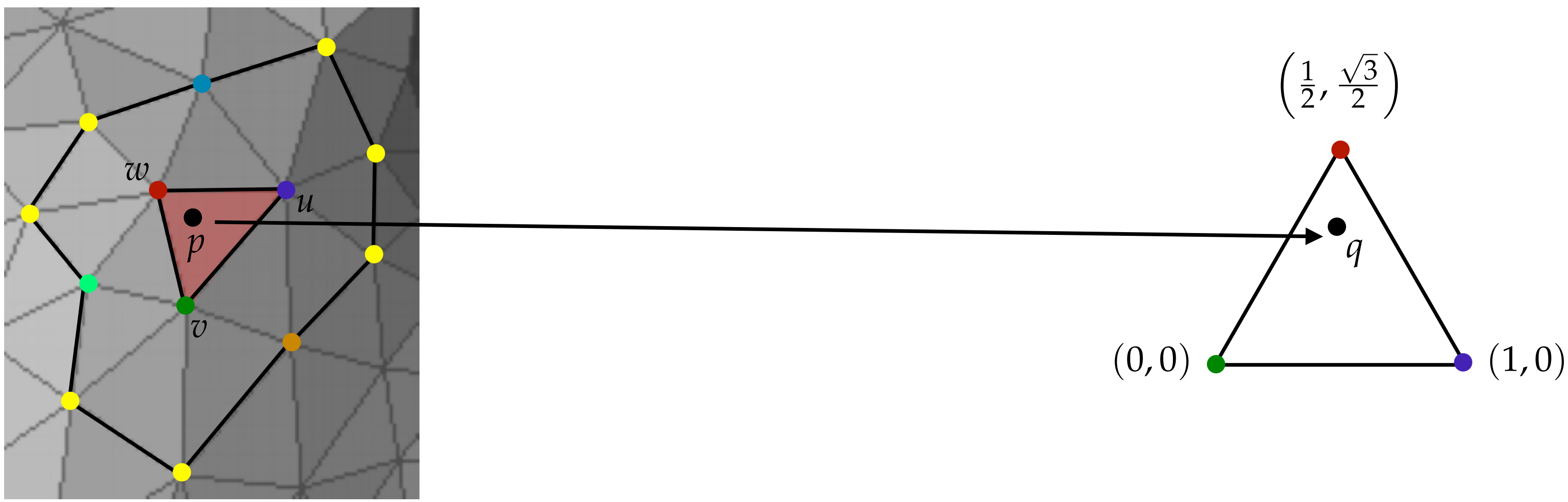
To map q to Ω_u, we use $r_{uv}^{-1} \circ g_u^{-1} \circ h$. In turn, we use $r_{wv}^{-1} \circ g_w^{-1} \circ h \circ r_{-\frac{\pi}{3}}$ to map q to Ω_w.



Parametric Pseudo-Manifolds

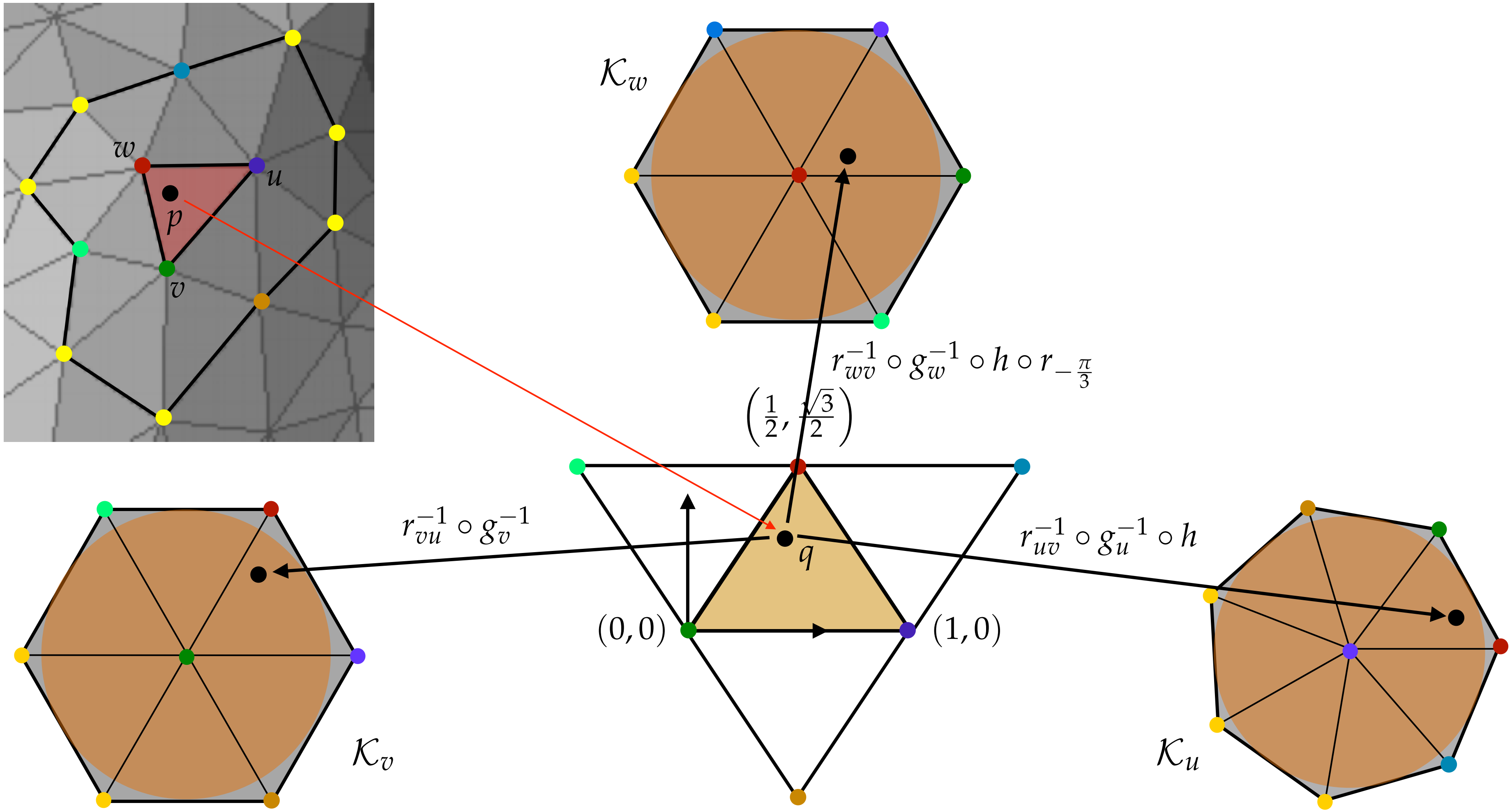
The User Perspective

Regardless of the p-domain where q will be mapped to, the important point is that q will be mapped to some p-domain, establishing a correspondence between $|\mathcal{K}|$ and the PPS.



Parametric Pseudo-Manifolds

The User Perspective



Parametric Pseudo-Manifolds

The User Perspective

KEEP IN MIND:
the user does not have to know about the existence of gluing data and parametrizations. All the user needs to interact with the PPS is a triangle, σ, in $\mathcal{K}$ and a point p in σ.

Once σ and p are given, we can take p to *some* p-domain and then to the image, S, of the PPS using the approach we just saw. So, all the user sees is still a "control mesh" (i.e., $|\mathcal{K}|$).

Let us now demo the code and discuss the implementation...

Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Now, we assume that we are given a quadrilateral surface mesh, $\mathcal{K}$.

A *quadrilateral mesh*, $\mathcal{K}$, in $\mathbb{E}^3$ is a set consisting of a finite number of (convex or non-convex) quadrilaterals, along with their edges and vertices, such that if σ_1 and σ_2 are any two quadrilaterals in $\mathcal{K}$, then $\sigma_1 \cap \sigma_2$ is either empty or a vertex or edge of both σ_1 and σ_2.

Here, we assume that the quadrilaterals of $\mathcal{K}$ are *planar* objects. This allows us to define the *underlying space*, $|\mathcal{K}|$, of $\mathcal{K}$ just like we did for simplicial complexes: the union

$$|\mathcal{K}| = \bigcup_{\sigma \in \mathcal{K}} \sigma \, .$$

Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

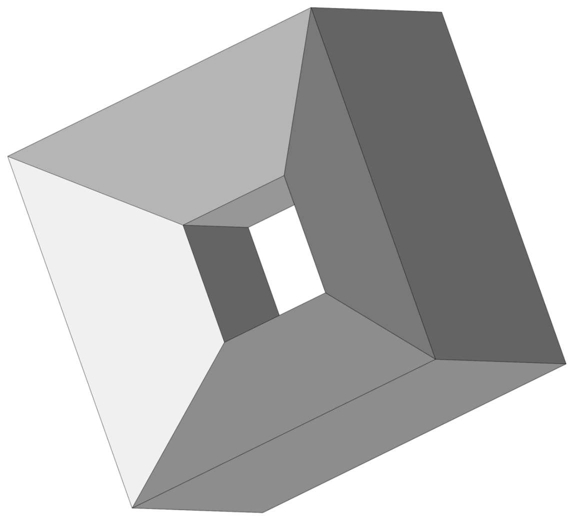
The notions of *star* and *link* of a vertex in $\mathcal{K}$ can be defined for quadrilateral meshes as well.

If σ is a vertex in $\mathcal{K}$, then the *star*, $\mathrm{st}(\sigma, \mathcal{K})$, of σ in $\mathcal{K}$ is the set of quadrilaterals that contain σ as a vertex, along with their edges and vertices. In turn, the *link*, $\mathrm{lk}(\sigma, \mathcal{K})$, of σ in $\mathcal{K}$ is the set of edges in $\mathrm{st}(\sigma, \mathcal{K})$ that do not contain σ as a vertex, along with their vertices.

Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

If every edge of $\mathcal{K}$ is incident with exactly two quadrilaterals of $\mathcal{K}$, and if the underlying space of $\mathrm{lk}(\sigma, \mathcal{K})$ is homeomorphic to the unit circle $S^1 = \{x \in \mathbb{E}^2 \mid \|x\| = 1\}$, for every vertex σ in $\mathcal{K}$, then we say that $\mathcal{K}$ is a *quadrilateral surface mesh* (without boundary).



The underlying space, $|\mathcal{K}|$, of a quadrilateral surface mesh, $\mathcal{K}$, is a topological surface in $\mathbb{E}^3$.

Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

We can now discuss the construction devised by Ying and Zorin (SIGGRAPH, 2003).

We start by the definition of the gluing data,

$$\mathcal{G} = \left((\Omega_i)_{i\in I}, (\Omega_{ij})_{(i,j)\in I\times I}, (\varphi_{ji})_{(i,j)\in K}\right).$$

We let I be the set of vertices of $\mathcal{K}$, as we assign a (distinct) p-domain to each vertex of $\mathcal{K}$.

Let v be any vertex of $\mathcal{K}$, and let n_v be the number of vertices of $\mathrm{lk}(v, \mathcal{K})$.

Parametric Pseudo-Manifolds

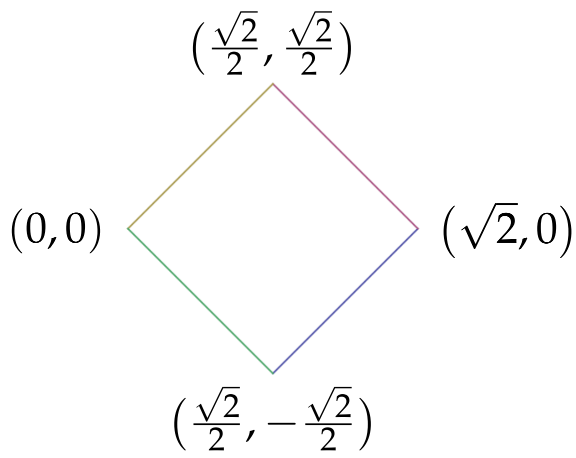
A Construction for Quadrilateral Meshes

Recall that

$$z^n = r^n \cdot (\cos(n \cdot \theta) + i \sin(n \cdot \theta)) \ ,$$

is the *polar form* of the complex number, $z = x + i\,y$, where $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

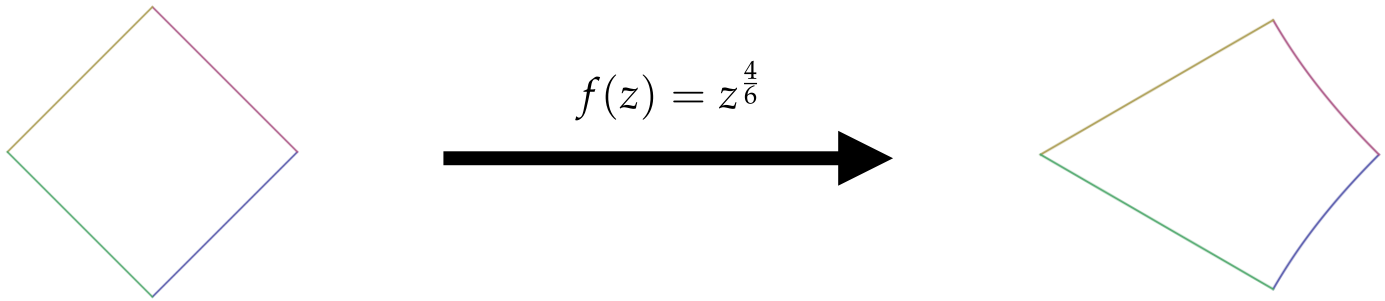
Let L be the square with vertices $(0,0)$, $(\sqrt{2}/2, -\sqrt{2}/2)$, $(\sqrt{2}, 0)$, and $(\sqrt{2}/2, \sqrt{2}/2)$.



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

If we apply the map $f(z) = z^{\frac{4}{n_v}}$ to L we obtain a "curved" quadrilateral (e.g., for $n_v = 6$):



As usual, the parameter n_v is the degree of the vertex v in $\mathcal{K}$. For a quadrilateral surface mesh, the value of n_v is half the number of vertices in the link, $\mathrm{lk}(v, \mathcal{K})$, of v in $\mathcal{K}$.

Parametric Pseudo-Manifolds

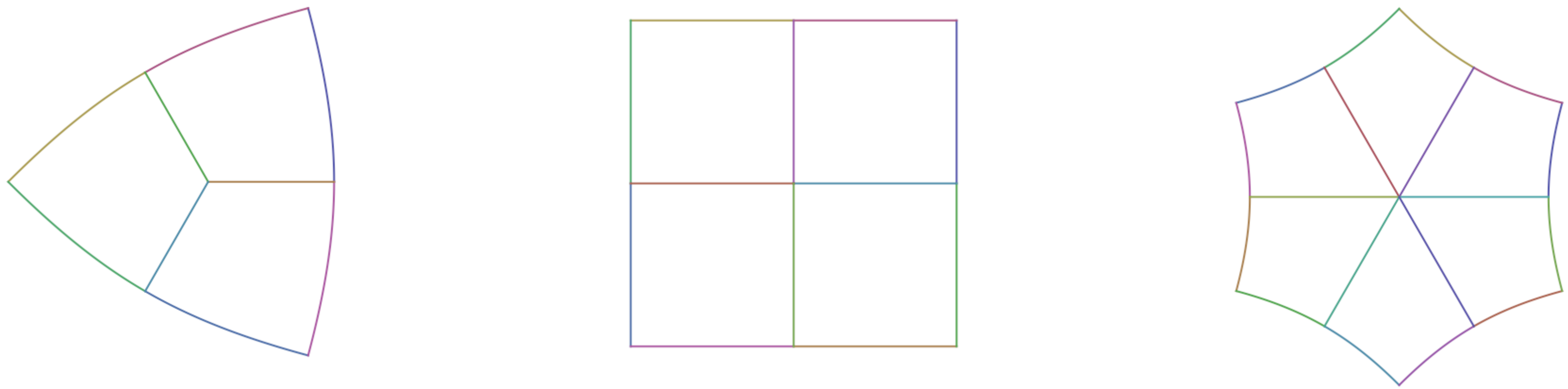
A Construction for Quadrilateral Meshes

We define the p-domain, Ω_v, as the interior of the union set

$$\bigcup_{k=0}^{n_v-1} \left(r_{\frac{(2k+1)\cdot\pi}{n_v}} \circ f \right)(L)\,,$$

where $r_{\frac{(2k+1)\cdot\pi}{n_v}}$ is a counterclockwise rotation around the origin by the angle $\frac{(2k+1)\cdot\pi}{n_v}$.

$n_v = 3$ $\qquad$ $n_v = 4$ $\qquad$ $n_v = 6$

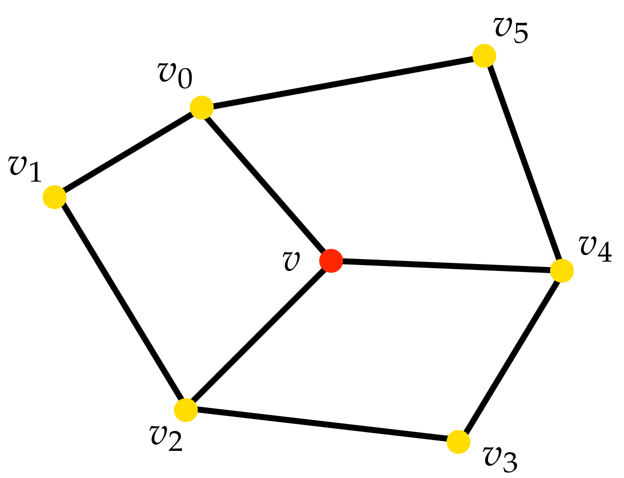


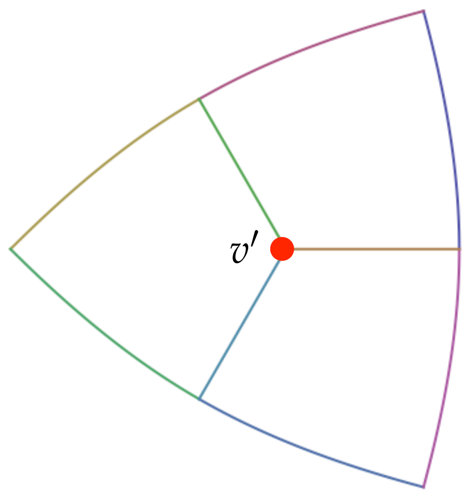
Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Fix a counterclockwise enumeration, $v_0, v_1, \ldots, v_{2 \cdot n_v - 1}$, of the vertices in $\mathrm{lk}(v, \mathcal{K})$.

We can then identify v with the point $v' = (0, 0)$ of Ω_v.





Parametric Pseudo-Manifolds

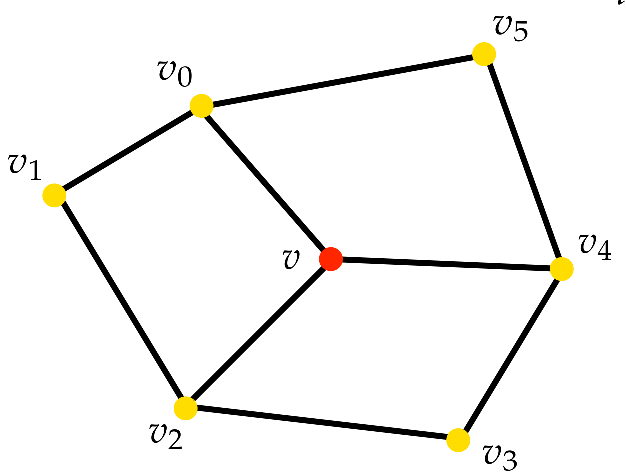
A Construction for Quadrilateral Meshes

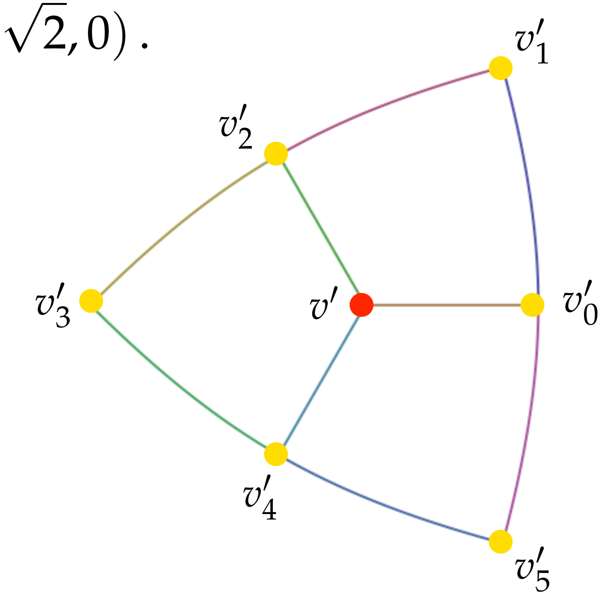
For each $i = 0, 1, \ldots, 2 \cdot n_v - 1$, we can also identify the point v_i with the point

$$v'_i = \left(r_{\frac{(i+1)\cdot\pi}{n_v}} \circ f \right) \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$$

if i is even; otherwise, we identify v_i with the point

$$v'_i = \left(r_{\frac{(i+1)\cdot\pi}{n_v}} \circ f \right) (\sqrt{2}, 0) \, .$$



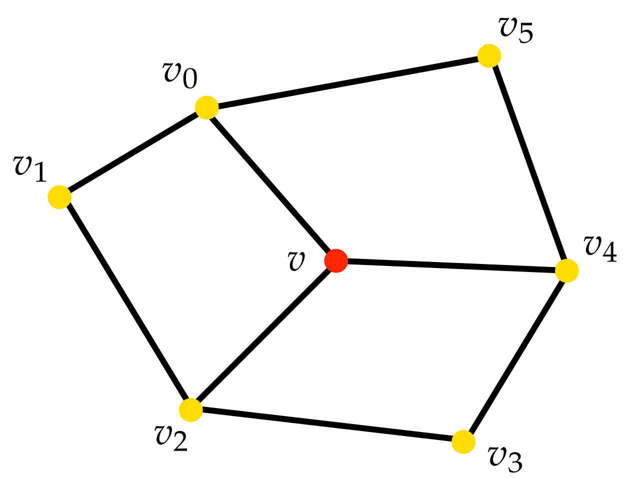


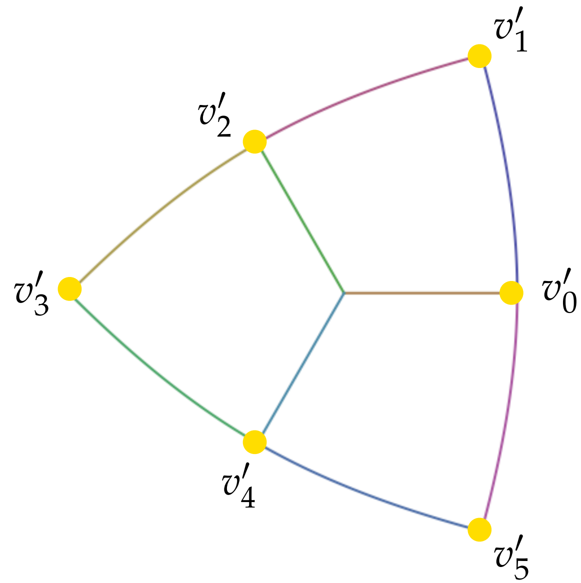
Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

The aforementioned identification can be represented by a function, which we name f_v.

So, $v' = f_v(v)$, $v'_0 = f_v(v_0)$, $v'_1 = f_v(v_1)$, and so on so forth.

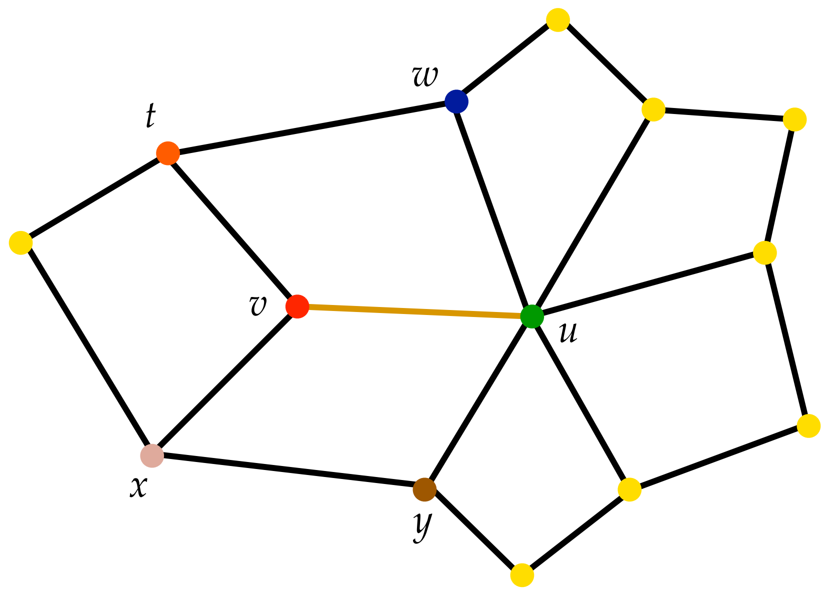




Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Suppose that v and u, with $v \neq u$, are the vertices of an edge, $[v, u]$, of $\mathcal{K}$. Then, the sets $\mathrm{st}(v, \mathcal{K})$ and $\mathrm{st}(u, \mathcal{K})$ share exactly two quadrilaterals, which in turn share an edge.

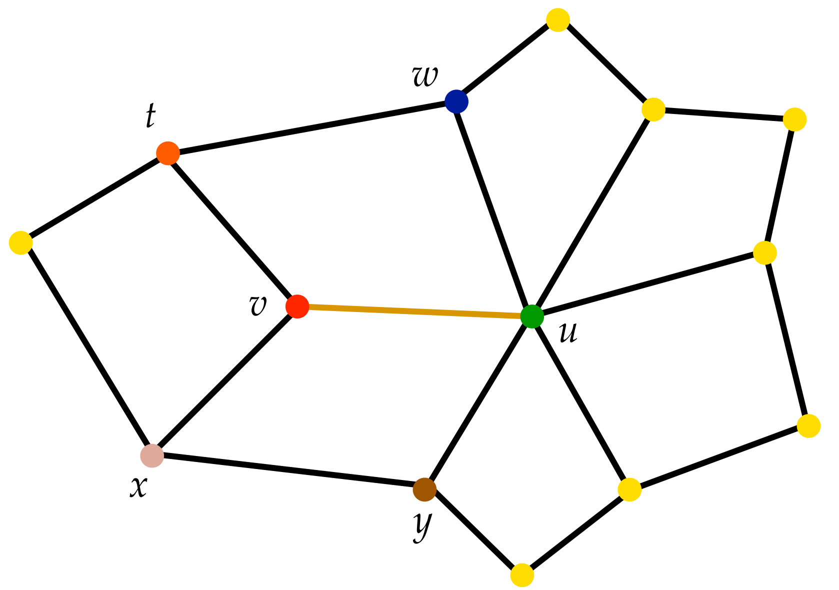


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

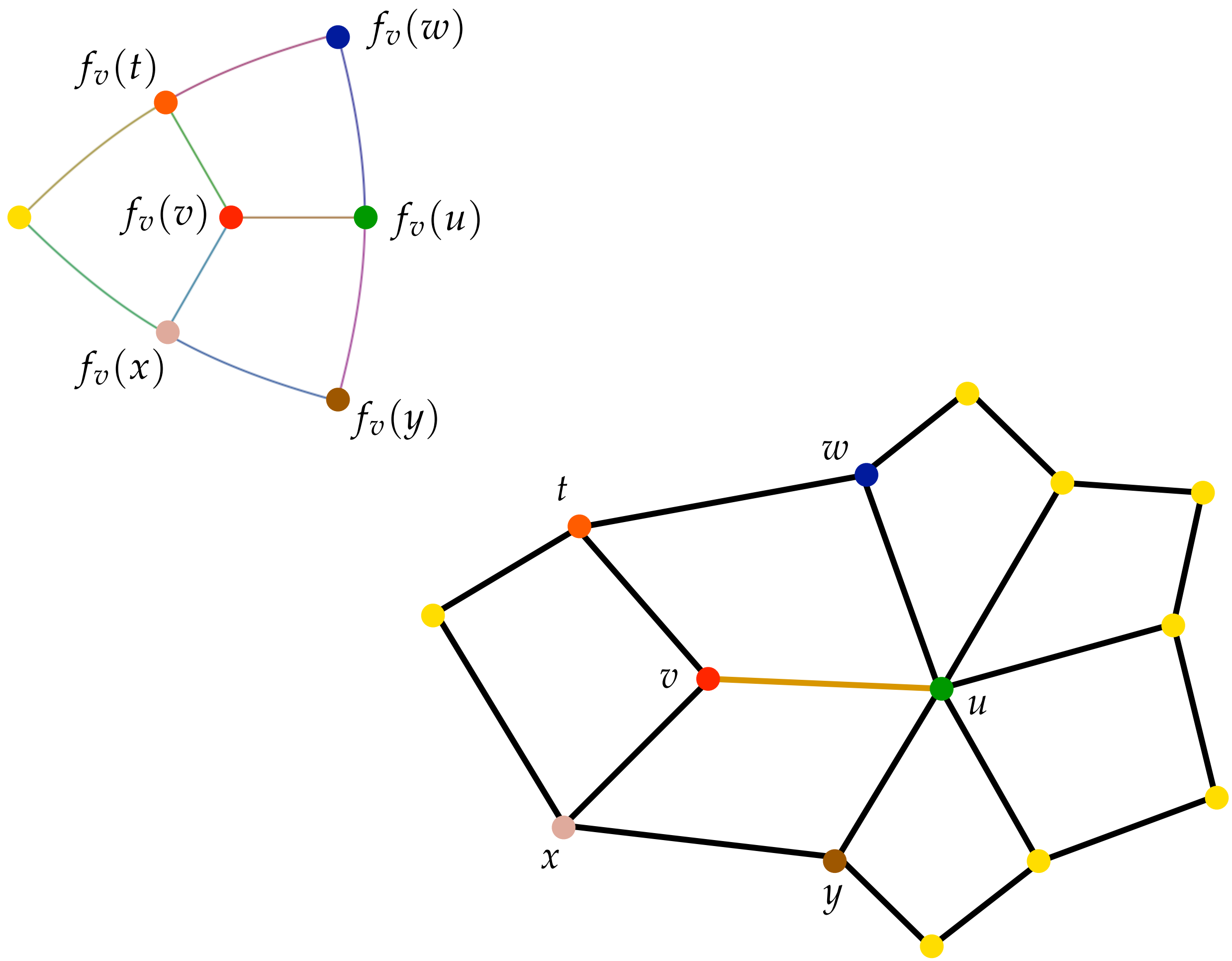
Let v, x, y, u, w, and t be the vertices of these two adjacent quadrilaterals.

We define Ω_{vu} as the subset of Ω_v corresponding to the interior of the union of the two "curved" quadrilaterals defined by the vertices $f_v(v)$, $f_v(x)$, $f_v(y)$, $f_v(u)$, $f_v(w)$, and $f_v(t)$.



Parametric Pseudo-Manifolds

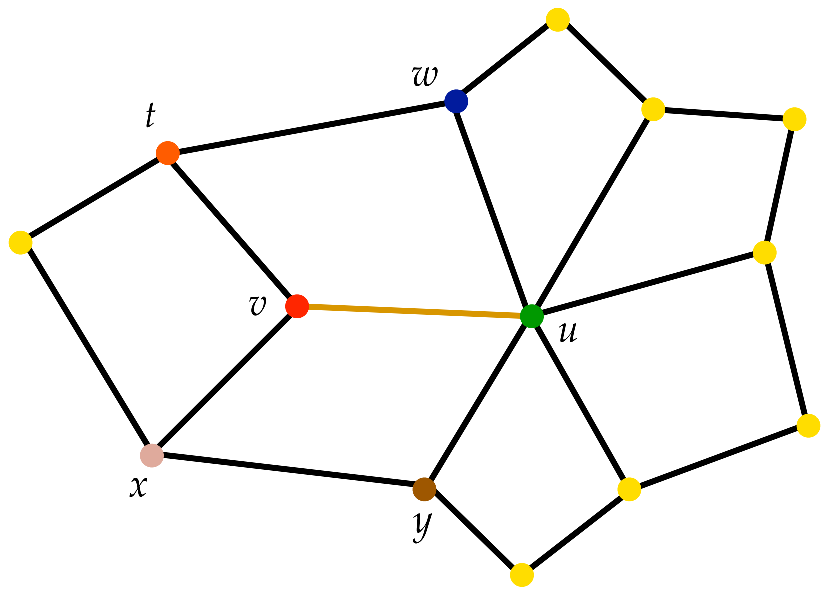
A Construction for Quadrilateral Meshes



Parametric Pseudo-Manifolds

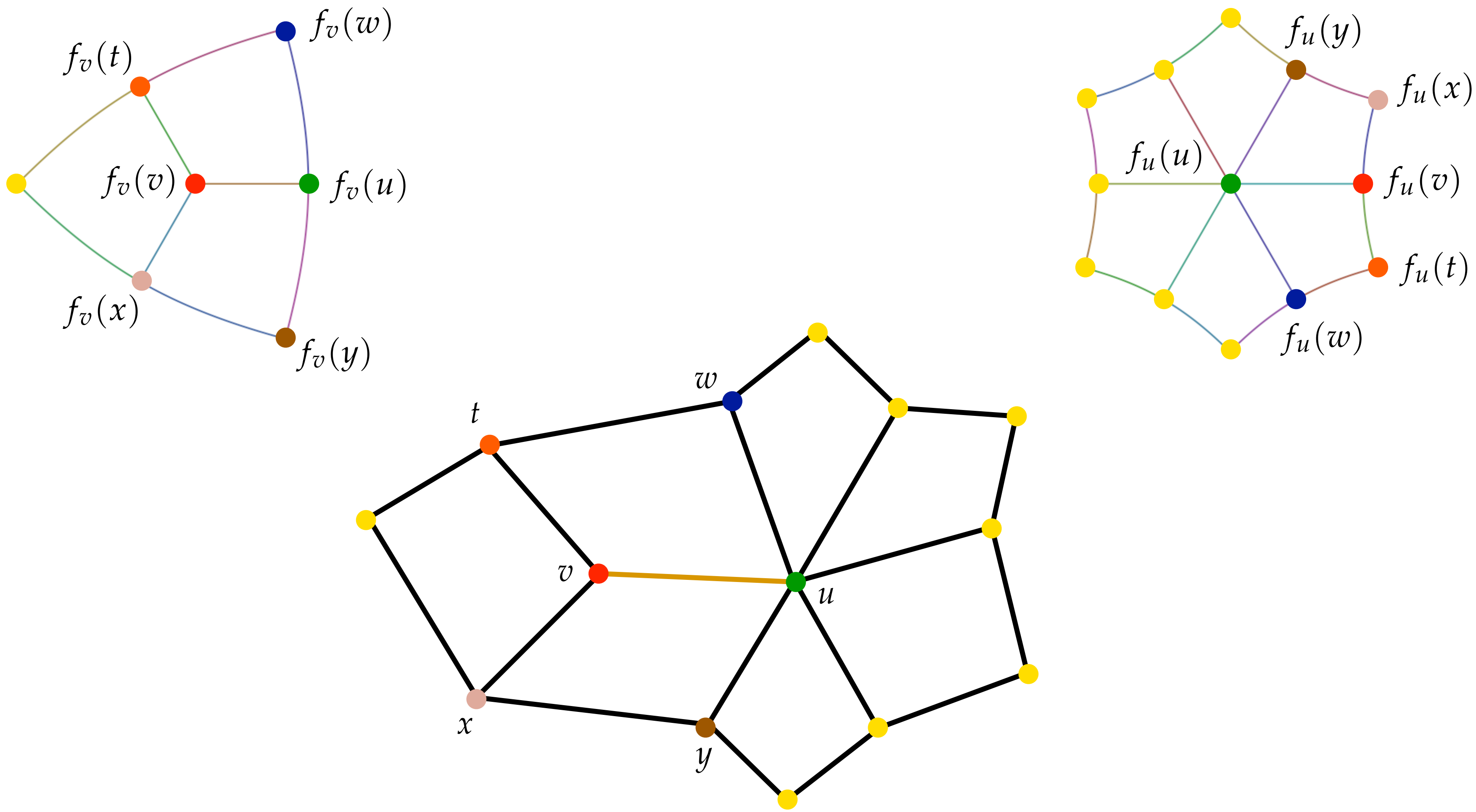
A Construction for Quadrilateral Meshes

We define Ω_{uv} in a similar fashion.



Parametric Pseudo-Manifolds

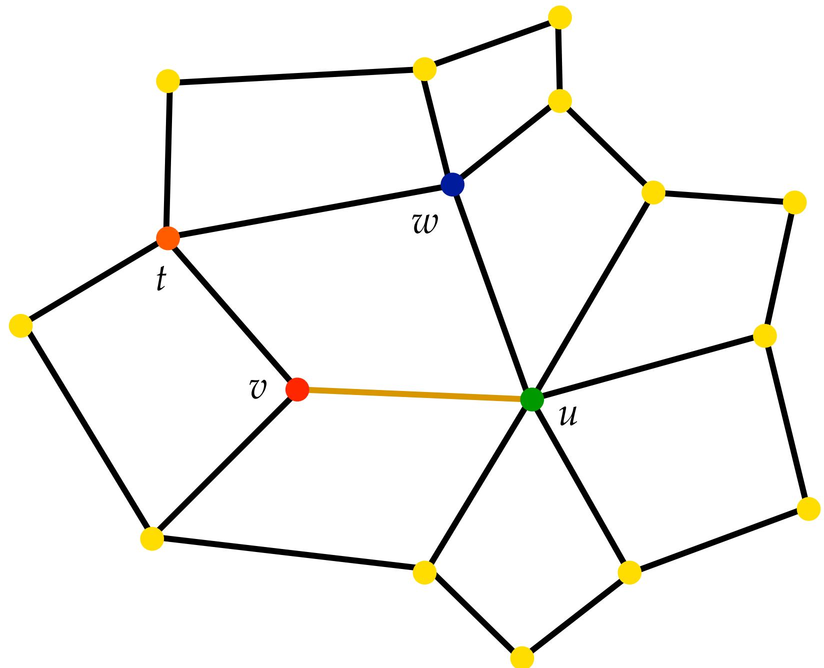
A Construction for Quadrilateral Meshes



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Suppose that v and w, with $v \neq w$, are the vertices of a quadrilateral of $\mathcal{K}$, but not vertices of an edge of $\mathcal{K}$. Then, the sets $\mathrm{st}(v, \mathcal{K})$ and $\mathrm{st}(w, \mathcal{K})$ share exactly one quadrilateral.

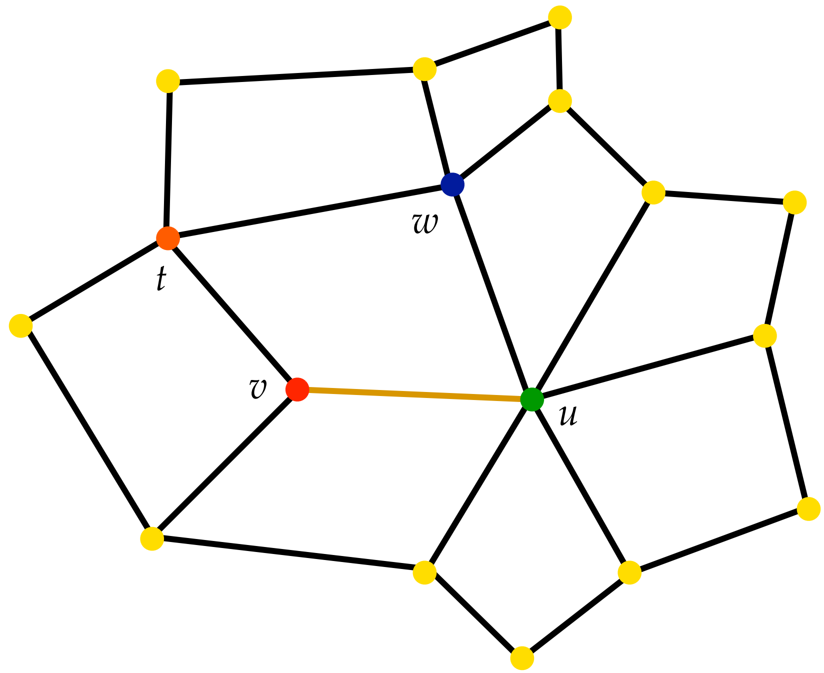


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

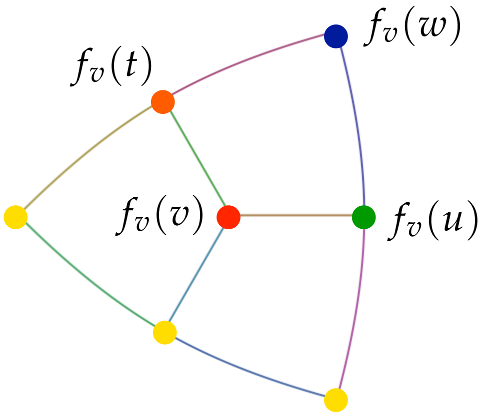
Let v, u, t, and w be the vertices of the common quadrilateral.

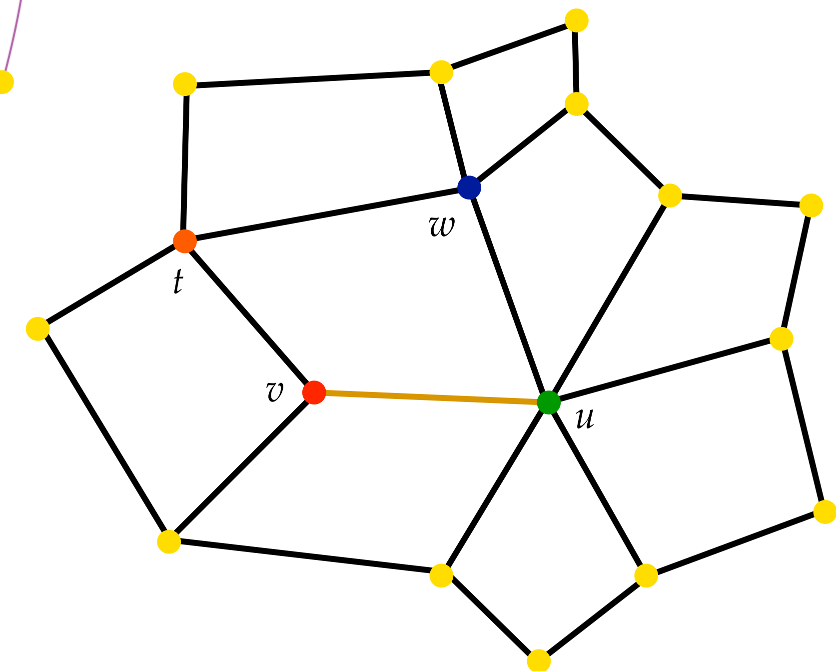
We define the gluing domain, Ω_{vw}, as the subset of Ω_v corresponding to the interior of the "curved" quadrilateral defined by the vertices $f_v(v)$, $f_v(u)$, $f_v(w)$, and $f_v(t)$.



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

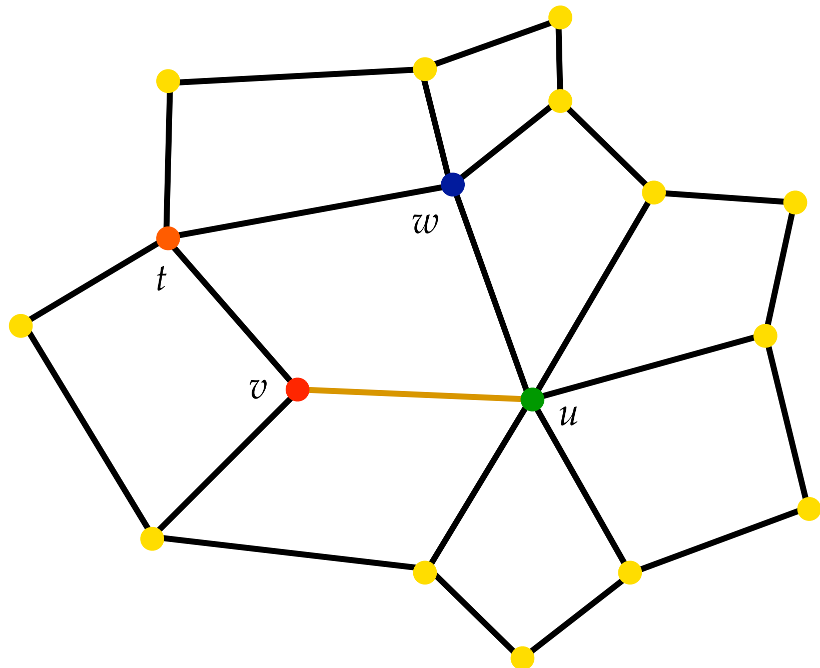




Parametric Pseudo-Manifolds

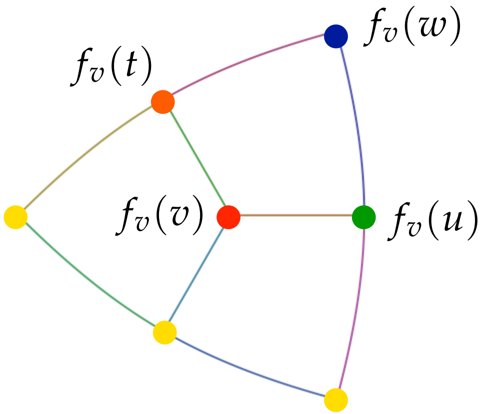
A Construction for Quadrilateral Meshes

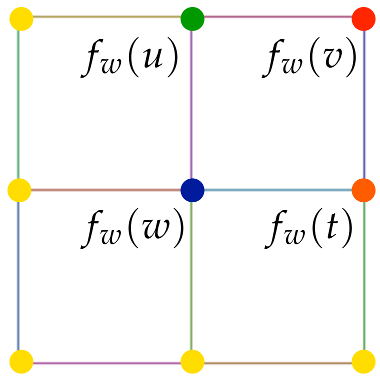
We define Ω_{wv} in a similar fashion.

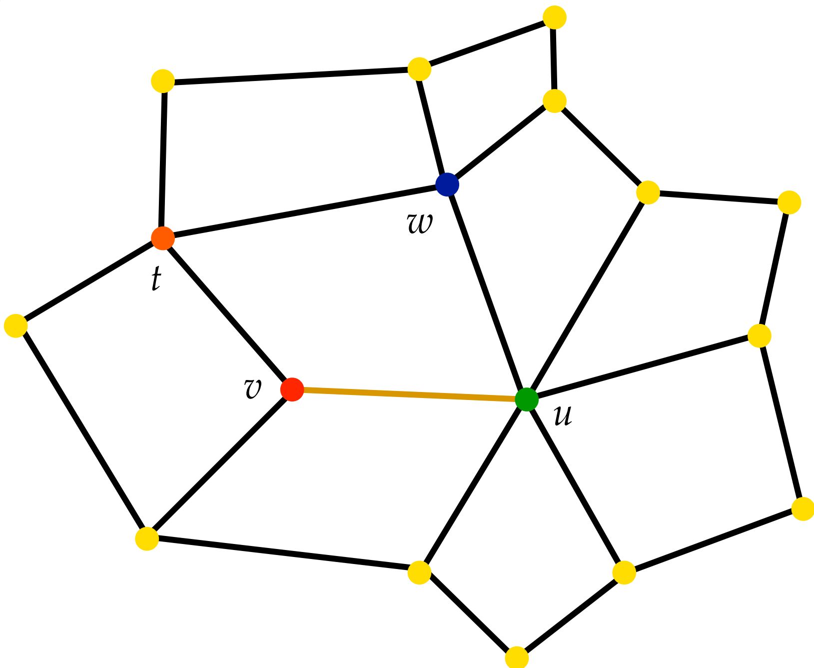


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes





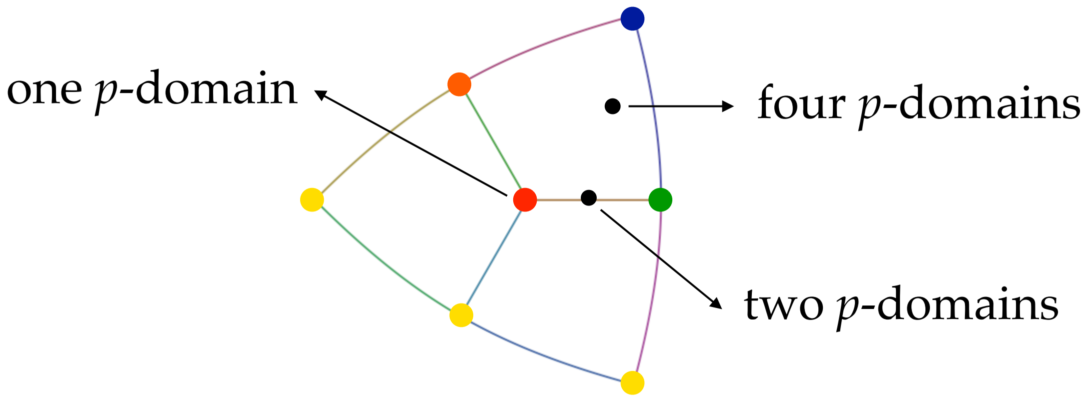


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

So, there are two types of gluing domains (the single and the double curved quadrilateral).

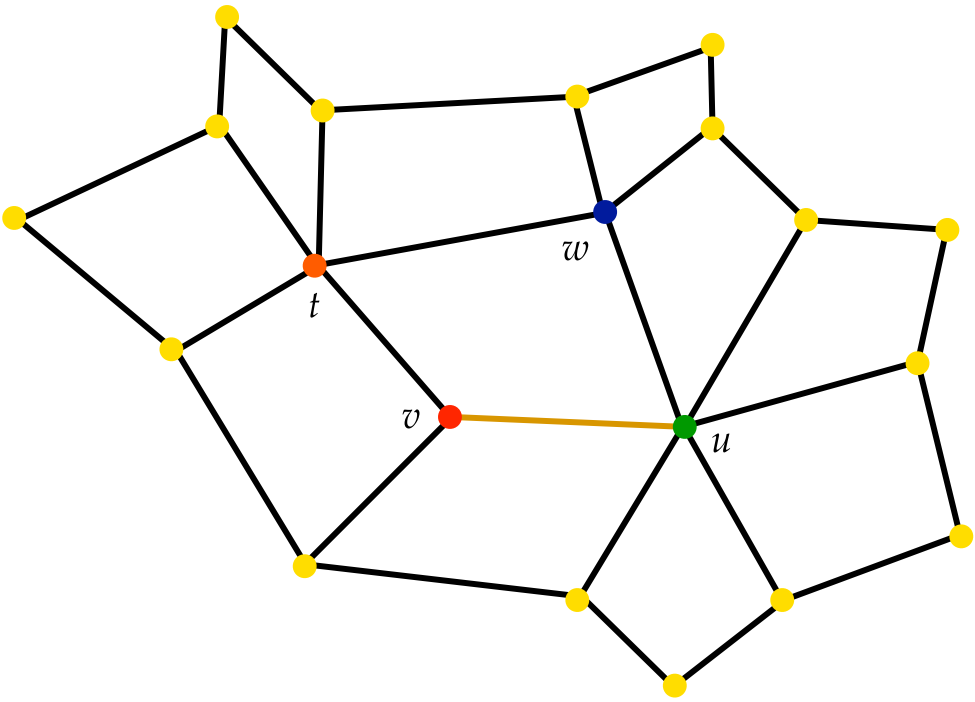
We notice that a point in a p-domain can be identified either with no other p-domain (*this is the case for the point located at* $(0, 0)$ *only*), with two p-domains, or with four p-domains.



Parametric Pseudo-Manifolds

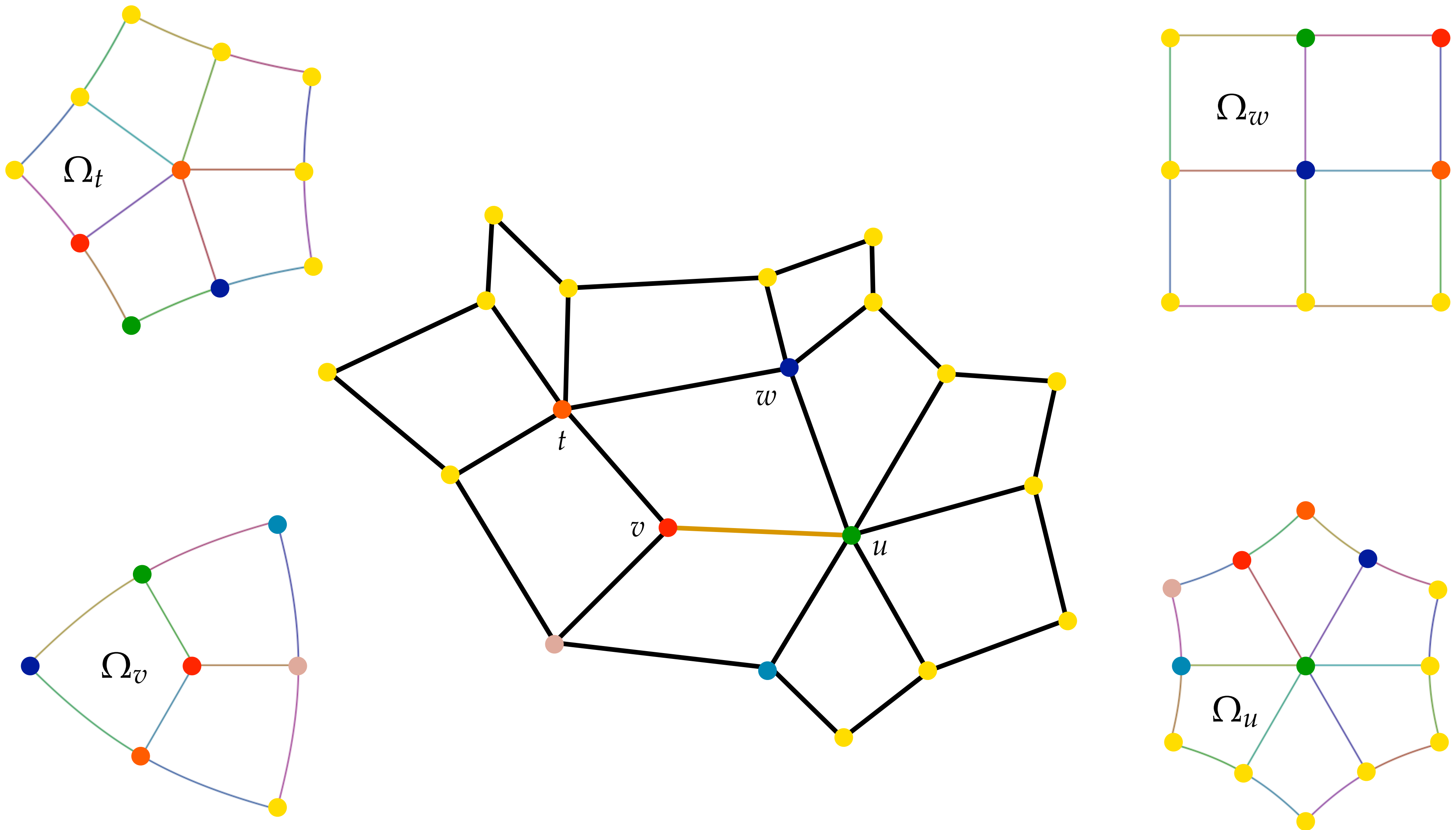
A Construction for Quadrilateral Meshes

How can we define the transition functions?



Parametric Pseudo-Manifolds

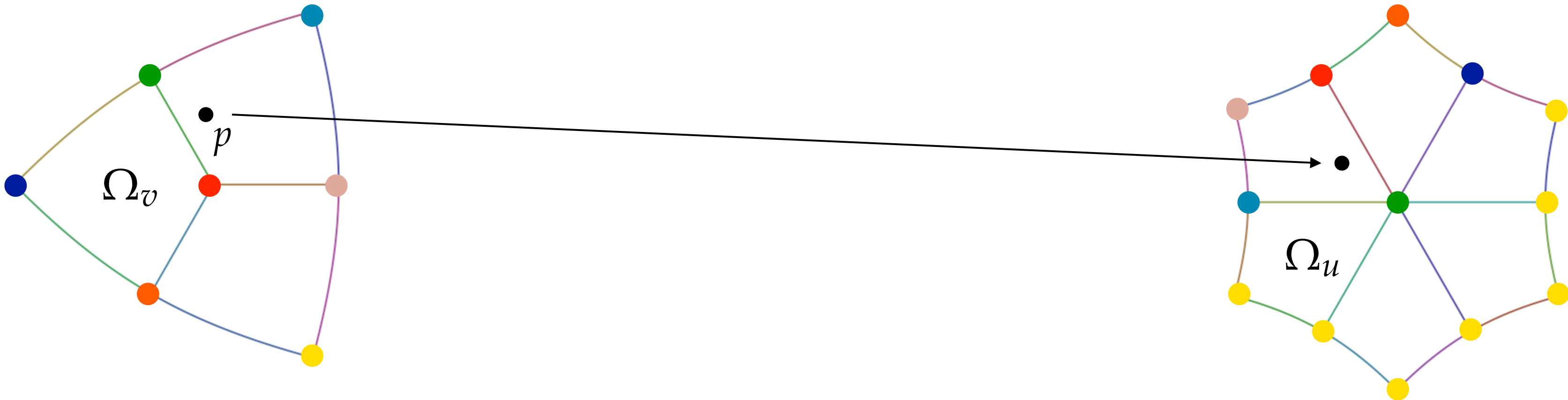
A Construction for Quadrilateral Meshes



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Consider the transition function $\varphi_{uv} : \Omega_{vu} \to \Omega_{uv}$ and let p be a point in Ω_{vu}.

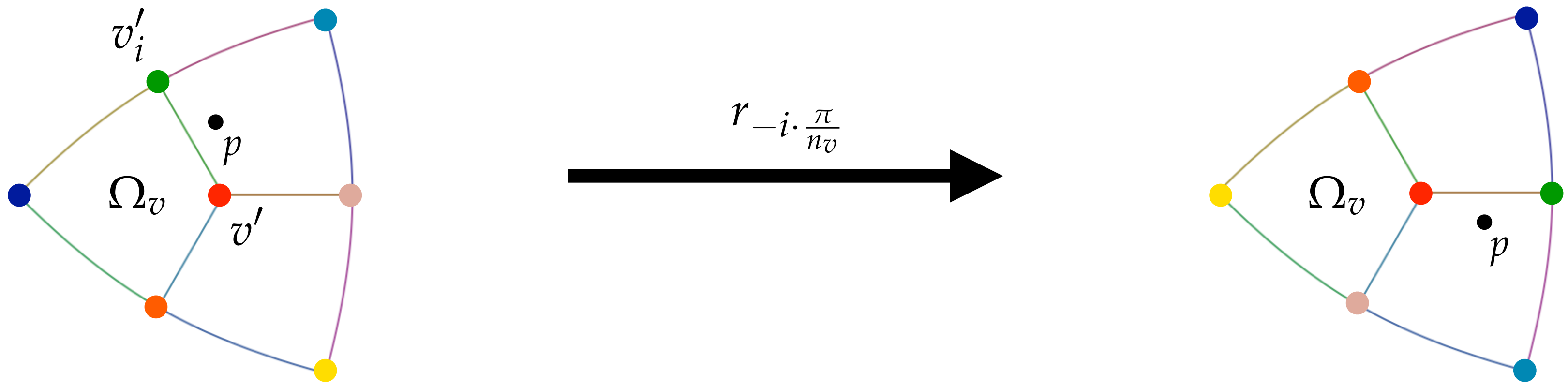


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Since $[v, u]$ is an edge of $\mathcal{K}$, we rotate Ω_v by an angle, θ, that makes $[f_v(v), f_v(u)]$ coincide with the x axis. If $f_v(u) = v'_i$, for some $i = 0, 1, \ldots, 2 \cdot n_v - 1$, with i even, then

$$\theta = -i \cdot \frac{\pi}{n_v} .$$

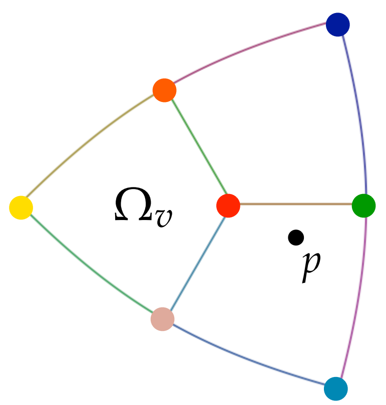


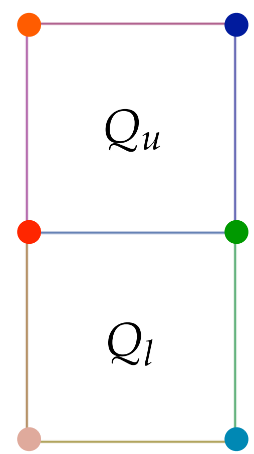
Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

We then take the set $r_{-i \cdot \frac{\pi}{n_v}}(\overline{\Omega_{vu}})$ onto a canonical set, say Q, which is the union of two squares:

$$Q_l = [(0,0),(0,-1),(1,-1),(1,0)] \qquad \text{and} \qquad Q_u = [(0,0),(1,0),(1,1),(0,1)] \, .$$





Parametric Pseudo-Manifolds

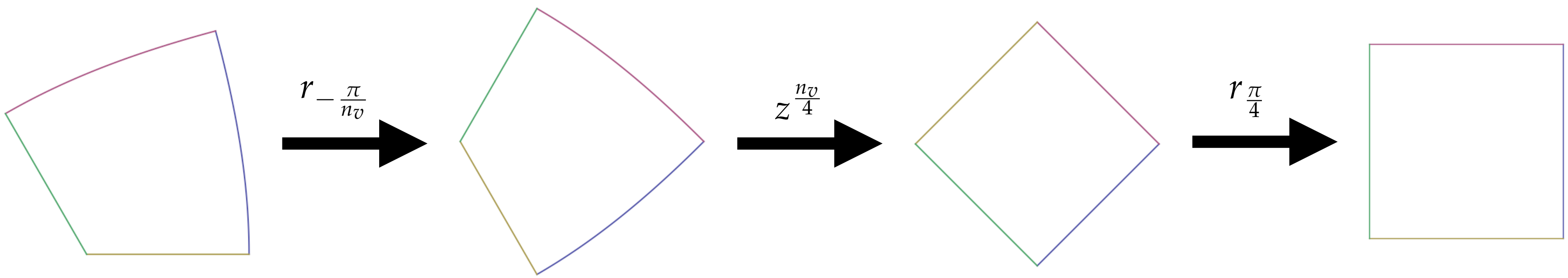
A Construction for Quadrilateral Meshes

This is done in a piecewise manner. More specifically, the upper curved quadrilateral of $r_{-i\cdot\frac{\pi}{nv}}(\overline{\Omega_{vu}})$ is mapped onto Q_u using the composition map $r_{\frac{\pi}{4}} \circ z^{\frac{nv}{4}} \circ r_{-\frac{\pi}{nv}}$. So, we get

$$r_{\frac{\pi}{4}} \circ z^{\frac{nv}{4}} \circ r_{-(i+1)\cdot\frac{\pi}{nv}} \,.$$

Similarly, the lower curved quadrilateral of $\overline{\Omega_{vu}}$ is mapped onto Q_l using the composition

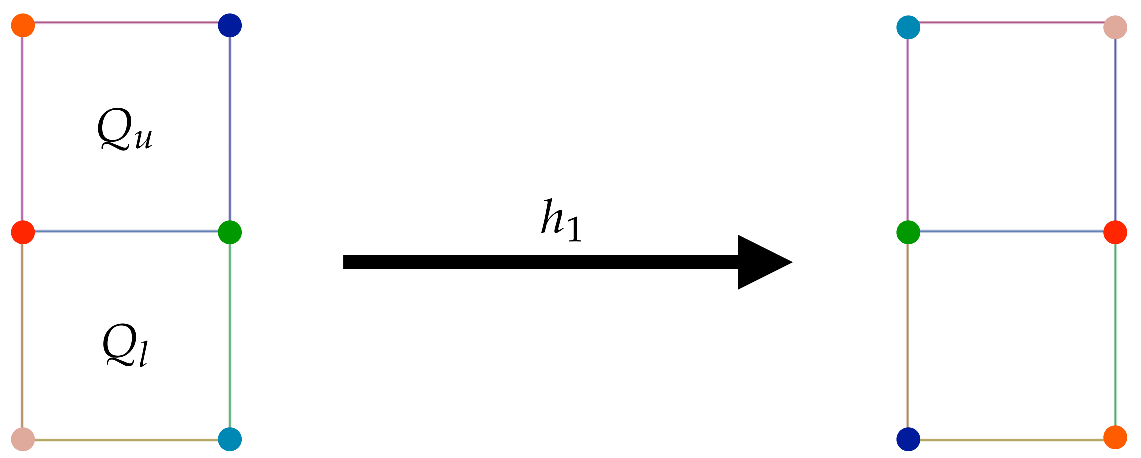
$$r_{-\frac{\pi}{4}} \circ z^{\frac{nv}{4}} \circ r_{-(i-1)\cdot\frac{\pi}{nv}} \,.$$



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Next, we apply a double reflection, $h_1(x, y) = (1 - x, -y)$, to Q, which is a reflection with respect to the vertical line $x = 0.5$ followed by a reflection with respect to the x axis.



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

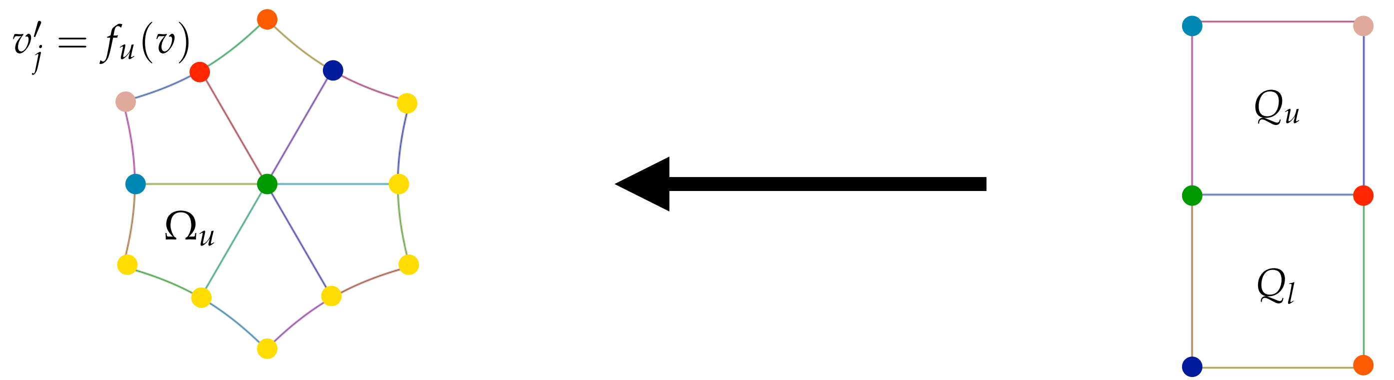
Finally, we map the interior of Q to Ω_u using the corresponding inverse functions. The interior of Q_u is mapped to the interior of the upper curved quadrilateral of $\overline{\Omega_u}$ by

$$r_{(j+1)\cdot\frac{\pi}{n_u}} \circ z^{\frac{4}{n_u}} \circ r_{-\frac{\pi}{4}} \, .$$

Similarly,

$$r_{(j-1)\cdot\frac{\pi}{n_u}} \circ z^{\frac{4}{n_u}} \circ r_{\frac{\pi}{4}}$$

is the map that takes the interior of Q_l to the interior of the lower curved quadrilateral of $\overline{\Omega_u}$.



Parametric Pseudo-Manifolds

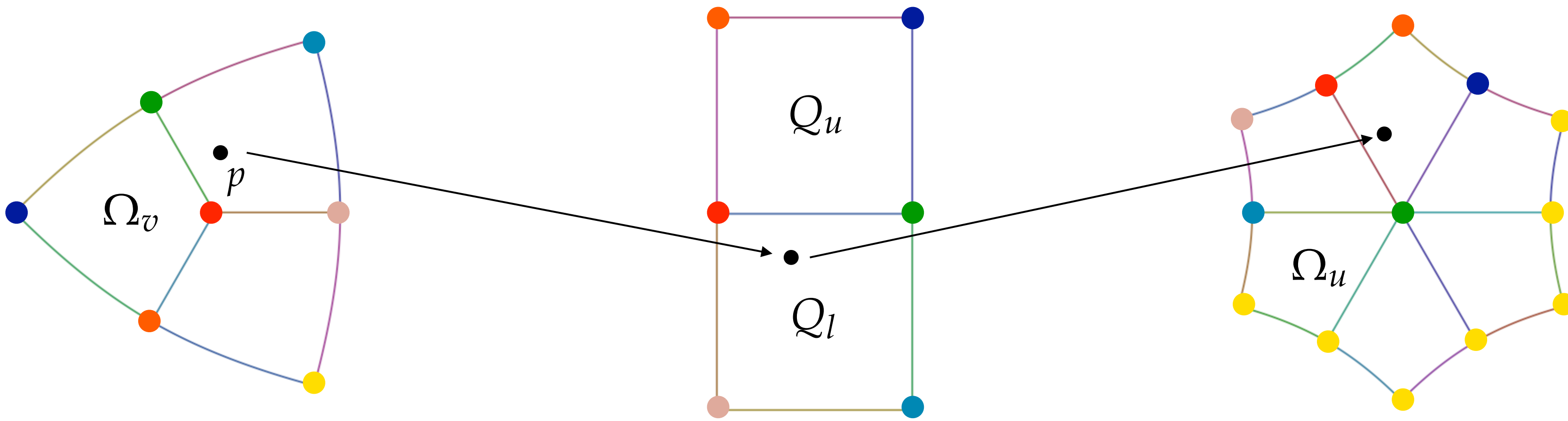
A Construction for Quadrilateral Meshes

So,

$$\varphi_{uv}(p) = \left(r_{(j-1)\cdot\frac{\pi}{n_u}} \circ z^{\frac{4}{n_u}} \circ r_{\frac{\pi}{4}} \circ h_1 \circ r_{\frac{\pi}{4}} \circ z^{\frac{n_v}{4}} \circ r_{-(i+1)\cdot\frac{\pi}{n_v}} \right)(p)$$

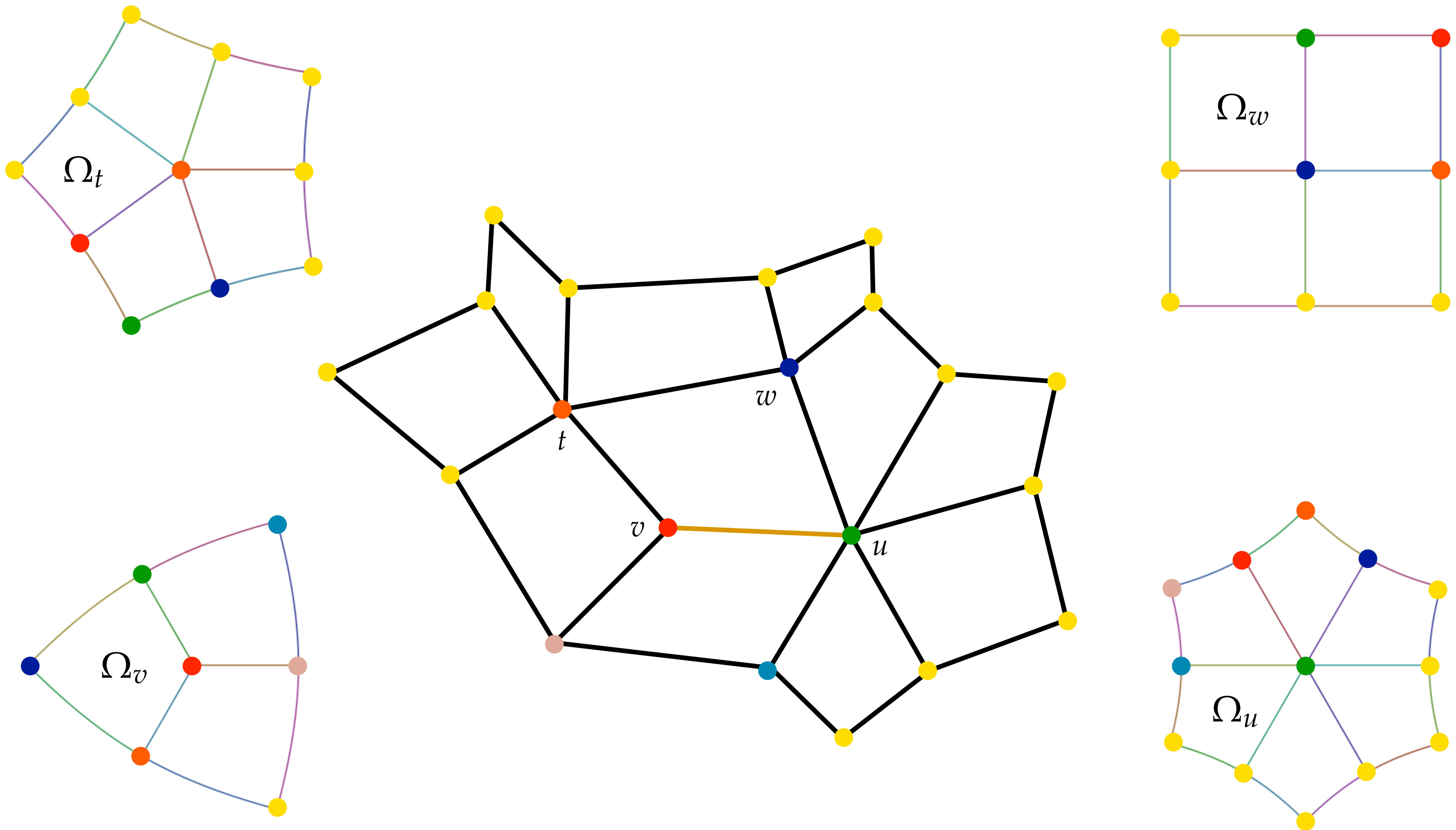
if p belongs to the upper curved quadrilateral of $\overline{\Omega_{vu}}$; otherwise, the transition function is

$$\varphi_{uv}(p) = \left(r_{(j+1)\cdot\frac{\pi}{n_u}} \circ z^{\frac{4}{n_u}} \circ r_{-\frac{\pi}{4}} \circ h_1 \circ r_{-\frac{\pi}{4}} \circ z^{\frac{n_v}{4}} \circ r_{-(i-1)\cdot\frac{\pi}{n_v}} \right)(p) \, .$$



Parametric Pseudo-Manifolds

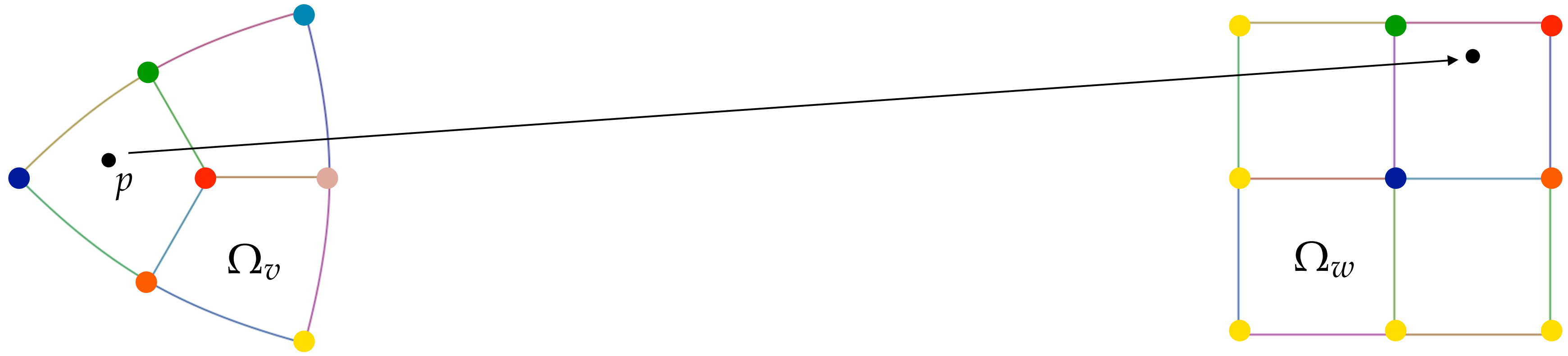
A Construction for Quadrilateral Meshes



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Now, consider the transition function $\varphi_{wv} : \Omega_{vw} \to \Omega_{wv}$ and let p be a point in Ω_{vw}.

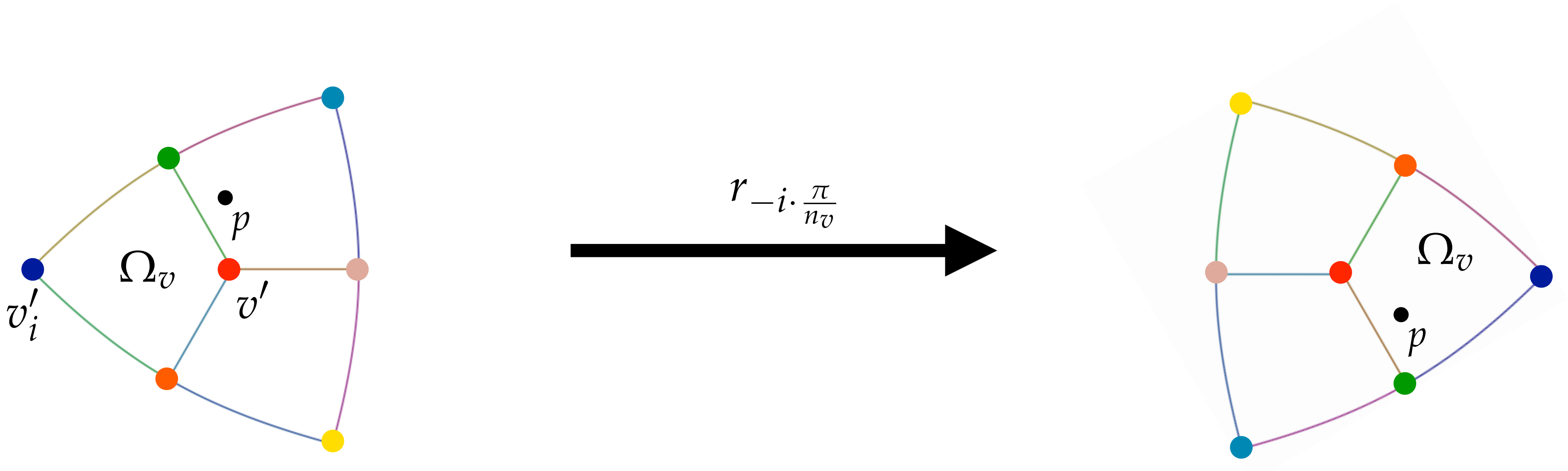


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Since $[v, w]$ is *not* an edge of $\mathcal{K}$, we rotate Ω_v by an angle, θ, that makes $[f_v(v), f_v(w)]$ coincide with the x axis. If $f_v(w) = v'_i$, for some $i = 0, 1, \ldots, 2 \cdot n_v - 1$, with i odd, then

$$\theta = -i \cdot \frac{\pi}{n_v}\,.$$

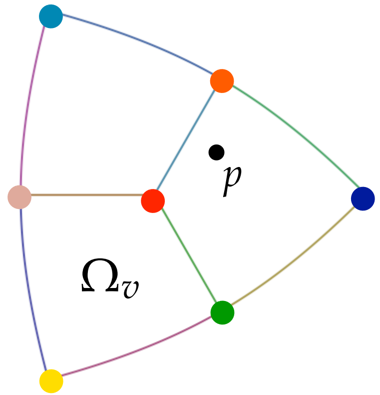


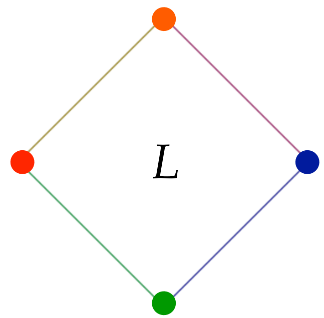
Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

We then take the set $r_{-i \cdot \frac{\pi}{n_v}}(\overline{\Omega_{vw}})$ onto the canonical quadrilateral, L, where

$$L = \left[(0,0), (\sqrt{2}, 0), \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \right].$$

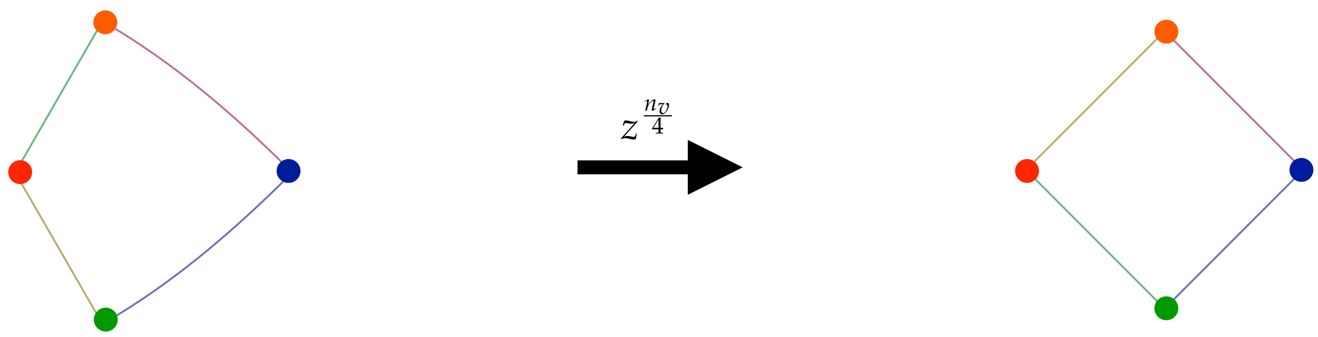




Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

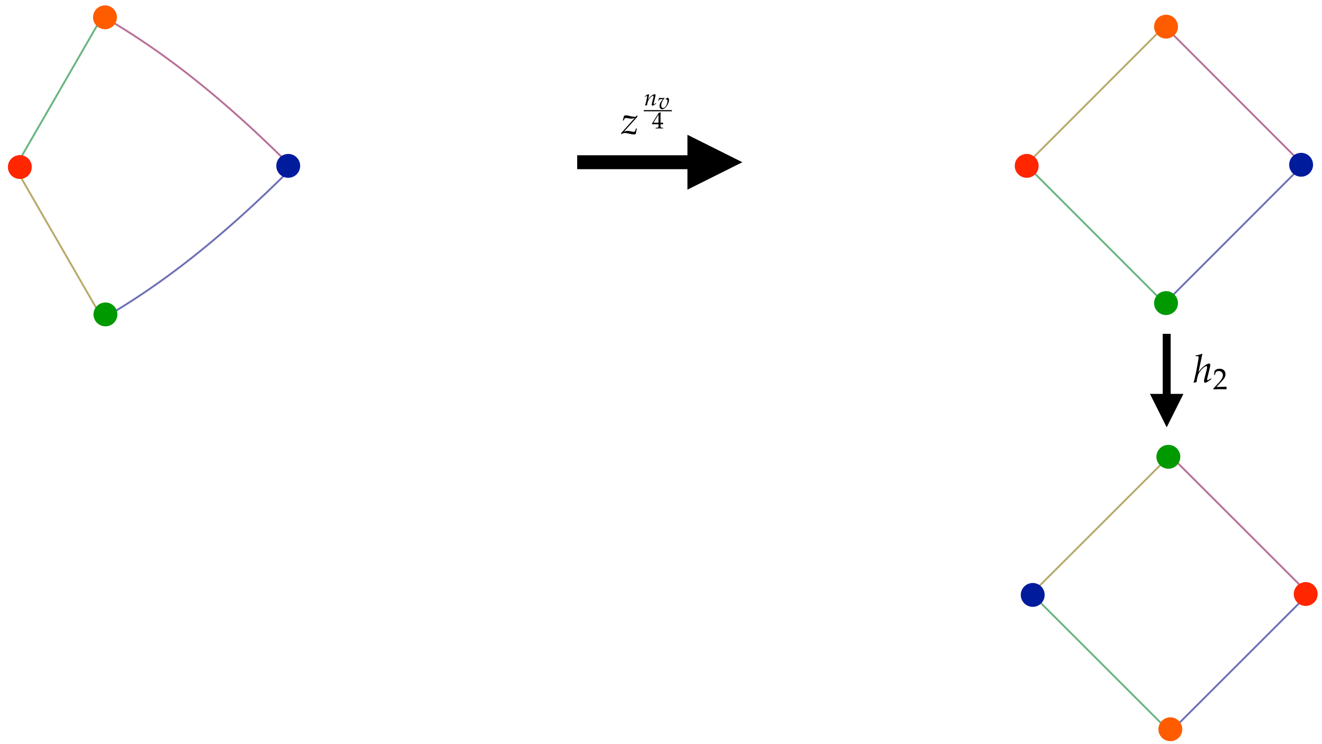
We use the map $z^{\frac{n_v}{4}}$ to take $r_{-i \cdot \frac{\pi}{n_v}}(\overline{\Omega_{vw}})$ onto L:



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

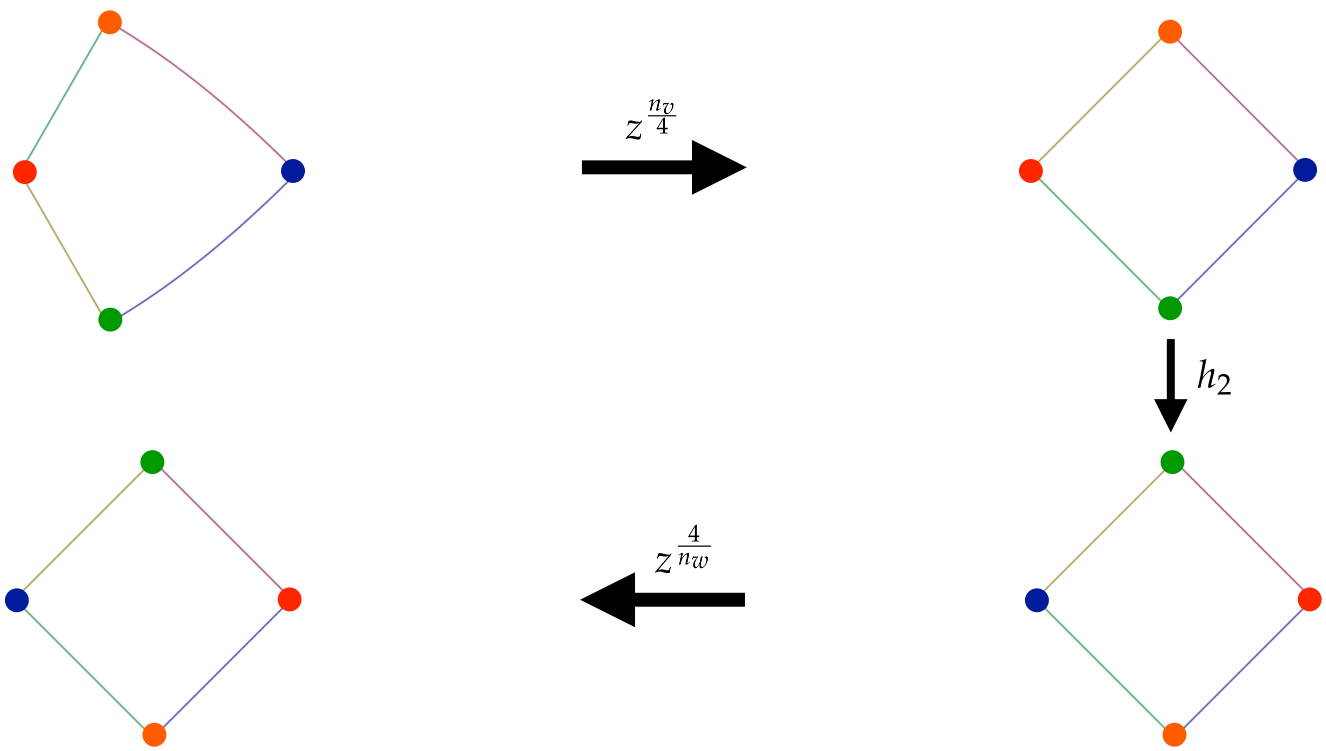
Next, we apply a double reflection, $h_2(x, y) = (\sqrt{2} - x, -y)$, to L, which is a double reflection: a reflection w.r.t the line $x = \sqrt{2}/2$ followed by a reflection w.r.t the y axis:



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

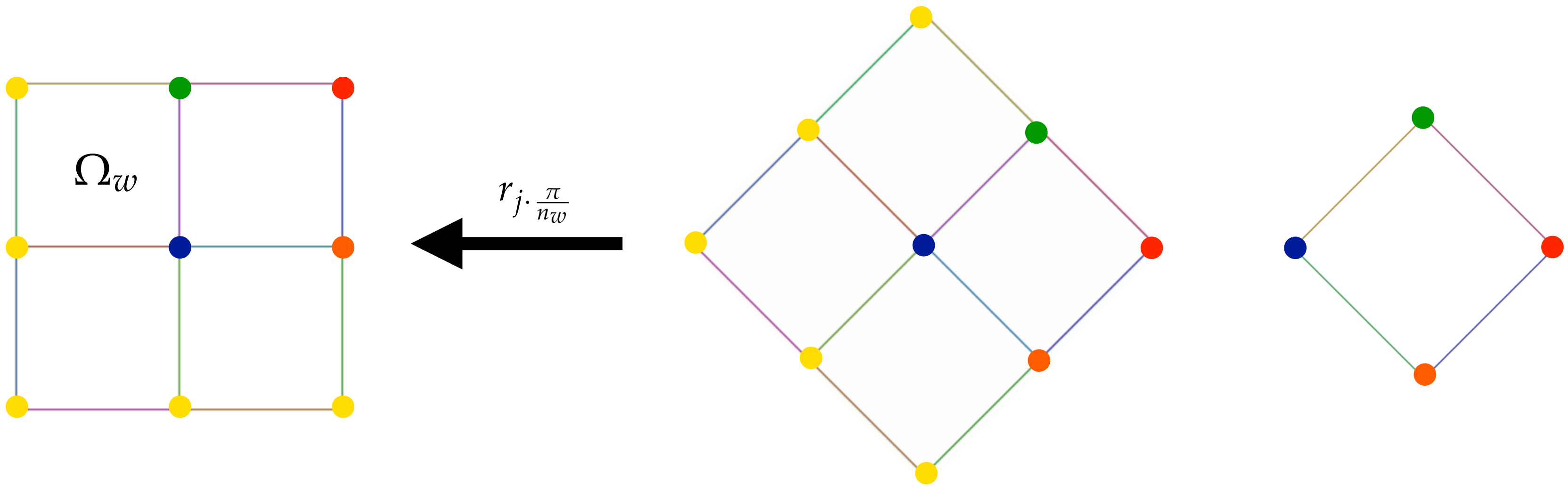
Next, we apply the map $z^{\frac{4}{n_w}}$ to $h_2 \circ z^{\frac{n_v}{4}} \circ r_{-i \cdot \frac{\pi}{n_v}}(\overline{\Omega_{vw}})$:



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Finally, we apply a rotation, $r_{j \cdot \frac{\pi}{n_w}}$, to the set $z^{\frac{4}{n_w}} \circ h_2 \circ z^{\frac{n_v}{4}} \circ r_{-i \cdot \frac{\pi}{n_v}}(\overline{\Omega_{vw}})$:

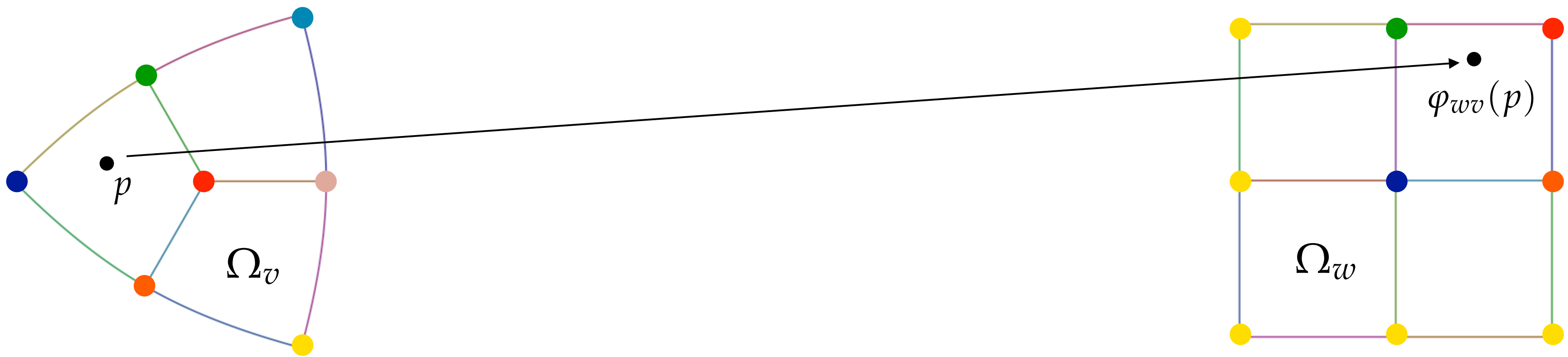


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

So,

$$\varphi_{wv}(p) = \left(r_{j \cdot \frac{\pi}{n_w}} \circ z^{\frac{4}{n_w}} \circ h_2 \circ z^{\frac{n_v}{4}} \circ r_{-i \cdot \frac{\pi}{n_v}} \right)(p) \, .$$

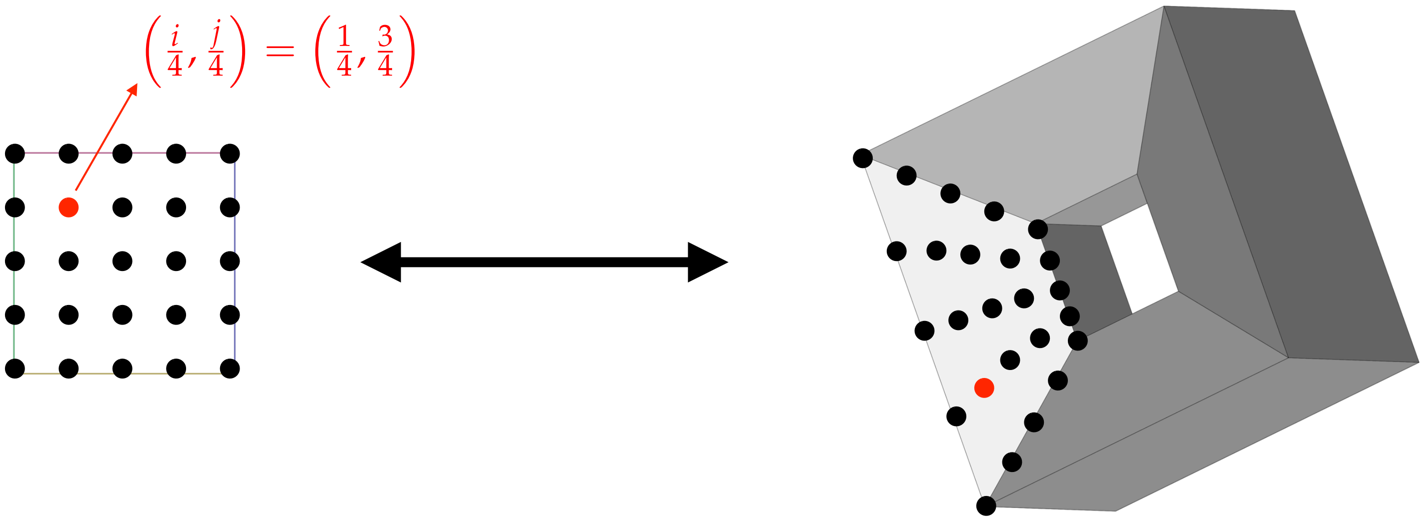


Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

What about geometry?

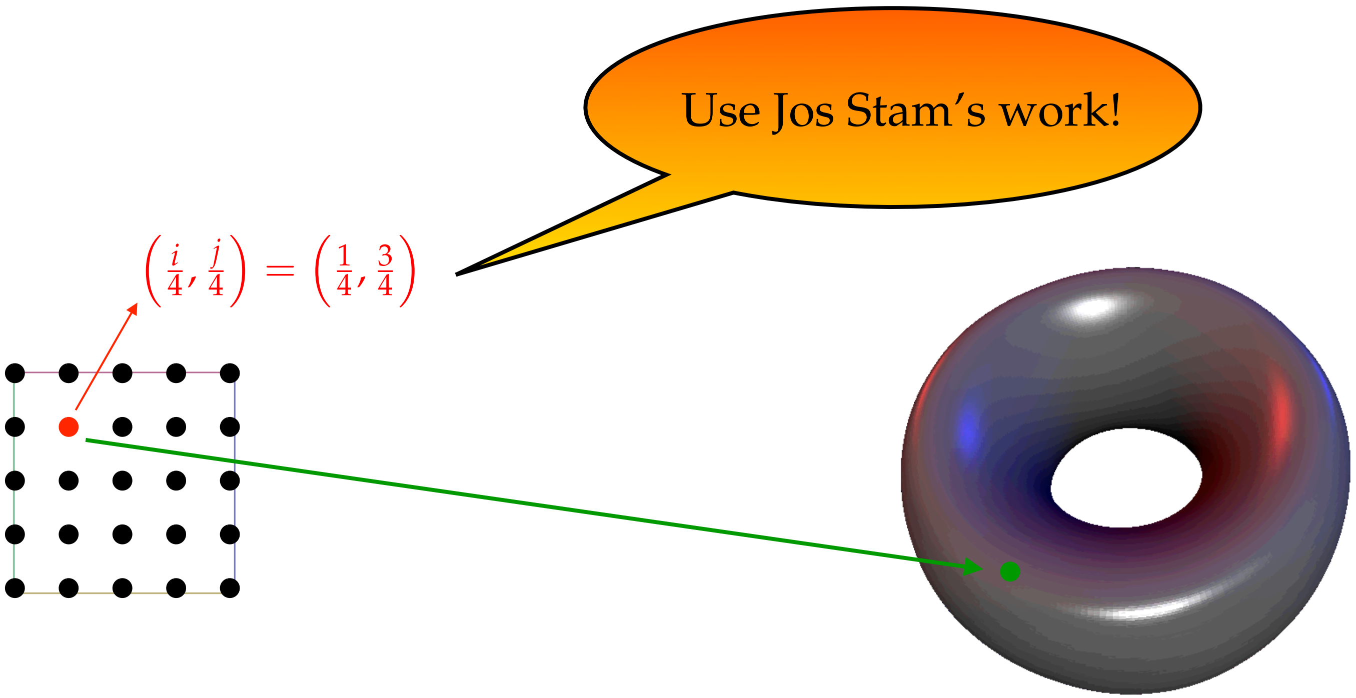
The idea is to uniformly sample the canonical quadrilateral corresponding to each quadrilateral face of the input mesh. This is done for each face of the input mesh at a time.



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

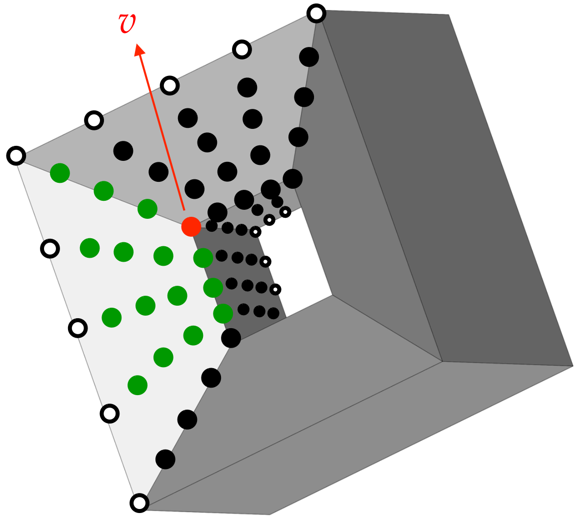
For each parameter point in the canonical quadrilateral, we compute the corresponding 3D point on the Catmull-Clark subdivision surface defined from the input mesh.

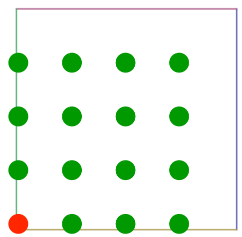


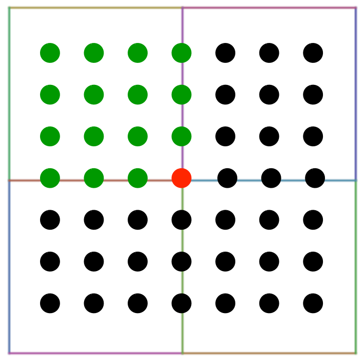
Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Finally, we map the points in the canonical quads to the p-domains. If n_v is the degree of v, then we map $12 \cdot n_v + 1$ points to Ω_v. These are the parameter points which can be viewed as being *inside* the underlying space of the star, $\mathrm{st}(v, \mathcal{K})$, of v in $\mathcal{K}$.





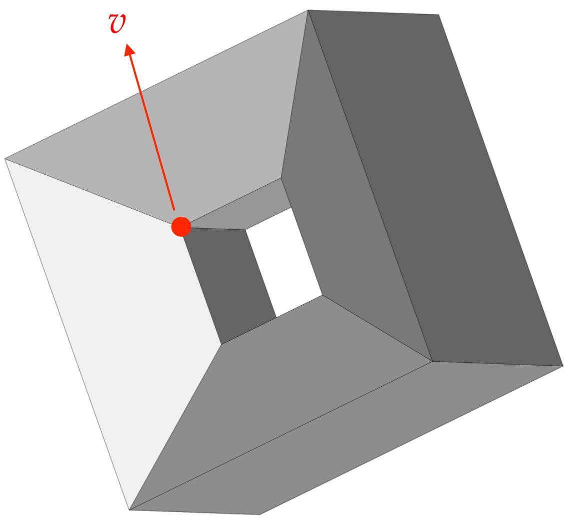


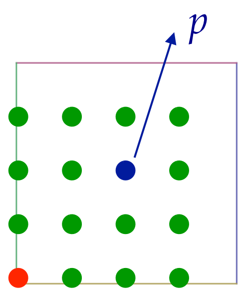
Ω_v

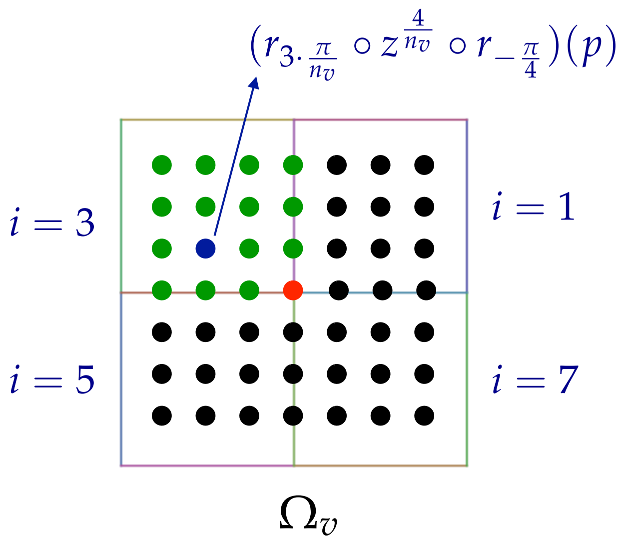
Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

To map the points from the canonical quadrilateral to Ω_v, we use the composite map $r_{i\cdot\frac{\pi}{n_v}} \circ z^{\frac{4}{n_v}} \circ r_{-\frac{\pi}{4}}$, where i identifies the curved quadrilateral in $\overline{\Omega_v}$ that will contain the point.



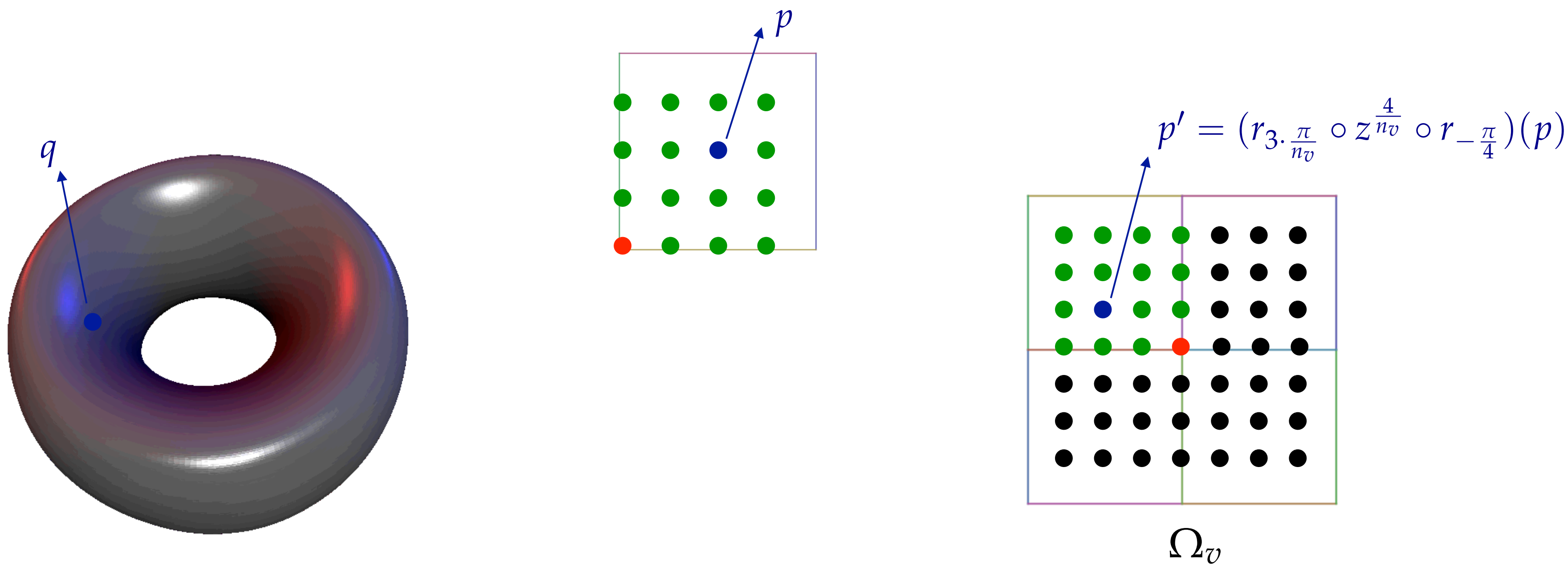




Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

For every point $p' = (r_{i \cdot \frac{\pi}{n_v}} \circ z^{\frac{4}{n_v}} \circ r_{-\frac{\pi}{4}})(p)$ in Ω_v, we define a pair of points, (p', q), where q is the point on the Catmull-Clark subdivision surface associated with the point p.



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

We are now in a position to compute the parametrization, $\theta_v : \Omega_v \to \mathbb{E}^3$, associated with v.

In order to do so, Ying and Zorin defined a basis of monomials, $(x^r y^s)_{r,s}$, of total degree, $r+s$, at most $d = \min\{14, n_v + 1\}$. Using this basis, they define a polynomial, $\psi_v(x,y)$, of degree at most d whose coefficients are computed by minimizing the differences

$$\|\psi_v(p') - q\|$$

in the least-squares sense, where (p', q) are the pairs of points we just computed before.

The polynomial ψ_v is the shape function associated with v. Note that these functions do not necessarily satisfy the convex hull property, as the Bézier patches we used before.

Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

Finally, we define

$$\theta_v : \Omega_v \to \mathbb{E}^3$$

as

$$\theta_v(p) = \frac{\sum_{z \in J_v(p)} (\psi_z \circ \varphi_{zv})(p) \cdot (\gamma_z \circ \varphi_{zv})(p)}{\sum_{z \in J_v(p)} (\gamma_z \circ \varphi_{zv})(p)}$$

for every $p \in \Omega_v$, where

$$J_v(p) = \{u \in I \mid p \in \Omega_{vu}\}$$

and $\gamma_v : \Omega_v \to \mathbb{R}$ is a bump function. The set $J_v(p)$ has at least one vertex and at most 4.

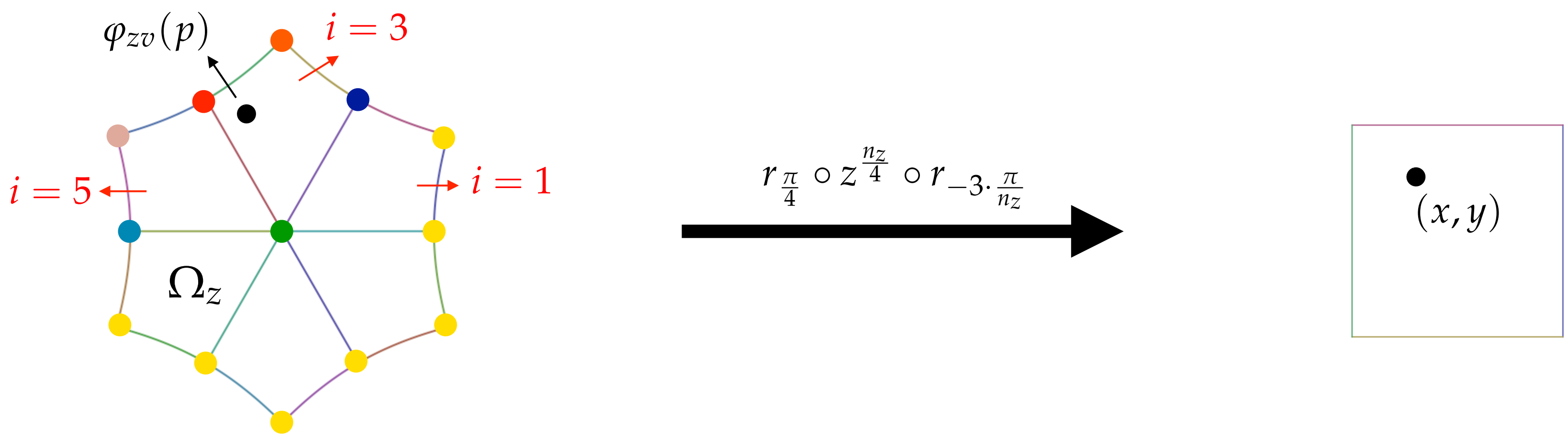
Note that θ_v is defined as a convex combination of the values of $(\psi_z \circ \varphi_{zv})(p)$ weighted by $(\gamma_z \circ \varphi_{zv})(p)$, which can be thought as the influence of $(\psi_z \circ \varphi_{zv})(p)$ on $\theta_v(p)$.

Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

The bump function, γ_z, is defined as a composition of a few maps. First, the point $\varphi_{zv}(p)$ is taken to the canonical domain using the map $r_{\frac{\pi}{4}} \circ z^{\frac{n_z}{4}} \circ r_{-i \cdot \frac{\pi}{n_z}}$, where i identifies the curved quadrilateral of $\overline{\Omega_z}$ that contains the point $\varphi_{zv}(p)$ in the p-domain Ω_z.

Let (x, y) be the coordinates of the point $(r_{\frac{\pi}{4}} \circ z^{\frac{n_z}{4}} \circ r_{-i \cdot \frac{\pi}{n_z}} \circ \varphi_{zv})(p)$ in the canonical quad.



Parametric Pseudo-Manifolds

A Construction for Quadrilateral Meshes

We compute the product $\xi(x) \cdot \xi(y)$, where $\xi : [0,1] \to \mathbb{R}$ is the function defined in the previous lecture. So, putting everything together, we define $\gamma_z : \Omega_z \to \mathbb{R}$ as follows:

$$\gamma_z(x,y) = \left(\eta \circ r_{\frac{\pi}{4}} \circ z^{\frac{n_z}{4}} \circ r_{-i \cdot \frac{\pi}{n_z}}\right)(x,y)\,,$$

where $\eta : [0,1]^2 \to \mathbb{R}$ is given by

$$\eta(x,y) = \xi(x) \cdot \xi(y)\,.$$