

Visualizing Thurston's geometries

Tiago Novello
Vinícius da Silva
Luiz Velho



33^o Colóquio
Brasileiro de
Matemática

Visualizing Thurston's geometries

Visualizing Thurston's geometries

Primeira impressão, julho de 2021

Copyright © 2021 Tiago Novello, Vinícius da Silva e Luiz Velho.

Publicado no Brasil / Published in Brazil.

ISBN 978-65-89124-21-4

MSC (2020) Primary: 68U05, Secondary: 65D18, 53A35, 57M60, 57M50, 57K35

Coordenação Geral

Carolina Araujo

Produção Books in Bytes

Capa Izabella Freitas & Jack Salvador

Realização da Editora do IMPA

IMPA

Estrada Dona Castorina, 110

Jardim Botânico

22460-320 Rio de Janeiro RJ

www.impa.br

editora@impa.br

Contents

1	Background on Manifolds and Orbifolds	1
1.1	History	1
1.1.1	Henri Poincaré	1
1.1.2	William P. Thurston	2
1.1.3	Grigori Perelman	3
1.2	2-Manifolds	3
1.2.1	Classification of compact surfaces	3
1.2.2	Geometrization of compact surfaces	4
1.3	3-Manifolds	5
1.3.1	Classification of compact 3-manifolds	5
1.3.2	Geometrization of compact 3-manifolds	6
1.4	8 Thurston Geometries	8
1.4.1	Classical geometries	9
1.4.2	Product geometries	12
1.4.3	“Twisted” product geometries	13
2	Immersive Visualization in Virtual Reality	21
2.1	3D Visualization	21
2.1.1	The Viewing Transformation Pipeline	21
2.1.2	Inside Views in Non-Euclidean Spaces	22
2.1.3	Types of Algorithms	23
2.1.4	Rendering Acceleration	25

2.2	GPU Ray Tracing using RTX / Falcor 3.2.1	25
2.2.1	RTX Ray Tracing	26
2.2.2	Falcor 3.2.1	28
2.3	Ray Tracing and Stereo Rendering	29
2.3.1	Simple Ray Tracer	29
2.3.2	Stereo Rendering	30
2.4	Integrating Ray Tracing and VR	30
2.4.1	Stereo Convergence	30
2.4.2	Ray Tracing Overhead	34
3	Riemannian Ray Tracing	36
3.1	Core Concepts	36
3.1.1	Geodesics and Fundamental Domain	36
3.1.2	Non-Euclidean Ray tracing	37
3.1.3	Riemannian Manifolds	38
3.2	Visualization of Riemannian manifolds	39
3.2.1	Visualization approaches	39
3.2.2	Riemannian ray tracing	40
3.3	Ray tracing in Riemannian manifolds	41
3.3.1	Overview of the Method	41
3.3.2	Algorithm in CPU	42
3.3.3	Ray Marching	43
3.3.4	RTX Pipeline	43
3.3.5	GPU Implementation	45
4	Visualization of Classical Non-Euclidean Spaces	47
4.1	Euclidean	48
4.1.1	Flat Torus	48
4.1.2	Mirrored Cube	50
4.2	Spherical	51
4.2.1	Poincaré sphere	51
4.3	Hyperbolic	53
4.3.1	Seifert–Weber dodecahedral space	53
4.3.2	Mirrored Dodecahedron	54
4.4	Analysis	55
4.4.1	Performance	56
4.4.2	Interaction	57
4.4.3	Space Perception	57

5 Visualization of Nil, $\widetilde{SL_2(\mathbb{R})}$, and Sol	60
5.1 Visualizing Nil space	61
5.2 Visualizing Sol space	64
5.3 Visualizing $\widetilde{SL_2(\mathbb{R})}$ space	66
5.4 Experiments and comparisons	66
Bibliography	70
Index	77



Instituto de
Matemática
Pura e Aplicada

ISBN 978-65-89124-21-4



9 786589 124214

