Introduction to Geometric Algebra

Extra I

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Tensor representation of products

$$C = \sum_{k=1}^{2^n} \gamma^k \mathbf{E}_k$$

A general multivector

k-th basis blade

$$C = A \circ B = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} \sum_{k=1}^{2^n} \alpha^i \beta^j \Gamma^k{}_{ij} \mathbf{E}_k$$

Any linear product from GA

$\Gamma^k{}_{ij} \in \mathbb{R}^{2^n \times 2^n \times 2^n}$ is a tensor encoding the product

$$\mathbf{E}_i \circ \mathbf{E}_j = \sum_{k=1}^{2^n} \Gamma^k{}_{ij} \mathbf{E}_k, \quad \forall i, j \in \{1, 2, \cdots 2^n\}$$

Tensor representation of products

$$C = \sum_{k=1}^{2^n} \gamma^k \, \mathbf{E}_k$$

$$C = A \circ B = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} \sum_{k=1}^{2^n} \alpha^i \, \beta^j \, \Gamma^k{}_{ij} \, \mathbf{E}_k$$

The relation among the coefficients and a given product is

$$\gamma^k = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} \alpha^i \, \beta^j \, \Gamma^k{}_{ij}$$

One may use such relation to perform multivector differentiation.

Matrix notation of products

The two Jacobi matrices of the product of multivector.

The relation among the coefficients and a given product is

$$\gamma^k = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} \alpha^i \, \beta^j \, \Gamma^k{}_{ij}$$

$$\frac{\partial \gamma^k}{\partial \alpha^i} = \sum_{j=1}^{2^n} \beta^j \, \Gamma^k{}_{ij} = \Gamma_L(\mathrm{b})$$

$$\frac{\partial \gamma^k}{\partial \beta^j} = \sum_{i=1}^{2^n} \alpha^i \, \Gamma^k{}_{ij} = \Gamma_R(\mathrm{a})$$

The product can now be written as

$$\mathrm{c} = \Gamma_R(\mathrm{a})\,\mathrm{b} = \Gamma_L(\mathrm{b})\,\mathrm{a}$$

where **a**, **b**, and **c** are column vectors,

e.g., $\mathrm{c} = \begin{bmatrix} \gamma^1 & \gamma^2 & \cdots & \gamma^{2^n} \end{bmatrix}^T$