QUADRILATERAL MESHING USING 4-8 CLUSTERING

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Key words: Tri-quad mesh conversion, Catmull-Clark subdivision.

Abstract: *In this short paper we describe a new technique for generating a quadrilateral mesh from an existing triangle mesh. Beginning with an initial triangulation, convex blocks of two triangles are identified based on the lenght of their common edge. These convex blocks, together with the remaining isolated triangles, are transformed into quadrilaterals by the application of hybrid 4-8 subdivision and clustering. The final quadrilateral mesh is adapted to the original triangulation and contains few irregular internal nodes.*

1 INTRODUCTION

Quadrilateral meshes are desirable in many applications. In order to construct a quadrilatera mesh it is necessaray to decompose a given a two-dimensional domain into complex of quadrilateral cells. Usually, the domain is specified by its boundary. There are two types of quadrilateral mesh generation methods: direct and indirect. Direct methods construct the domain decomposition from its specification [3], while indirect methods first triangulate the domain and convert the simplicial mesh into a quadrangulation [1]. In this paper we describe an indirect method thet generates a quadrilateral mesh using clustering of triangle pairs.

2 DESCRIPTION OF THE METHOD

The problem to be solved is transforming an arbitrary triangle mesh into a quadrangulation, making just few modifications to the mesh. Our solution is based on the fact that applying Catmull-Clark subdivision [2] to an arbitrary tessellation, produces a mesh containing only four-sided faces [4]. Figure 1 shows an example of this subdivision.

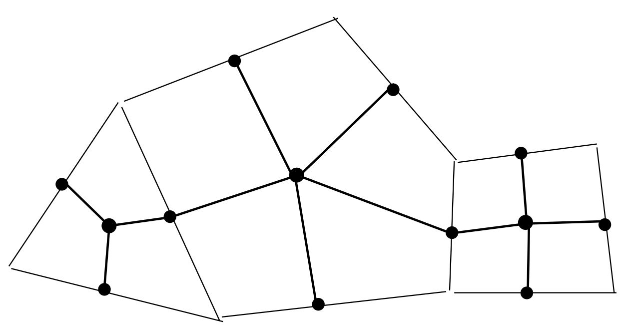

Fig. 1. Example of first step of Catmull-Clark subdivision.

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Since the above result is valid for generic triangulations, we could just use Catmull-Clark subdivision to construct the quadrangulation. Nonetheless, we pursue a different strategy that minimizes changes to the original mesh, produces well shaped faces, and avoids increasing vertex valences.

We use the following strategy. First, we select a set of two triangle face clusters Q, that covers most of the mesh. At the same time, we identify the set of remaining triangle faces $T = \overline{Q}$. Then, we apply one step of Catmull-Clark subdivision to $T \cup Q$, but treating the two-triangle clusters in Q, as quadrilateral faces.

The complete procedure is summarized below:

(1) Find an independent set of two-triangle clusters, and identify the remaining isolated triangle faces;

(2) Perform a hybrid binary subdivision step;

(3) Perform one step of binary subdivision;

(4) Remove internal edges of the subdivided triangulated quadrilateral blocks to obtain the quad-mesh.

To implement pass (1) we select clusters based on edge length. This heuristics guarantees that we obtain convex quadrilateral blocks, and a triangle mesh with good aspect ratio. The pseudo-code is shown in Algorithm 1

Algorithm 1 : find_clusters

```
sort_edges (E)
store e ∈ E in priority queue Q
while Q ≠ ∅ do
  get e from Q
  if e not marked then
    mark_cluster (e)
```

The routine sort_edges, sorts edges by decreasing length. The routine mark_cluster(e) marks an edge e and the edges sharing a face with e. This is illustrated in Figure 2. Marking ensures that we obtain an independent set of two-triangle face clusters.

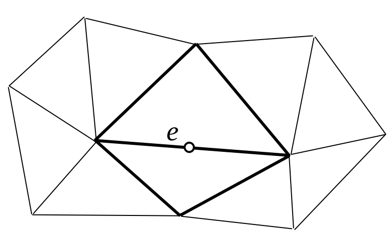


Fig. 2. Two-face cluster marking.

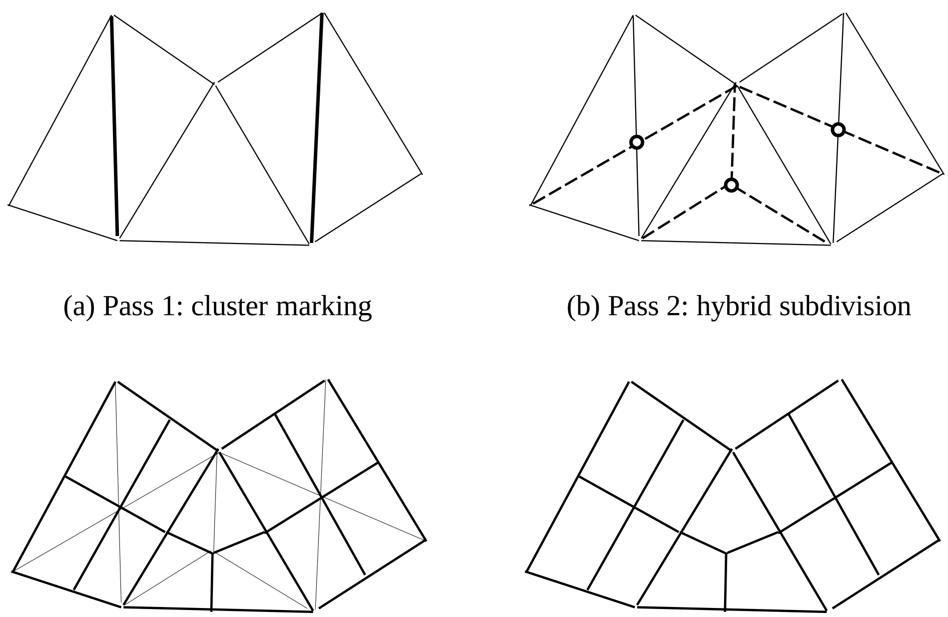
In general, it is not possible to cover the whole mesh with these two-triangle face clusters. There will be a few isolated triangular faces remaining.

The purpose of the hybrid subdivision step in pass (2) is to make these isolated triangle faces compatible with the two-triangle face clusters, such that after binary subdivision in pass (3) we get a quadrangulation that covers the whole mesh.

The hybrid subdivision procedure applies distinct refinement rules to two-triangle face clusters and isolated triangles. Two-face clusters are subdivided through binary subdivision along their internal edge. Isolated triangles are subdivided using barycentric subdivision: three new faces are created by linking the barycenter to each old vertex of the triangle.

After the pre-processing, two-face clusters subdivide into four blocks of two triangles, and isolated triangles subdivide into three blocks of two triangles. The union of these blocks covers the mesh, providing the desired triangulated quadrangulation structure. Finally, in pass (4) the triangulated quadrilateral blocks are transformed into quadrilateral faces by removing the internal edge.

Figure 3 illustrates the four passes of the method.



(a) Pass 1: cluster marking

(b) Pass 2: hybrid subdivision

(c) Pass 3: final subdivision

(d) Pass 4: tri-quad conversion

Fig. 3. Steps of the method.

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