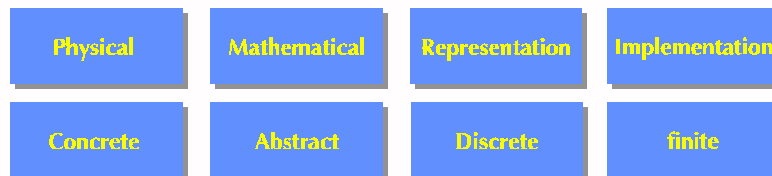


Color Theory and Systems

Jonas Gomes
IMPA, Rio de Janeiro

Mathematical Modeling

- 4 Paradigm Universes

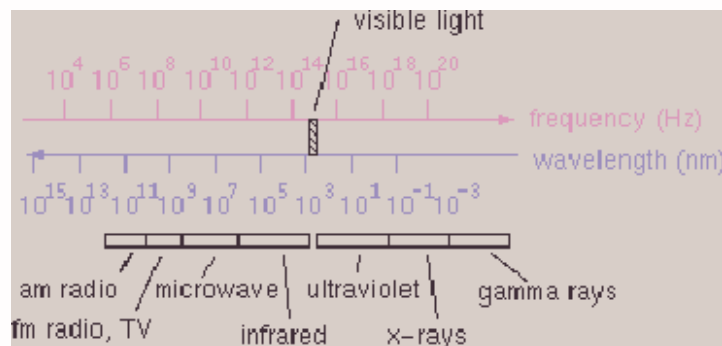


Mathematical models of color

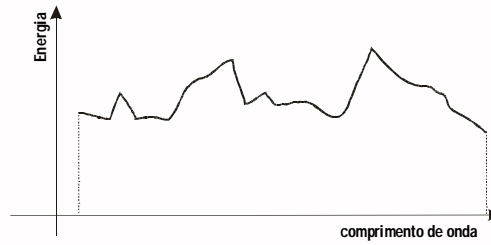
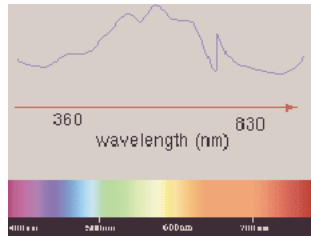


Electromagnetic Radiation

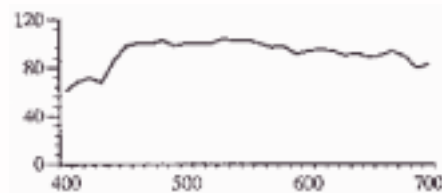
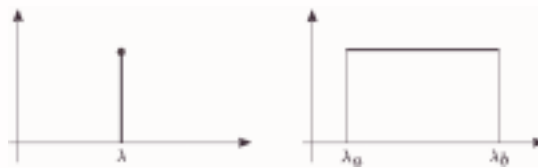
- **Color: The visible radiation**



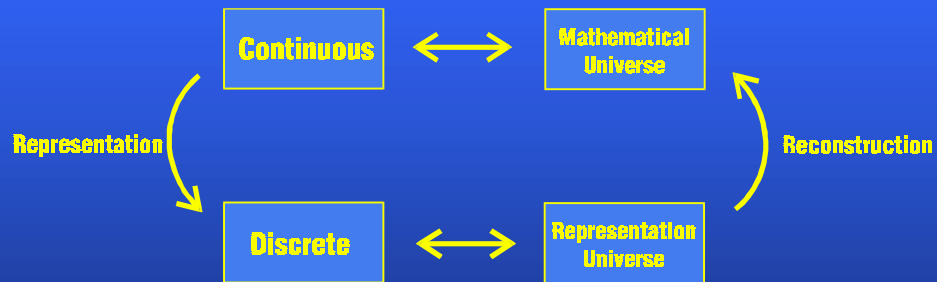
Spectral Distribution Function



Some examples

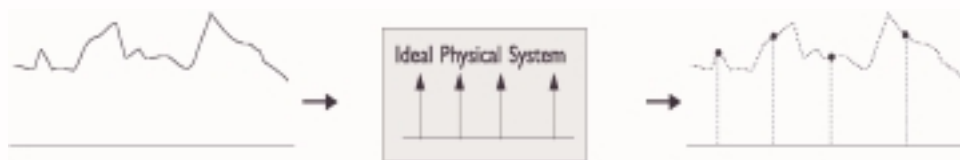


Representation and Reconstruction



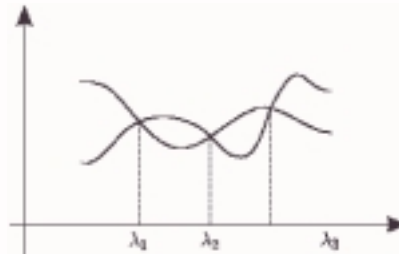
- *Problems: How to represent a color?
How to reconstruct a color?*

Color Representation



- **Representation = sampling**
- **Reconstruction = Interpolation**

Sampling and Reconstruction



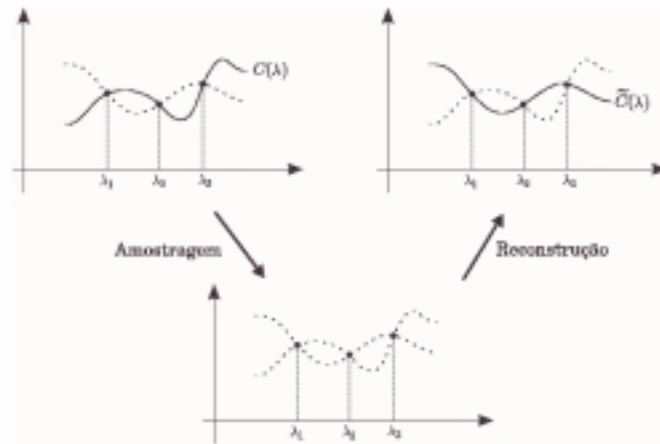
- Distinct colors with same representation
- Metamerism



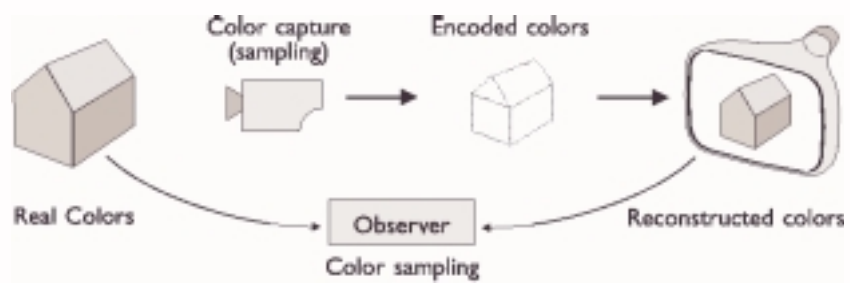
Sampling and Reconstruction

- How many samples?
 - Young-Helmholtz RGB theory
- Exact reconstruction?
 - Perceptual reconstruction

Perceptual Reconstruction



Sampling and Perceptual Reconstruction



Color Physical Systems

- Receptive systems
 - Sampling
 - Eye, scanner, video camera
- Emissive systems
 - Reconstruction
 - Monitors, printers

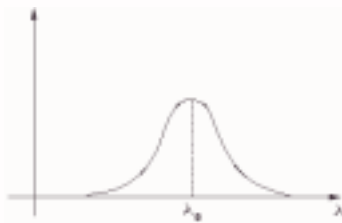


A bit of Mathematics

- Receptive systems

$$c(\lambda) \rightarrow (c_1, c_2, \dots, c_n),$$

$$c_i = \int_0^{+\infty} c(\lambda) s_i(\lambda) d\lambda,$$



Spectral response function

A bit of Mathematics

- Emissive systems
 - Primary colors

$$P_1(\lambda), \dots, P_n(\lambda)$$

$$c_r(\lambda) = \sum_{k=1}^n c_k P_k(\lambda).$$



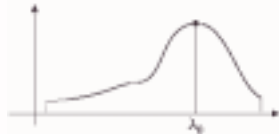
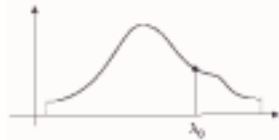
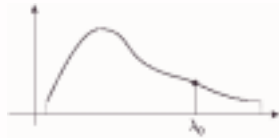
Metameric Reconstruction

- Spectral color C
- Color reconstruction system R
 - primaries $\{P_1, P_2, P_3\}$
- Determine metameric color

$$\tilde{C} = c_1 P_1 + c_2 P_2 + c_3 P_3$$

A Solution

$$\delta(\lambda) = c_1(\lambda)P_1 + c_2(\lambda)P_2 + c_3(\lambda)P_3.$$

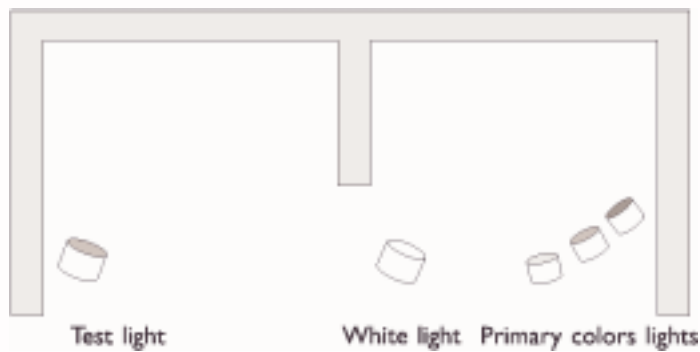


$$c_i = \int_{\mathbb{R}} C(\lambda)c_i(\lambda)d\lambda.$$

Color matching functions

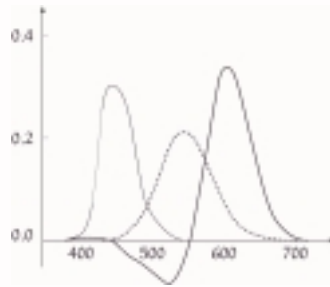


Computing the Color Matching Functions



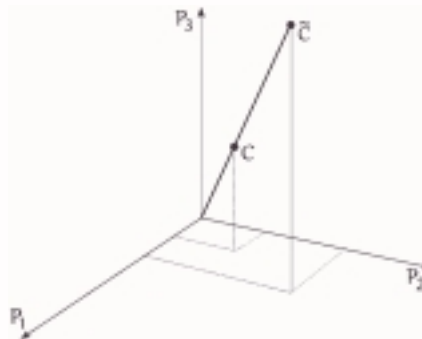
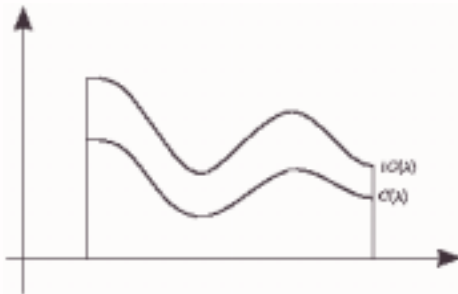
CIE-RGB standard

- Basis of primaries
 - Red 700 milimicrons
 - Green 546 milimicrons
 - Blue 435.8 milimicrons



Geometry of Color Space

- Luminance
- Chrominance



Chrominance and Luminance



Full Color Image



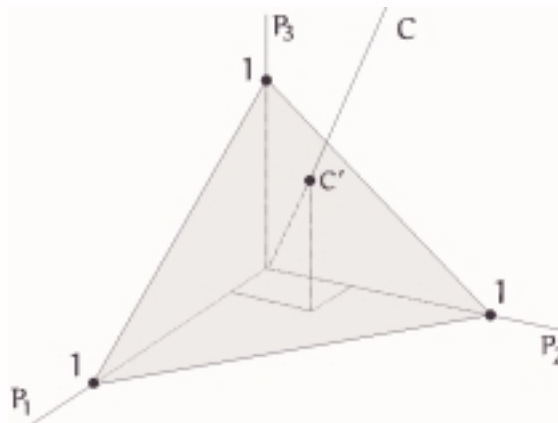
Image Luminance



Image Chrominance

Maxwell Plane and Triangle

- $x + y + z = 1$



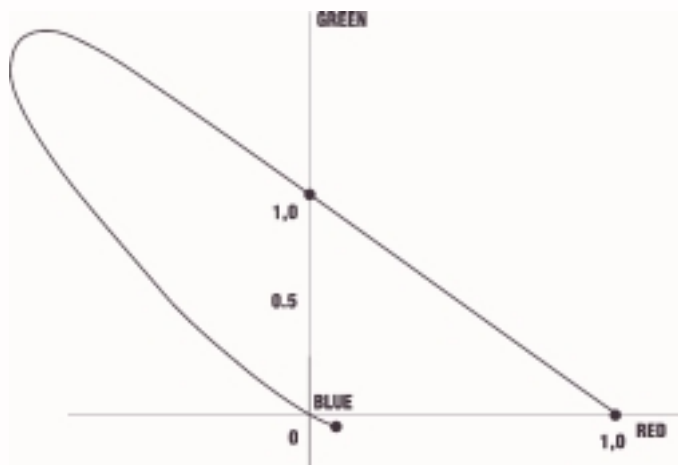
Chromaticity Coordinates

$$c' = (c'_1, c'_2, c'_3)$$

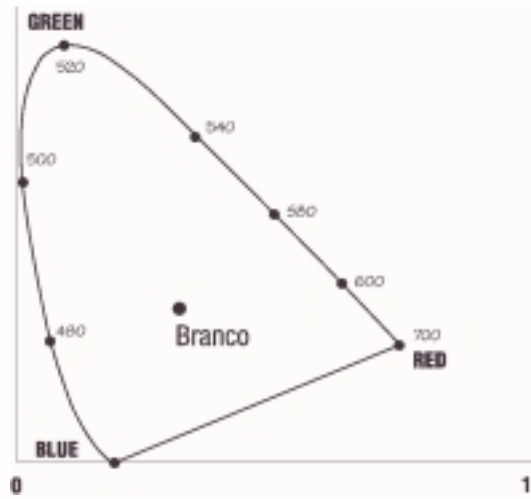
$$c'_i = \frac{c_i}{(c_1 + c_2 + c_3)}.$$



Chromaticity diagram



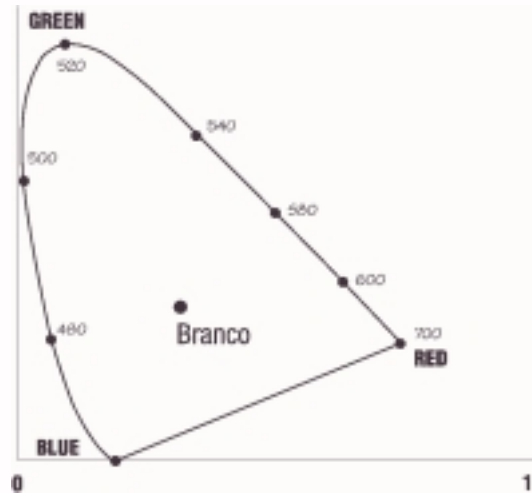
CIE-XYZ Standard



Chromaticity Diagram CIE-XYZ



Complementary Colors and Dominant Wavelength

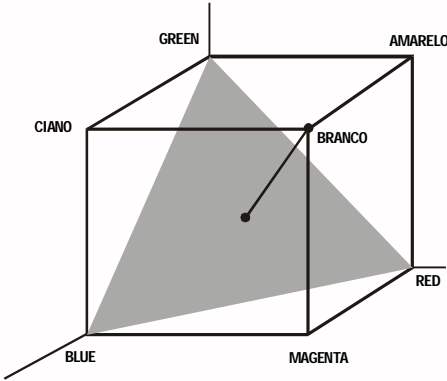


Color Systems and Computer Graphics

- Device Color systems
- Standard Systems
- Computational systems
- Interface systems

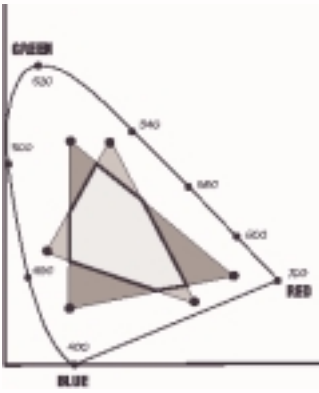
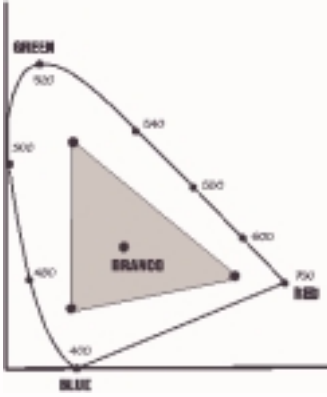
Device Color Systems

- RGB Color system



RGB Color System

- RGB gamute



Device Color Systems

- **Color Printer Systems**
 - **Complementary colors**
 - **CMY**
 - **CMYK**
 - **CMYKRGB**

Device Color Systems

- **Chrominance-luminance systems**
 - **Video and television**
 - **Multimedia**

$$Y = 0.299R + 0.587G + 0.114B.$$

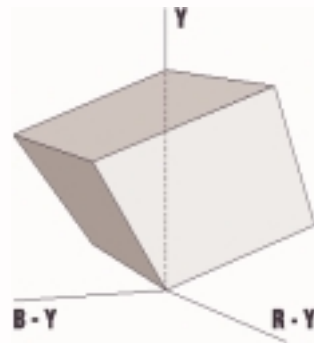
$$Y = 0.299R + 0.587G + 0.114B;$$

$$R - Y = 0.711R - 0.587G - 0.114B;$$

$$B - Y = -0.299R - 0.587G + 0.99B,$$

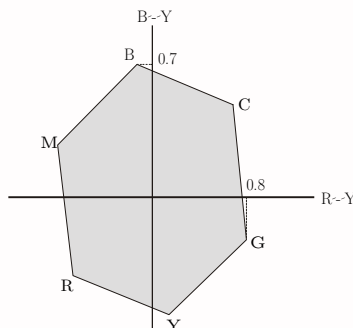
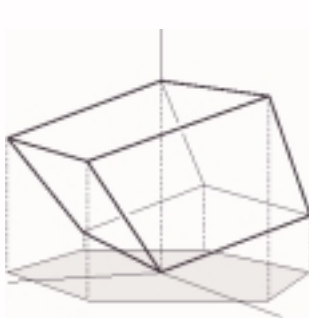
From RGB to Y,R-Y,B-Y

$$\begin{pmatrix} Y \\ R-Y \\ B-Y \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.11 \\ 0.711 & -0.587 & -0.11 \\ -0.299 & -0.587 & 0.99 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$



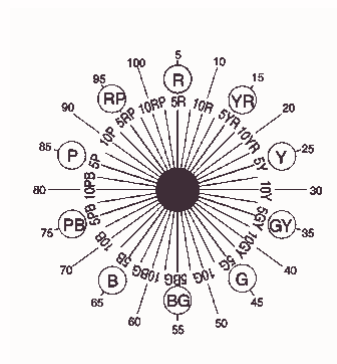
Y, R-Y, G-Y

- Chromaticity Diagram

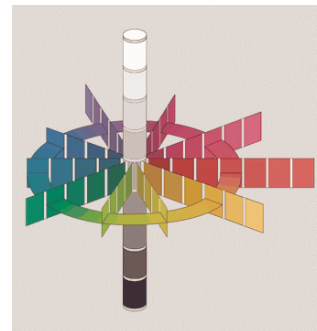
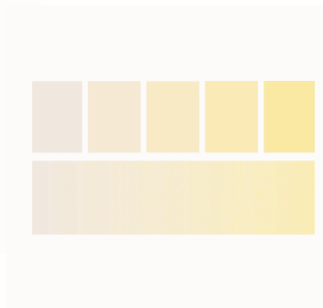


User Interface Systems

- The Munsell System
- A. Munsell, art teacher
 - A Color Notation, 1905

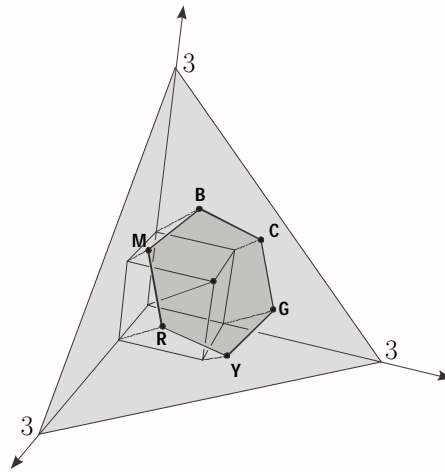


The Munsell Solid

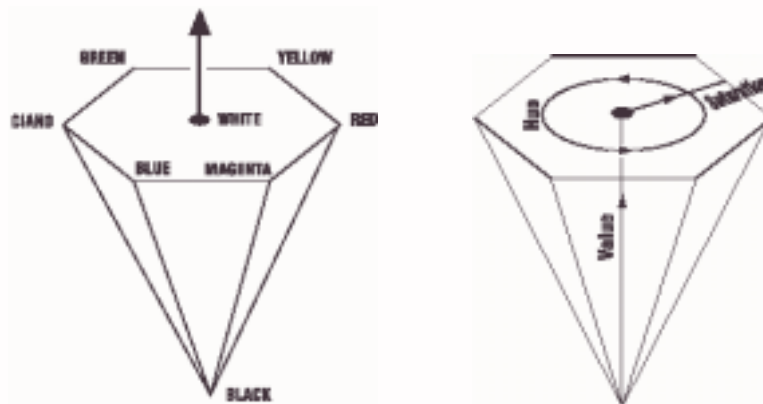


User Interface Systems

- HSV (Alvy Ray Smith)



HSV Solid



Computing With Colors

- **Image Processing**
 - **Photoshop**
- **Shading Computation**
 - **Image Synthesis**
- **Computational Color Systems**
- **Quantization**

Quantization

- **Numerical representation of color**
 - **finite number of bits**
 - **finite number of colors**
- **Problems**
 - **Loss of information**
 - **Perceptual problem**

Quantization

- **Perceptual problem**



Sites about Color

- www.munsell.com
- <http://ziggy.derby.ac.uk/colour/>
 - **Colour & Imaging Institute**
- www.hike.te.chibau.ac.jp/ikeda/CIE/home.html
 - **CIE home page**