Adaptive Fitting of C^∞ Surfaces to Dense Triangle Meshes

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The use of triangle meshes for the representation of surfaces with highly complex geometry and arbitrary topology has become the *de facto* standard in many different areas of computer graphics and geometry processing. This is mainly due to (1) their geometric simplicity, (2) the existence of efficient algorithms for displaying, editing, smoothing, simplifying, remeshing, parametrizing and compressing them, and (3) the advent and progress of 3D laser range scanner technologies [1]. Also, most techniques for manipulating traditional industry standard smooth surface representations, such as NURBS [2], are now available for triangle meshes.

On one hand, triangle meshes can represent surfaces with arbitrary topology much more easily than smooth surfaces can. In addition, triangle meshes have always been used in several stages of a typical pipeline in geometric design applications. So, it seems attractive to replace smooth surfaces with triangle meshes in all stages, avoiding inter-stage, error-prone representation conversion and accelerating the production pipeline. On the other hand, in certain applications, smooth surface representations are still preferable to triangle meshes for reasons of compactness, manufacturability, appearance, and continuity degree.

More specifically, smooth surface representations offer a much more compact way of representing fine geometric features, which would typically require a large amount of small triangles to be represented with the same level of detail [3, 4, 5, 6, 7]. Also, many shapes designed by CAD systems are physically realized by numerical controlled (NC) machines, and are often represented by smooth surface to meet aesthetic and functional requirements [8]. Finally, C^2-continuity is important for visual quality, as it guarantees smooth normal variation, whereas higher degrees of continuity are desirable for numerical purposes. For instance, to enable smooth calculations in the quantification of joint kinematics and distances between joint surfaces [9], and to achieve high-order convergence rates in the numerical solution of boundary integral equations [10]. Hence, algorithms to convert a triangle mesh representation into a smooth surface one and vice-versa are still necessary in the processing pipeline of certain applications related to computer graphics and scientific computing.

In particular, the conversion of triangle meshes into smooth surface representations can be regarded as a *surface fitting* problem, whose solution is a smooth surface, with the same topology as the triangle mesh, that interpolates or closely approximates the mesh vertices. Here, we introduce a new solution for the aforementioned fitting problem, which is catered to dealing with dense triangle meshes, i.e., triangle meshes with hundreds of thousands of vertices. Our solution consists of three main steps as described below:

1. Given a dense triangle mesh, M, we apply the mesh simplification algorithm in [11] to obtain a mesh, M', with the same topology as M but a smaller number of triangles and vertices. An important feature of the algorithm in [11] is that the set of vertices of M' is a subset of the set of vertices of M.

2. We identify the vertices of M' with their counterparts in M, and then embed the edges of M' in the input mesh, M, using an algorithm for computing geodesic curves over triangle meshes [12]. This algorithm computes a geodesic curve on M between each pair of vertices of M (resp. M') connected by an edge in M'. The resulting network of geodesic curves and their endpoints induces a triangulation, T, on M whose triangles are curved, triangle-shaped regions of M called *macro patches*.

3. We construct a C^∞-continuous surface, S, from T and M. This is done by an extension of a recent manifold-based approach for fitting C^∞-continuous surfaces to triangle meshes [13]. Roughly speaking, this manifold-based approach defines a set of local parametrizations of M that are combined by convex sums to define the surface S. Each local parametrization is associated with exactly one macro patch of T. The topology of T defines the topology of S, and the vertices of M inside each macro patch of T defines the geometry of the local parametrizations associated with the patch.

The size complexity of S is proportional to the number of local parametrizations, which is three times the number of macro patches of T. So, by carefully and adaptively simplifying the dense input mesh M in step 1, we can obtain a significantly more compact and yet accurate smooth approximation to M. Furthermore, by being based on the manifold-based approach in [13], the resulting surface S is C^∞ everywhere, has fixed-sized local support for basis functions, is guaranteed to be in the convex hull of all control points that define the local parametrizations, and its geometry can be locally controlled. All these features together make our solution more attractive than previous solutions to the same fitting problem [3, 6, 7].

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