Computational Applications in Complex Dynamics

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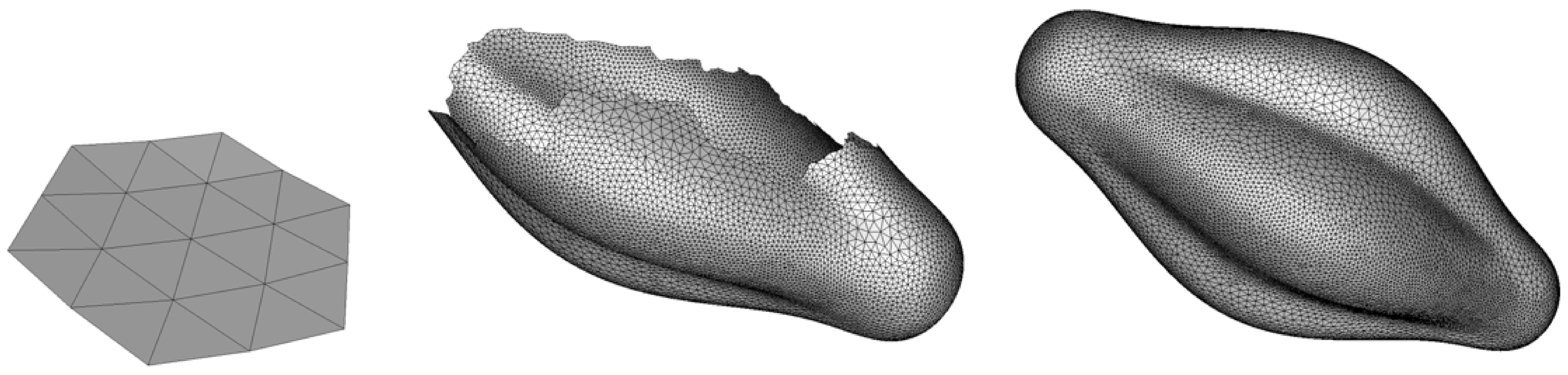
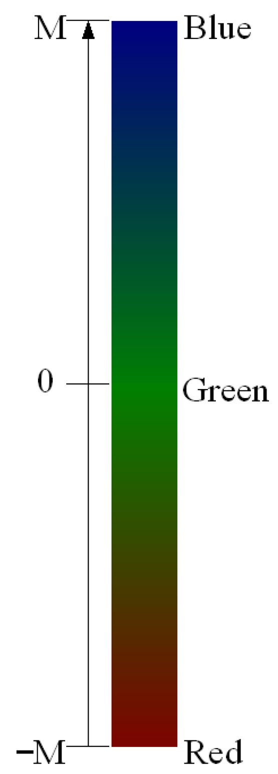
Technique of advancing fronts in the construction of a polyhedral surface in $\mathbb{R}^3$



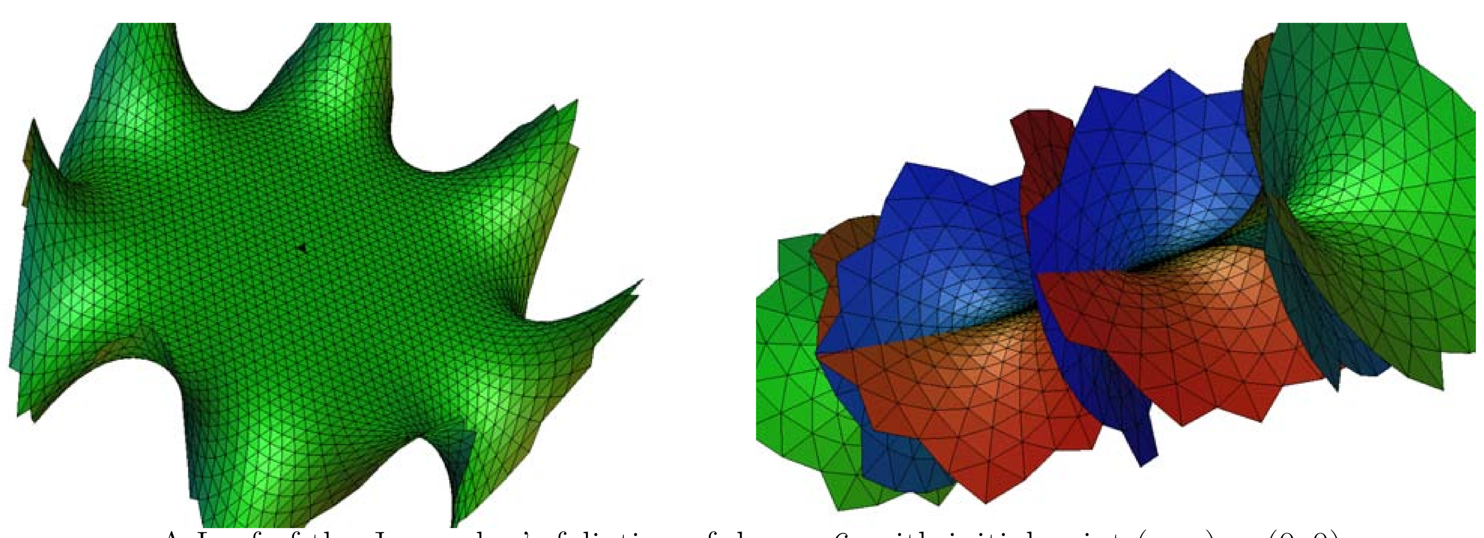
Extension of the Euler's Method to complex ODEs using techniques of advancing fronts

The leaves of a holomorphic foliation for curves can be approached by a polyhedral surface of triangular faces. When the foliation is in $\mathbb{C}^2$, we can visualize this polyhedral surface substituting one of the four coordinates of $\mathbb{C}^2$, namely $\mathrm{Re}(x)$, $\mathrm{Im}(x)$, $\mathrm{Re}(y)$ and $\mathrm{Im}(y)$ (considering $\mathbb{C}^2$ as $\mathbb{R}^4$), for a color coordinate t. Thus the visualization will be a colored polyhedral surface in $\mathbb{R}^3$, where the color in each point gives us the fourth coordinate of this point. In this work the colors have been chosen in the following form: If $M > 0$ is such that $-M \leq t \leq M$ then, a point on the surface will be painted with green if its coordinate t is equal to 0, red if $t = -M$ and blue if $t = M$, varying gradually from green to red if t varies from 0 to $-M$, and varying gradually from blue to green if t varies from 0 to M.

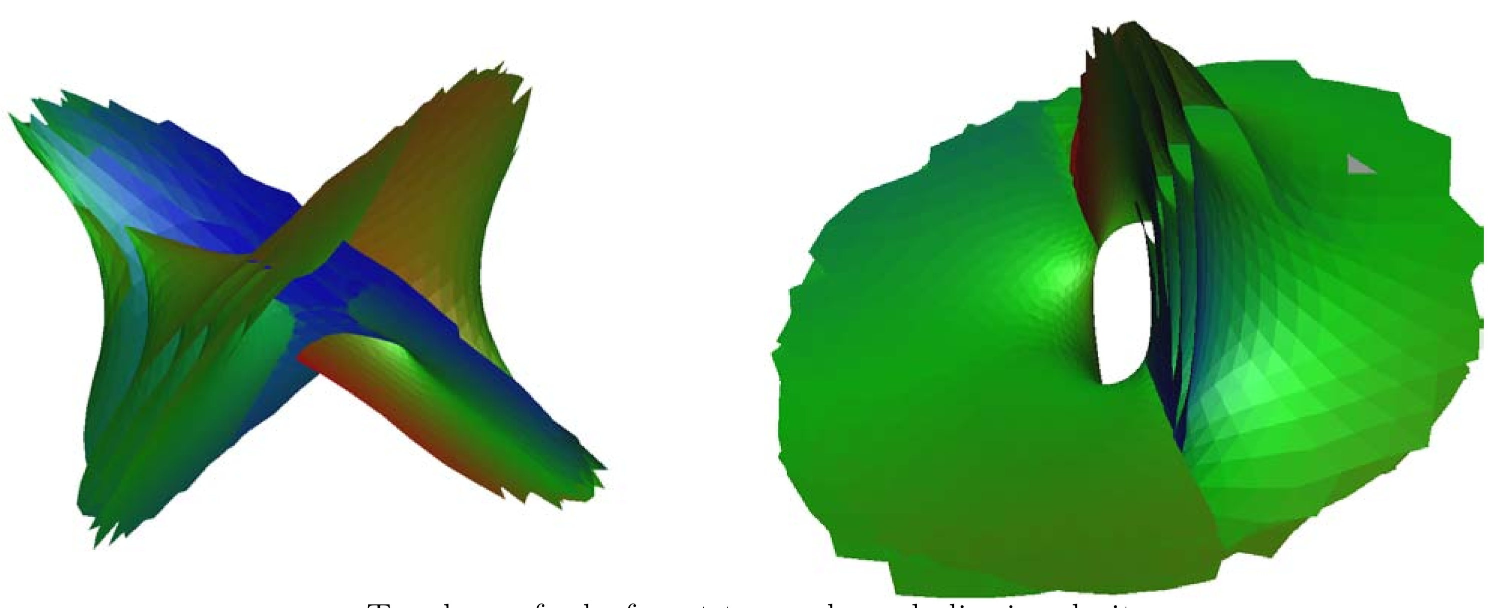


Examples

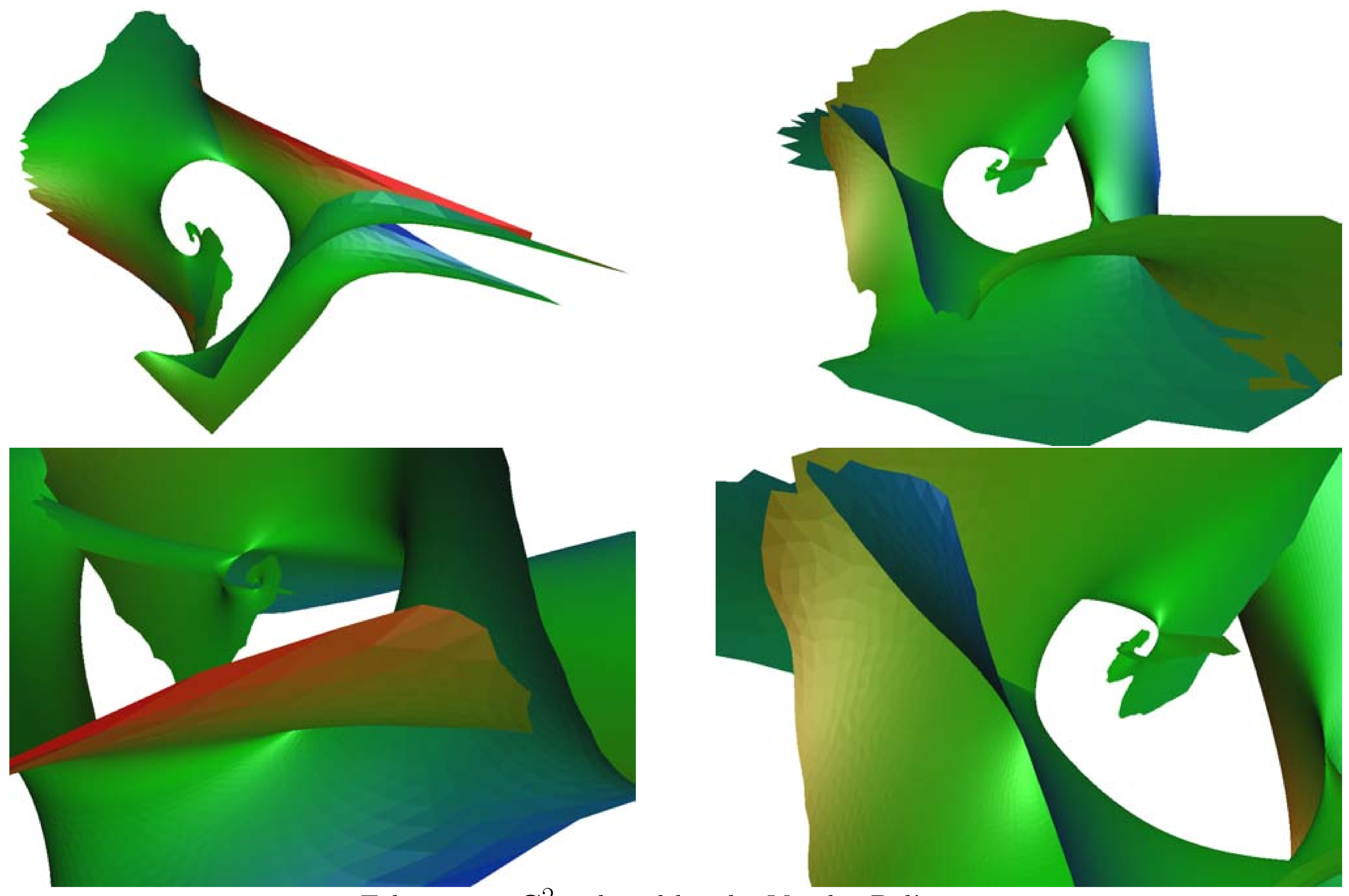
The examples below have been constructed under two view points: $t = \mathrm{Im}(x)$ (on the left) and $t = \mathrm{Im}(y)$ (on the right). In all examples the polyhedral surfaces approache "locally"(in the sense of polyhedrals surfaces) the algebraic curves of degree 10 which it approach the leaf locally.



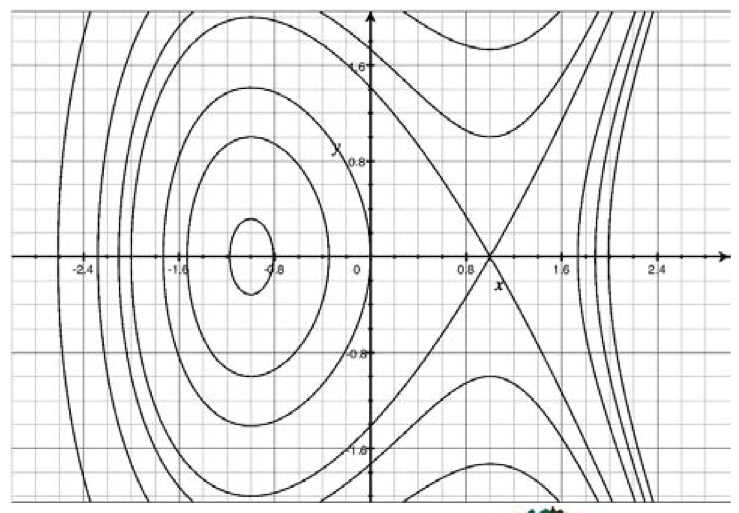
A Leaf of the Jouanolou's foliation of degree 6, with initial point $(x, y) = (0, 0)$



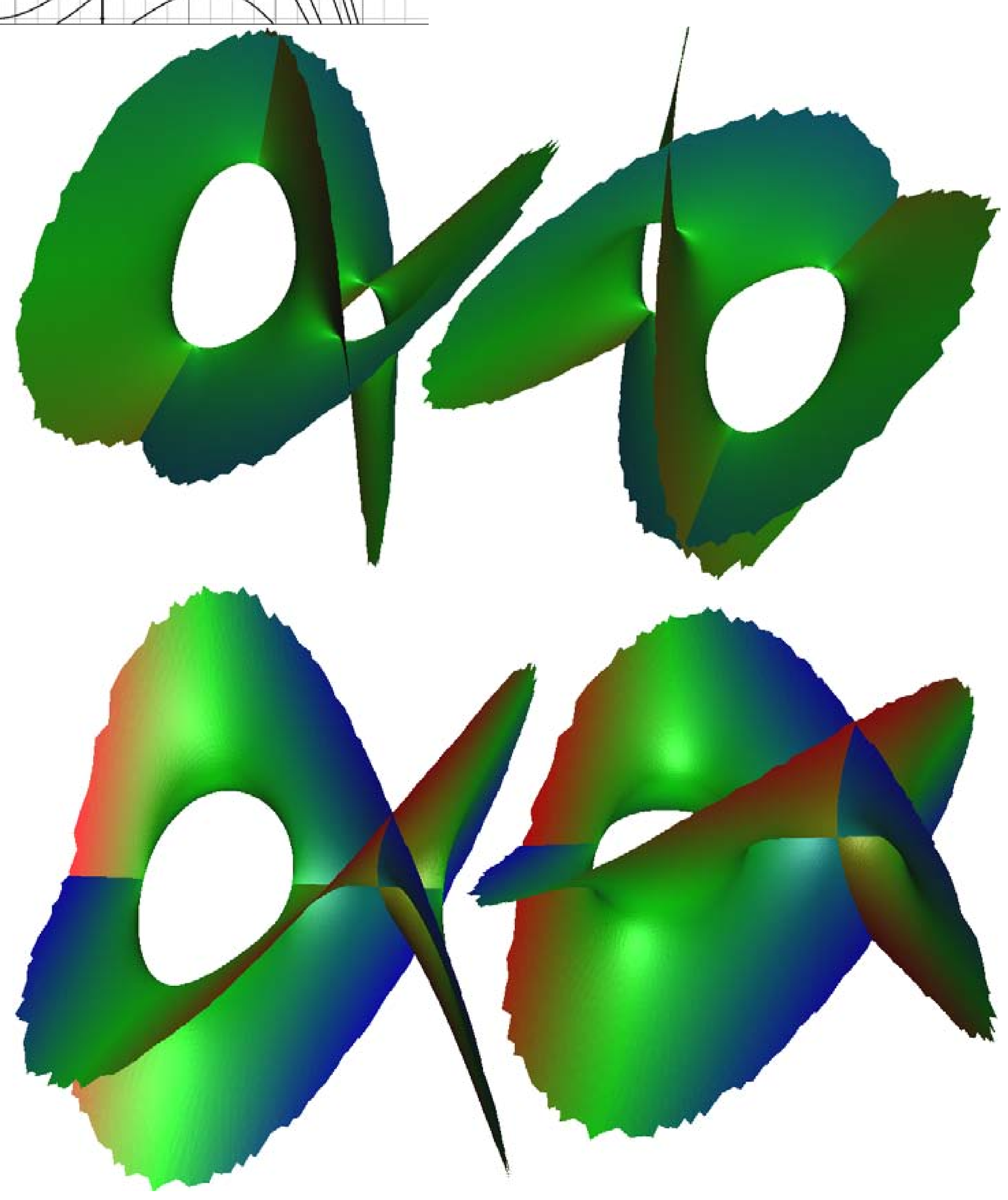
Topology of a leaf next to one hyperbolic singularity



Foliation in $\mathbb{C}^2$ induced by the Vander Pol's equation.



On the left, the elliptic curves $y^2 - x^3 + 3x = a$, $a = -1.9, -1, 0, 2, 3, 5, 10$ in $\mathbb{R}^2$. Bottom the elliptic curve $y^2 - x^3 + 3x = 0$ in $\mathbb{C}^2$. We use $t = \mathrm{Im}(x)$ in the first couple of images and $t = \mathrm{Im}(y)$ in the second one.



Foliation in $\mathbb{C}^2$ gived by elliptic curves

Applications

Searching for leaves that have an atractor in the holonomy group.
(The Black triangle is the initial triangle)