Interactive manipulation of multiresolution curves

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15th April 1999

Abstract

A new method for manipulation of curves in several degrees is presented. When a point of curve is moved, it affects a relative local segment, which size could depends of the propagation of the point through their lower resolution curves. So, for the same displacement of the point, could be generated relative local segments of different sizes, only varying a local parameter such stiffness constant. We direct manipulation to produce local deformation on each resolution curve, so does not necessary to use energy terms for deformation. We have tested our ideas in a prototype system for modelling uniform B-spline curves and the B-spline wavelets. Using this method, the designer can interactively modifies, in a natural manner, the curve by controlling the size of segment affected.

Key words: Interactive manipulation, Design curves, Direct manipulation, multiresolution curves, Local deformation, Controled deformation, Uniform B-splines curves.

1 Introduction

Manipulation of curves and surfaces is necessary in many geometric modelling systems. In many cases a designer requires to model an object (curves or surfaces) of arbitrary form, complex morphology, that could be obtained by manipulating some region of the object. So, the designer could obtain by a natural manner, the desired complex geometric form deforming one existing object sufficiently similar to the former.

Producing an object deformation by a natural manner of a region isn't simple. A designer many times wishes to modify a region superficially, or only modify a pronounced small region -as an elastic material-, or even a large modification in the nearby region of it point shifted, as a metallic material such as a wire with certain stiffness. The figure 1, as example, showes some cases of deformation.

The recent works in the interactive modelling has provided tools for deformation of objects based in different approachs. So, control point manipulation [PieTil95, Farin90] is inappropriate for this case, because the user does not know how and what points to move in order to obtain a desired form. Knots points operations, in B-splines objects, [ForBar88, PieTil95, LycMor87] offer the same problem as manipulating control points. This technique solves the problem trough tedious solution, with unnecessary knots operations. Direct manipulation technique [BarBea89, FowBar93, Fowler92] permits the natural deformation of objects, with implicit movement of control points, when the designer shifts a point on object; nevertheless, that technique doesn't solve the problem once it isn't possible to control the region size affected by the deformation. The deformation based in variational theory [Wesseli96, Welch95, WesVel97] is another alternative to design object interactively minimizing the energy functional. The variational design using multiresolution theory [Takaha98, TaShKu98], editing the effects in continuos levels, is another technique for controlling the smoothness of objects. In these cases, it is also necessary to manipulate the energy functions. Visually real effects are obtained using the physics-based deformation of objects [GuLiTa+97], but the isn't explicit the effect of the force applied on the deformable object.

We formulate a new approach to manipulate some segments of objects, specifically of curves, controlling one local attribute that relates with the size of region segment to be modified. Is extended the direct manipulation technique to produce a local deformation on segment of multiresolution representation, so that the movement of number of control points will depend on the degree of deformation determined by the designer. We dedicate the section 2, for preliminary theory, where the definitions of the curve used and the multiresolution representation, particularly the biothogonal wavelets, are established. Section 3, treats the direct manipulation in multiresolution curves; for that, we first define the local deformation, and apply it in multiresolution curve. In section 4, we formulate the technique to relate the segment deformation through their low-resolution. In section 5 we define the distribution function that permits the distribution of displacement segments in all low-resolution curves, and Section 6 concludes this paper.

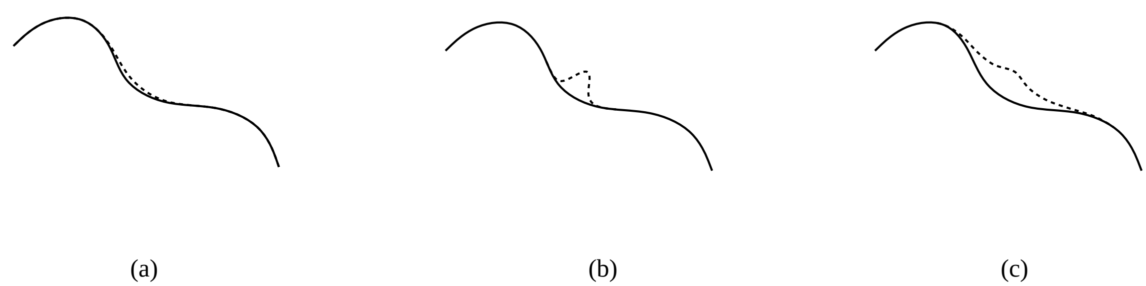

Figure 1: Some examples of curves deformation:(a) superficially deformation, (b) pronounced deformation in small region, (c) large deformation.

2 Representation and multirresolution of curves

In this section, we deal with the type representation of curve to be use in other sections of this work. Also, we describe the formulation of the formal framework of multiresolution representation of curves, we introduce the Biorthogonal wavelets that permits, with flexibility, the manipulation in multiresolution of B-spline curves.

2.1 B-spline curves

Is well known that the basis function B-splines, $N_i^k(t)$, of order k, is defined such

$$N_i^1(t) = \begin{cases} 1 & \text{se } x_i \leq t < x_{i+1} \\ 0 & \text{other} \end{cases}$$

and

$$N_i^k(t) = \frac{(t - x_i).N_i^{k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t).N_{i+1}^{k-1}(t)}{x_{i+k} - x_{i+1}}. \qquad (1)$$

The values x_i, known such *knots*, are elements of vector of knots X, with the condition $x_i \leq x_{i+1}$ ($[x_i, x_{i+1})$ is a domain interval).

All points of a curve's segment j are generated by k basis function with k constant control points C_j^k, only varying the parameter t in a domain interval. Is convenient to reparameterize the basis function to interval $[0, 1)$, because the periodic uniform B-splines basis function are all translates of each other (see figure 2).

The segment j of an uniform B-spline curve of order k is defined as

$$f_j(t) = \tilde{N}^k(t).C_j^k, \tag{2}$$

where the matrix $\tilde{N}^k(t)$ is obtained reparameterizing the k basis, being of the form $\tilde{N}^k(t) = \frac{1}{(k-1)!}[t^{k-1}\, t^{k-2} \ldots 1][m_{i,r}^k]$ with $0 \leq t < 1$ and for $j = 1, ..., n$. The generalized form of $\tilde{N}^k(t)$ could be obtained by the formula of Cohen and Riesenfeld (see [RogAda90] for more detail). Specifically for $k = 4$, the matrix expression of reparameterized basis function is

$$\tilde{N}^k(t) = \frac{1}{6}[t^3\, t^2\, t\, 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}. \tag{3}$$

Each column r $(0 \leq r \leq 3)$ of the matrix is formed by coefficients of the reparameterized basis function $\tilde{N}_r^4(t)$. So, to make a segment of curve $f_j(t)$, according to expression (2), the basis is $\tilde{N}^4(t) = [N_0^4(t)\ N_1^4(t)\ N_2^4(t)\ N_3^4(t)]$, and the matrix column C_j^4 is the form $C_j^4 = [c_{j-1}\ c_j\ c_{j+1}\ c_{j+2}]^T$.

A B-spline curve is called uniform if the spacing between the knots is constant. In figure 2, the spacing of a domain interval is 1. When the function is reparameterized, the same pieces of function are repeated in each interval, for example, as in the pieces of function generated in the interval $[4, 5]$, where a segment of curve is affected by four basis function.

2.2 Multiresolution curve

Let $f(t)$ be a function that defines a curve with 2^n control points as shown in column matrix $C^n = [c_0^n\ c_1^n\ c_2^n\ \ldots\ c_{2^n-1}^n]^T$, where each element c_k^n, in the plane, is the x and y-coordinate point $(c_k^n = (x_k, y_k)\, \epsilon\, R^2)$. For effect of multiresolution representation of the curve, we denote by $f^n(t)$ the refined or limit curve, where n represents the

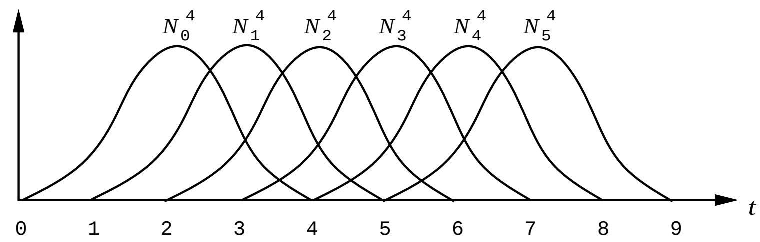

Figure 2: Periodic uniform B-splines basis functions.

highest-resolution level. The notation $f^j(t)$, for $j < n$, is low-resolution representation of the curve. In other words, f^j is a projection of f in some space V^j, and exist another function o^j that is the projection of f in another space W^j, such as $f^{j+1} = f^j + o^j$. See more information about this in [GomVel98].

2.2.1 Decomposition and composition of curve in multiresolution

In order to decompose one curve f^j in another low-resolution f^{j-1} and difference o^{j-1} it is necessary to use some form of linear filtering and down-sampling on the entries of C^j, which are the control points of f^j, to obtain the low-resolution *scaling coefficients* C^{j-1} and *detail coefficients* D^{j-1}.

The low-resolution scaling coefficients are obtained using the *scaling filter matrix* A^j, such that

$$C^{j-1} = A^j C^j. \tag{4}$$

The detail coefficients, of this resolution, $D^{j-1} = [d_0^{j-1}\ d_1^{j-1}\ d_2^{j-1}\ \ldots\ d_{2^{j-1}-1}^{j-1}]^T$, are obtained using the *detail filter matrix* B^j, such that

$$D^{j-1} = B^j C^j. \tag{5}$$

Both matrixes, A^j and B^j, are known as *analysis filter matrixes*. For composing the object from their low-resolution coefficients, we use the *synthesis filter matrixes* P^j and Q^j, being their formulation such

$$C^j = P^j C^{j-1} + Q^j D^{j-1}. \tag{6}$$

Figure 3 shows the decomposition of coefficients C^j in C^{j-1} and D^{j-1}, and the composition or reconstruction of C^{j-1} and D^{j-1} in C^j.

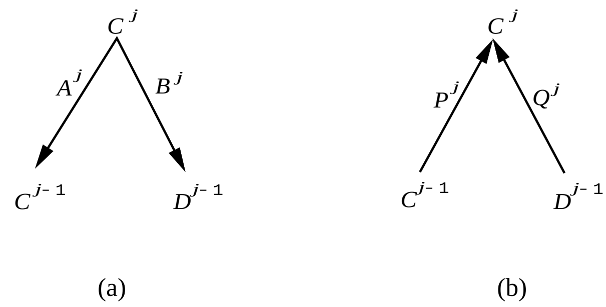

Figure 3: Direct and inverse transform structure:(a) decomposition; (b) composition.

The total elements of C^n is 2^n, so, in each recursion of the decomposition process, we always gave half of the total number of elements for each group of coefficients. There is an approach in [SweSch95] for manipulating arbitrary number of coefficients considering their limits, but here we're not considering limits of curves.

The filter matrixes A^j, B^j, P^j and Q^j are calculated associating the *scaling basis function* $\Phi^j(t)$ that defines $f^j(t)$ with the scaling coefficients C^j, such that

$$f^j(t) = \Phi^j(t)C^j, \tag{7}$$

where the vector $\Phi^j(t) = [\phi^j_1(t), ..., \phi^j_{2^j}(t)]$. In this case, each $\phi^j_i(t)$ is a cubic B-splines function defined in the previous subsection. The $\Phi^j(t)$ is refined by synthesis filter matrix P^j as

$$\Phi^{j-1}(t) = \Phi^j(t)P^j, \tag{8}$$

and *wavelets* $\Psi^{j-1}(t)$ at resolution level $j-1$, that is the basis function of W^{j-1}, is refined using matrix Q^j, as

$$\Psi^{j-1}(t) = \Phi^j(t)P^j. \tag{9}$$

When W^{j-1} is the orthogonal complement of V^{j-1} in V^j, it is called orthogonal wavelets. The spaces $\{V^j\}$, such $... \subset V^{j-1} \subset V^j \subset ...$, define the multiresolution of f.

2.2.2 Relation between coefficients and filters

A scaling function $\phi^j_k(t)$ is defined by shifting and scaling one single function $\phi(t)$, called *mother function* (see for more detail in [StDeSa96, SweSch95, GomVel98]). In general, it is expressed as

$$\phi^j_k(t) = \phi(2^j t - k). \tag{10}$$

The refinement of a scaling function, expression (8), could be re-written, with p_m of P^j, as

$$\phi(t) = \sum_m p_m \phi(2t - m). \tag{11}$$

From the two equations above and changing the index, the refinement expression could be such

$$\phi_k^j(t) = \sum_m p_{m-2k} \phi_m^{j+1}(t). \tag{12}$$

Similar to the operations above is made with expression (9) to deduce the refinement of wavelets function,

$$\psi_k^j(t) = \sum_m q_{m-2k} \phi_m^{j+1}(t). \tag{13}$$

Such $f^{j+1} = f^j + o^j$, it could be expressed using the expressions (12) and (13) and operating as

$$\begin{aligned} f^{j+1} &= \sum_m (\sum_k p_{m-2k} c_k^j + \sum_k q_{m-2k} d_k^j) \phi_m^{j+1} \\ &= \sum_m c_m^{j+1} \phi_m^{j+1} \end{aligned}$$

In conclusion, the relation between coefficients and filters for composition (expression 6) is

$$c_m^{j+1} = \sum_k p_{m-2k} c_k^j + \sum_k q_{m-2k} d_k^j. \tag{14}$$

Similar criteria is used to obtain the relation in decomposition (expression 4 and 5),

$$c_m^j = \sum_k a_{k-2m} c_m^{j+1} \quad \text{and} \quad d_m^j = \sum_k b_{k-2m} c_m^{j+1}. \tag{15}$$

Observe that, in calculus process of coefficients c_m^j and d_m^j, we capture relative elements to even position $(2m)$ of C^{j+1}. This means that, in each decomposition recursion half the number of elements of C^{j+1} are generated. The elements a, b, p and q are elements of matrices A, B, P and Q respectively, and do not depend of level j. For more detail, see [StDeSa96, GomVel98].

2.2.3 Biorthogonal wavelets

Biorthogonal wavelets are a generalization of orthogonal wavelets; they have attributes of orthogonal wavelets and are much flexible than others. These attributes are essencial to express geometric objects in multiresolution and their manipulations with the fast transformation using B-splines basis functions.

In biorthogonal multiresolution, the biorthogonal basis is used in places of orthogonal basis: one for analysis and another for synthesis. In analysis, to calculate C_k^{j-1} and D_k^{j-1}, the *primal* basis Φ^j and Ψ^j are used, respectively; and in synthesis *dual* basis $\tilde{\Phi}^j$ and $\tilde{\Psi}^j$ are used. The functions Φ^j and $\tilde{\Phi}^j$ are basis of spaces V^j and $\tilde{V}^j$, respectively; and the Ψ^j and $\tilde{\Psi}^j$ are basis of W^j and $\tilde{W}^j$, respectively. So, we have two multiresolution spaces

$$... \subset V^{j-1} \subset V^j \subset ...$$

$$... \subset \tilde{V}^{j-1} \subset \tilde{V}^j \subset ...$$

with $V^{j-1} + W^{j-1} = V^j$ and $\tilde{V}^{j-1} + \tilde{W}^{j-1} = \tilde{V}^j$. The orthogonality is given between primal and dual basis (see for more detail [StDeSa96, GomVel98, SweSch95]). So, the basis functions are related with the filters, with the same formulation as in expression (11), as

$$\phi(t) = \sum_k h_k \phi(2t-k) \quad \text{and} \quad \psi(t) = \sum_k g_k \phi(2t-k),$$

similarly

$$\tilde{\phi}(t) = \sum_k \tilde{h}_k \tilde{\phi}(2t-k) \quad \text{and} \quad \tilde{\psi}(t) = \sum_k \tilde{g}_k \tilde{\phi}(2t-k).$$

Finally, considering the orthogonality condition of basis and operating, we obtain the relation between coefficients and filters could re-written as, for decomposition

$$c_m^j = \sum_k h_{k-2m} c_k^{j+1} \quad \text{and} \quad d_m^j = \sum_k g_{k-2m} c_k^{j+1} \tag{16}$$

and for composition

$$c_m^{j+1} = \sum_k \tilde{h}_{m-2k} c_k^j + \sum_k \tilde{g}_{m-2k} d_k^j. \tag{17}$$

The expressions (15) and (16) are similars, for decomposition, and (14) and (17) for composition. In that way, for effect of decomposition and composition process, the filter coefficients are considered such $h \in A$, $g \in B$, $\tilde{h} \in P$ and $\tilde{g} \in Q$, that could be computed using frequency domain [GomVel98] or spatial domain using *Lifting method* [SweSch95]. The filter coefficients could be interchanged, so these could be used for analysis process such as synthesis process.

For decomposition process, expression 16, we use the *fast transform of wavelets* formulated in some publications, as [GomVel98], such

$$c_k^{j-1} = a_0 c_{2k}^j + \sum_{i=1}^r a_i (c_{2k-i}^j + c_{2k+i}^j) \quad \text{and} \quad d_k^{j-1} = b_0 c_{2k-1}^j + \sum_{i=1}^s b_i (c_{2k-i}^j + c_{2k+i}^j).$$

For the composition process, simplifying the expression (17) in linear operations such

$$c_{2k}^{j} = p_0 c_k^{j-1} + \sum_{i=1}^{v} p_{2i}(c_{k-i}^{j-1} + c_{k+i}^{j-1}) + \sum_{i=0}^{w} q_{2i-1}(d_{k-i}^{j-1} + d_{k+i+1}^{j-1})$$

and

$$c_{2k+1}^{j} = \sum_{i=0}^{v} p_{2i-1}(c_{k-i}^{j-1} + c_{k+i+1}^{j-1}) + q_0 d_{k-1}^{j-1} + \sum_{i=1}^{w} q_{2i}(d_{k-i+1}^{j-1} + d_{k+i+1}^{j-1}).$$

3 Direct manipulation of multiresolution curves

Given a B-spline curve of order k, we could modify interactively a segement of that by moving one of their point to target position. This action will permit the minimal movement of their k relative control points, that is evident in the equation (2). The k control points influence in $k+(k-1)$ segments, these also undergo some modification. The form to compute the minimal movement of k control points of B-splines curves was given by Bartels and Beaty [BarBea89], and Bartels and Fowler [FowBar93], and for tensor product surfaces by Fowler [Fowler92]. Deformation by using this techniques does not require refinement of the control points, the use of knot operations such insertion or remotion, and another tedious calculus.

Finally, the operation consists in computing the variations Δc_{j+i-1} of each control point $\{c_{j+i-1}\}_{i=0,\dots,k-1}$ relative to segment j of the curve, with the follow expression:

$$\Delta c_{j+i-1} = \Delta f_j(t) \frac{N_{j+i-1}^{k}(t)}{\sum_{l=0}^{k-1}(N_{j+l-1}^{k}(t))^2} \quad \text{for } i = 0, ..., k-1. \tag{18}$$

The new position of point c_{j+i-1} is given by

$$c'_{j+i-1} = c_{j+i-1} + \Delta c_{j+i-1}. \tag{19}$$

We noted that the basis functions $N_{j+i-1}^{k}(t)$ are the same of the expressions (3), so each variation of control point (Δc_{j+i-1}) is ease to compute. If the manipulation is to produce a local deformation, after actualizing the new positions of k control points relative to point $f_j(t)$, we recalculate all modified curves. But our purpose is to manipulate the curve controlling its segment size affected by a natural manner, taking advantages of the multiresolution curves.

3.1 Local deformation

In direct manipulation of curve or surface (object), the selected point of object is moved by shifting with the mouse to another position. The effect of this movement is immediate and could be seen in the vicinity of the point. When the control points of the object are very close one from the others, the effect of the manipulation does not look like natural deformation. This effect is a local deformation, when the manipulation of one point affects a small segment of objects that is influenced by the moviment of k control points.

In modelling, the user could desire to deform a piece of the object with size greater than the size of a local deformation, that will require a movement greater than k number of control points. There exist some approaches that solve this problem, such [ForBar88], that refine the object's control points hierarchically. To that end they use operations with knots, but this is not very intuitive for one modelling user.

The size of the segment affected by the manipulation of a select point of object could be controlled by using their multiresolution version. So, when the local deformation is applied in each low-resolution object, the movement of theirs relative control points will be the greatest, and it will affect large segment on refined object. With this focus, we could move more than k control points to obtain the real deformation different to the local deformation.

3.2 Local deformation effects in low-resolution curve

Here, we use the multiresolution curve edition idea, commented briefly in [FinSal94, KazElb97]. In order to make some modifications of curve's form using local deformation in multiresolution curve, we manipulate proportionally all low-resolution curves. These modifications will be propagated to the highest-resolution curve after the composition process.

One alteration of $f^j(t)$ in $\Delta f^j(t)$ affects the highest resolution curve ($f^n(t)$), in composition process, such

$$\tilde{f}^j(t) = f^j(t) + \Delta f^j(t)$$
$$\tilde{f}^{j+1}(t) = \tilde{f}^j(t) + o^j(t)$$
$$\dots$$
$$\tilde{f}^n(t) = \tilde{f}^{n-1}(t) + o^{n-1}(t).$$

The $\Delta f^j(t)$ represents the alteration made in some coefficient of C^j. The basis functions for spaces V^j and W^j are inalterable. So, in the composition process, the variations reflect in their high-level coefficients ($\tilde{C}^{j+1}$).

$$\text{If} \quad \tilde{C}^j = C^j + \Delta C^j \quad \text{then} \quad \tilde{C}^{j+1} = C^{j+1} + P^{j+1}\Delta C^j.$$

Suppose that still exists one variation in the detail coefficients in ΔD^j, then

$$\tilde{C}^{j+1} = C^{j+1} + P^{j+1}\Delta C^j + Q^{j+1}\Delta D^j. \tag{20}$$

The variations in this level propagates by projection on the higher levels coefficients, such follow mode:

$$\begin{aligned} \tilde{C}^{j+2} &= C^{j+2} + P^{j+2}(P^{j+1}\Delta C^j + Q^{j+1}\Delta D^j) \\ &\dots \\ \tilde{C}^{n} &= C^{n} + P^{n}...P^{j+3}P^{j+2}(P^{j+1}\Delta C^j + Q^{j+1}\Delta D^j). \end{aligned} \tag{21}$$

Finally, we observe that some variations of any point of curve of level j determinates the degree of deformation in the refined curve. The expression (21) shows that, when j is close to zero, the variation of the segment of curve is greatest, while if j is close to n the segment affected is least. So, the depth of level being manipulated determine the degree of deformation, that could be classified as a small or big deformation.

3.3 Manipulating in decomposition process

The degree of deformation of the curve f^n could be controlled propagating the displacement vector $\Delta\mathbf{d}$ of point $f^n(t_i)$, picked up by one of method as formulated in [RiCaVe99], for all low-resolution curves.

If $\Delta\mathbf{d}_j$ is the corresponding displacement applied to any point of f^j, the resulting curve will be $\tilde{f}^j = f^j + \Delta f^j$. The $\Delta\mathbf{d}_j$ is determined from $\Delta\mathbf{d}$ by a distribution function defined in section 5. The modified curve, $\tilde{f}^j$, affects theirs low and high-resolution levels. When the modification is made in decomposition process, the coefficients generate from this modified curve include these variations.

The method consists of, given the refined curve's control points C^n, we deform it locally, applying $\Delta\mathbf{d}_n$ to obtain $\tilde{C}^n = C^n + \Delta C^n$. These new elements will be different to the ones obtained by composition, because the lower level elements are also modified locally. In general, we could denote the scaling coefficients to be decomposed by $\tilde{C}^j_d$, and the composition result by $\tilde{C}^j_a$. After decomposing $\tilde{C}^j_d$ we obtain $\tilde{C}^{j-1}$ and $\tilde{D}^{j-1}$. We then apply local deformation over them to obtain $\tilde{C}^{j-1}_d$ and $\tilde{D}^{j-1}_d$. This process is applied recursively until level m (minimum level permitted for decomposition). When $\tilde{C}^j$ is modified by $\Delta\tilde{C}^j$, theirs detail orientations are updated in $\Delta\tilde{D}^j$. When $m = 2$, for example, the process should be as the following sequence:

Modification: $\tilde{C}^n_d = C^n + \Delta C^n$.
Decomposition: $\tilde{C}^{n-1} = C^{n-1} + A^n\Delta C^n$,
$\tilde{D}^{n-1} = D^{n-1} + B^n\Delta C^n$.
Modification: $\tilde{C}^{n-1}_d = C^{n-1} + A^n\Delta C^{n-1} + A^n\Delta C^n + \Delta(A^n\Delta A^n)$,
$\tilde{D}^{n-1}_d = D^{n-1} + B^n\Delta C^{n-1} + B^n\Delta C^n + \Delta(B^n\Delta C^n)$.

After the first stage, decompositions and modifications, we reconstruct recursively the curve. In this second stage, it is necessary, prior to each composition, to convert each detail coefficients from polar representation in local coordinate system to vector representation in global coordinate system, detailed in subsection 3.4. Having both elements expressed in the same system, we can then reconstruct the curve recursively as $\tilde{C}^{j+1}_a = P^{j+1}\tilde{C}^j_a + Q^{j+1}\tilde{D}^j_a$. Finally, we obtain the highest resolution coefficients as $\tilde{C}^n_a = C^n + \Delta C^n + \Delta_r C^n$, where $\Delta_r C^n$ is the projection on level n of all variations in lower levels. Considering $m = 2$, in the sequence of decomposition and deformation above, the composition would be as

$$\begin{aligned}\tilde{C}^n_a &= P^n\tilde{C}^{n-1}_d + Q^n\tilde{D}^{n-1}_d \\ &= C^n + \Delta C^n + P^n\Delta C^{n-1} + Q^n\Delta C^{n-1} + P^n\Delta(A^n\Delta C^n) + Q^n\Delta(B^n\Delta C^n)\end{aligned}$$

Finally, the modified curve would be as

$$\begin{aligned}\tilde{f}^n &= \Phi^n[C^n + \Delta C^n + P^n\Delta C^{n-1} + Q^n\Delta C^{n-1} + P^n\Delta(A^n\Delta C^n) + Q^n\Delta(B^n\Delta C^n)] \\ &= f^n + \Delta f^n + proj_n\Delta f^{n-1} + proj_n(\Delta proj_{n-1}\Delta f^n) \\ &= f^n + \Delta f^n + \Delta_r f^n,\end{aligned}$$

where Δf^n is the local variation of f^n plus the low variations projected in level n ($\Delta_r f^n$). The $proj_n\Delta f^{n-1}$ is the projection of Δf^{n-1} over resolution level n.

Figure 4 shows several degrees of manipulations of one point on the closed curve. The degree of modification of each segment depends of the distribution form of the displacement vector in each resolution level of the curve.

3.4 Orientation of detail

Since that each detail coefficient d_k^j of D^j is, in this case,composed by two elements $(\Delta x_k^j, \Delta y_k^j)$, it could be treated as a two-dimensional vector δ_k^j. For each δ_k^j it will be assigned a Local Coordinate System (LCS). So any operation with the scaling coefficients could produce some local operations with relative detail coefficients, such as local rotation.

When an object suffers a deformation, it is necessary to keep up with detail characteristics controling their orientations. In this way a real deformation effect is achieved, which would be difficult to obtain if the operation of rotation was executed in the Global Coordinate System (GCS) form. These principles were suggested in [StDeSa96, FinSal94, ForBar88, ThuWut97], each one with different purpose. So, the detail coefficients expressed in polar notation are the best way of updating the orientation when they are affected by the movement of theirs relative scaling coefficients.

One alternative for the representation of LCS for each δ_k^j is to use a similar method, formulated by Forsey and Bartels [ForBar88], considering the x-axis tangent to the curve of level $j-1$ in t_0, where the wavelets function $\psi_i^j(t_0)$ has the maximum value. In this work, we considered the axis of LCS of each element δ_k^{j-1}, such being the x-axis parallel to the edge k of the control polygon of f^{j-1} formed from point c_k^{j-1} to c_{k+1}^{j-1}. The y-axis is an extern vector orthogonal to the edge k.

The orientation of each detail, in polar notation, is calculated in each decomposition process, being stored in the same field used by the detail. The polar notation for detail δ_k^{j-1} is composed by the pair $(r_k^{j-1}, \theta_k^{j-1})$, where r_k^{j-1} is the module of δ_k^{j-1} and θ_k^{j-1} is the angle formed by δ_k^{j-1} with the x-axis of their LCS. The calculus are made by using one coordinate transform process (coordinates change). This process is linear relative to scaling coefficients number, so they do not increment the computational complexity of algorithmic.

4 Deformation distribution in segments of multiresolution curves

The variation Δf^{n-i} on each of the i low-levels curves is determined by the movement of k control points, in each level, relative to the highest level segment f_k^n. In this section, we relate the affected coefficients of level j to other ones of low-level $j-1$, to distribute the deformation in lower level curves, recursively.

A point on the segment f_k^n influenced by the relative control points $B_k^n = [c_{k-1}^n \, c_k^n \, c_{k+1}^n \, c_{k+2}^n]^T$, converges, just as the decomposition, to any point of segment f_h^{n-1}, where the indexes relation is $h = \lfloor k/2 \rfloor$. These criterion permit us to relate two adjacent segments, as even and odd positions, of the same level n with another segment of level $n-1$. All points of the segment f_h^{n-1} are influenced by their relative control points matrix $B_h^{n-1} = [c_{h-1}^{n-1} \, c_h^{n-1} \, c_{h+1}^{n-1} \, c_{h+2}^{n-1}]^T$. With these criterions is possible to establish the correspondence between segments and control points of multiresolution curves, recursively.

So, we have in figure 5, that the segment f_k^{j-1} with reference point c_k^{j-1}, is the convergence segment of the high-level adjacent ones f_{2k}^j and f_{2k+1}^j, whose reference points are c_{2k}^j and c_{2k+1}^j respectively. The relation between the control points of C^j and C^{j-1} was established in the section 2.2.2, where c_{2k}^j and their neighbourhood generate c_k^{j-1}.

To relate the detail coefficients of level j with level $j-1$, we could deduce the expression as the same form to one obtained for the scaling coefficients. Some action that affect in some point of segment f_{2k}^j or f_{2k+1}^j, is transmitted to one vicinity of d_k^{j-1}, it being the best representative. Such as presented in the detail coefficients calculation, d_k^{j-1} is related with the scaling coefficient c_{2k+1}^j (odd position). In general, f_k^j and d_h^{j-1} are related by the index expression $h = \lfloor (k+2)/2 \rfloor$.

How is determinated a point p_{h_m} on f_h^j which is the convergence from the point p_{k_i} of f_k^{j+1}?. The answer to this question could be found in the criterion considered in the segments propagation in multi-levels, explained above. As two adjacent segments of curve of level j converge to one segment of level $j-1$, we could deduce that the point $p_{h_m} = f_h^j(t_{h_m})$ and their symmetric vicinity converge to one point $p_{k_i} = f_k^{j-1}(t_{k_i})$ and their vicinity. Being both points representatives in the propagation from the level j to the low-level $j-1$, respectively.

The relation between the index h and k is given, as described in the propagation of segments, by $k = \lfloor h/2 \rfloor$. The parameter t_{h_m} is known, while t_{k_i} is calculated by analyzing the convergence relation of segments from level j to $j-1$. This is, we compute t_{k_i} using the propagation proportion, because two adjacent segments, even and odd, of f^j converge to another segment of f^{j-1}. So, the parameter t_{k_i} is calculated as

$$t_{k_i} = t_{h_m}/2 + (h/2 - k), \text{ with } k = \lfloor h/2 \rfloor,$$

because the parameter t varies as $0 \leq t_{k_i} < 1$ in all segments.

5 Distribution function for deformation

A vector $\Delta \mathbf{d}$, which indicate the displacement of one point of the curve from their initial position to the target position, can be distributed in determined proportions between the m curve's low-levels. The proportion of displacement is determined by a distribution function, that could be called also as the filter function of distribution, because it atuates such a filter to assign a piece of displacement to one level of curve. So, we have limited in $[0,1]$ the domain for the filter function, where the area defined by the integral of the function is 1, representing the amount of the proportion filters.

Let $g(t)$ be the distribution function, such that

$$\int_0^1 g(t)dt = 1.$$

Then, the function $g(t)$ is a normalized function $h(t)$:

$$g(t) = \frac{h(t)}{\int_0^1 h(t)dt}.$$

In order to propagate the total variation of the highest resolution curve between their low-resolutions, is logical to think that the highest resolution level has local modification in greater proportion than the lowest resolution level. Then, the function $h(t)$ has negative exponential form as

$$h(t) = e^{-kt},$$

where the constant k is the weakness degree of the function $g(t)$, that could be considered as the stiffness of the curve f^n. Then, the function $g(t)$, with these restriction, will be

$$g(t) = \frac{ke^{-kt}}{1 - e^{-k}}, \tag{22}$$

because

$$\int_0^1 h(t)dt = \int_0^1 e^{-kt}dt = \frac{1 - e^{-k}}{k}.$$

We devide amount area into m vertical subareas of the same base size, for that the domain interval $[0,1]$ of $g(t)$ is subdivided into m equal subintervals $\{t_0, ..., t_i, t_{i+1}, ..., t_m\}$, as shows figure 6. So, each vertical subarea a_i is calculed as

$$a_i = \int_{t_i}^{t_{i+1}} g(t)dt.$$

Each a_i corresponds to one level of integer resolution of curve (exist techniques for editing not integer level curves [TaShKu98]). With these criterion, we consider that

a_0 will be the portion corresponding to the resolution curve of level n, a_1 corresponds to the low-resolution of level $n-1$, so successively until a_{m-1} that corresponds to the resolution curve of level $n-m+1$.

The displacement applied in the level $n-i$ will be given by $\mathbf{\Delta d_i} = a_i \mathbf{\Delta d}$.

6 Conclusions and future works

Continuos B-splines basis were used to modelling curves in order to describe the technique of manipulation of multiresolution curves. This technique could be used for manipulating other types of curves and surfaces, because it is possible to represent these curves and surfaces in multiresolution, and it is possible to apply the direct manipulation on them.

In general, the distribution form of the control points determinates the complex characteristic of the object (curves or surfaces). So, if the control points are distributed with some density, the deformation of a object's region generates visually a real effect, when it is made with direct manipulation propagating on their low-resolution levels, because the size on that region deformed depends of the number of control points of the object. With these deformations, an object could adopt the complex characteristic wanted by the modeler.

This method conserves the initial number of control points, because does not use any operation of knots points. So, the same spaces used to store the control points are used in the manipulation of object in multiresolution. It isn't necessary to use some additional space beyond the used by original control points, because one advantage of the wavelet transform is the conservation of the space used in all decomposition and composition process.

The extern agent, which in this case is action with the mouse, could be a force for automatic deformation, comprises the initial point of the application, the module and the orientation, to produce a controlled real deformation varying the object affected segment size. The detail coefficients of the multiresolution object have essential participation to manipulate the object in a natural manner, because these details conserve the orientation of the object characteristic which must preserved after each alteration of low-resolution object.

The technique described in this work, could be generalized to automatic deformation of object, considering the extern agent as a force that could be generated as consequence of collision between objects, as the application proposed in [HsHuKa92], to produce semi-phisic effects.

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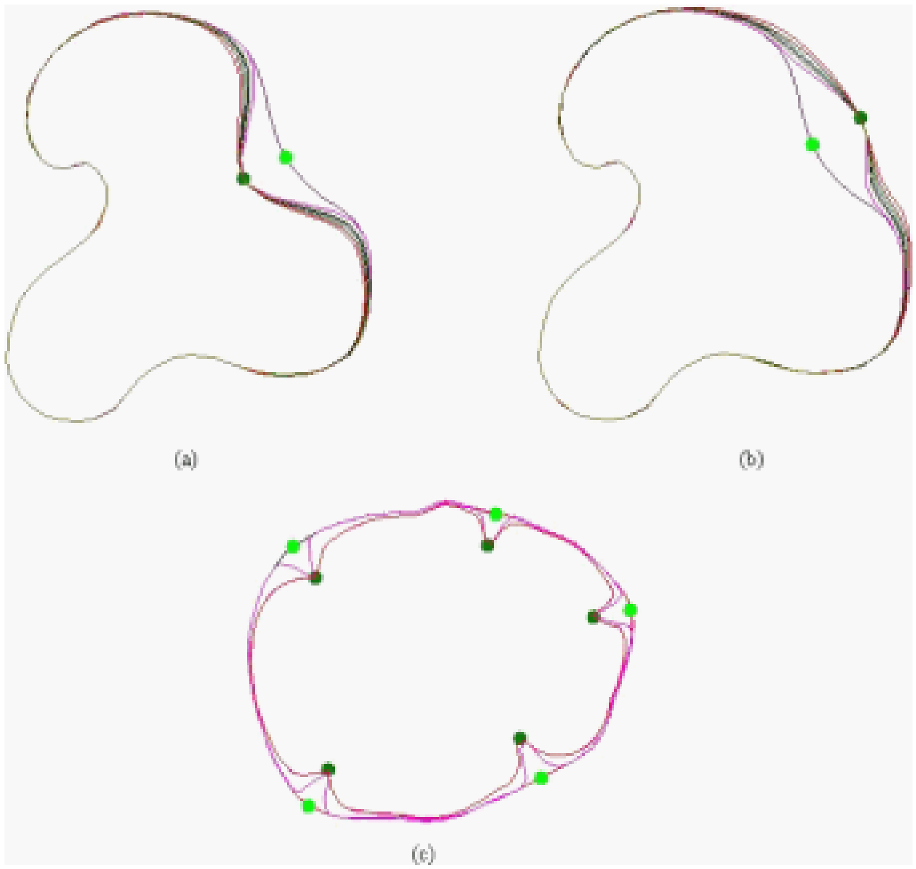

Figure 4: A closed curve deformed in several levels of resolution in different degree. (a) distribution of deformation from exterior to interior, the lower level with coefficient $k = 1$; (b) distribution of deformation from interior to exterior, the highest exterior level with coefficient $k = 1$; (c) deformation of curve with five types of distribution: left superior $k = 1$, left inferior $k = 2$, next others, in these direction, $k = 3$, $k = 4$ and $k = 5$ respectively.

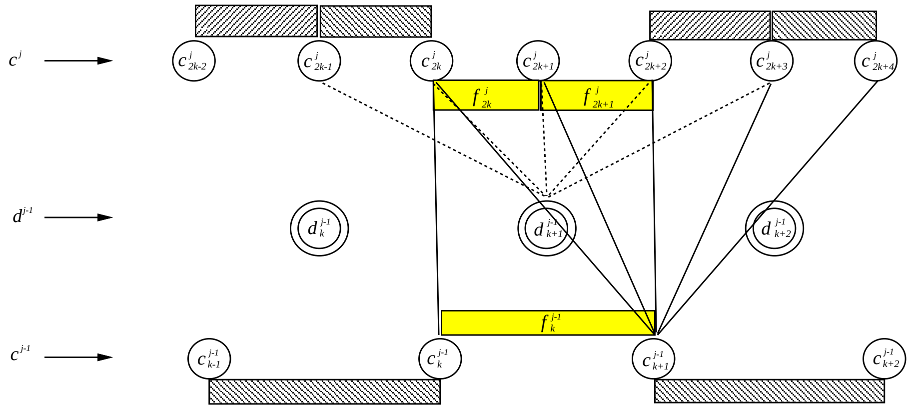

Figure 5: Convergence of segments of curve from resolution j to resolution $j-1$, and the relation of their coefficients.

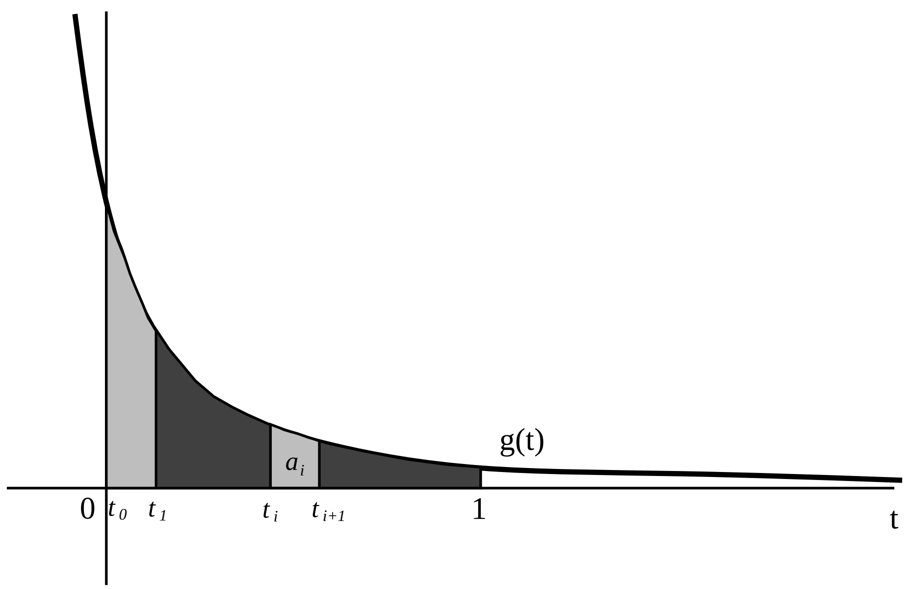

Figure 6: Distribution function of sliding vector in low-levels of resolution curves.