Towards A Unified Framework For Geographical Data Models

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Abstract. This paper describes a unified framework for the problems of modelling and processing geographical entities. We propose a general definition of *geographical objects*, and show that the different types of geographical data can be expressed as particular cases of this definition. Furthermore, we present a taxonomy for the various types of GIS operations, defined in terms of the properties of this definition. Our goal is to argue that GIS data types and operations can be defined based on a single formal notion, which encapsulates the GIS concepts of both fields and objects, with important consequences for system and interface design, interoperabilty issues and language proposal.

1 Introduction

The literature on GIS usually considers two broad classes of models of geographic information: *field-based* and *object-based* (Coucelis, 1992). The *field model* represents the geographical data as a set of spatial distributions over the geographical space. The *object model* represents the world as a surface occupied by discrete, identifiable entities, with a geometrical representation and descriptive attributes.

Some authors have already pointed out that the *field* and *object* views have their roots in different cognitive perceptions of the geographical space and that they are extreme, idealised notions from a more complex taxonomy (Burrough and Frank, 1995; Couclelis, 1996).

Notwithstanding the importance of the conceptual debate, it is important to consider the challenges of representing these concepts in a computer environment. We consider that the computer-based modelling of geographical reality requires a level of formalization and generalization (either explicit or implicit), which is necessary to define precisely the nature of operations to be performed in a GIS.

In this perspective, we propose a general formal definition of *geographical objects*, and show that the different types of geographical data can be expressed as particular cases of this definition. Furthermore, we present a taxonomy for the various types of GIS operations, in terms of properties of this definition. Our goal is to show that GIS data types and operations can be defined based on a single formal notion, with important consequences for system and interface design, interoperabilty issues and language proposal.

One of the most relevant aspects of this definition is its impact on the questions of interoperability. It has been recognized that interoperability in GIS requires a level of semantic modelling to account for the correspondence of concepts between different systems (Câmara et al, 1999). By establishing a formal notion and referring the concepts presented in GIS literature to this definition, we could provide a basis for a unified semantic framework for GIS.

The paper is organized as follows. Section 2 introduces the abstraction paradigm used as a basis for our concepts. Section 3 proposes a general definition for geographical objects, discusses the representation of these objects and gives a classification. Section 4 shows how traditional GIS concepts are related with the definition of a geographical object. Section 5 summarizes the typical operations on geographical data, as seen in the perspective of our framework. Section 6 outlines some consequences of this approach, both from a theoretical and a practical point of view.

2 Abstraction Paradigms For Geographical Modelling

Geographical data modelling for GIS can be viewed as a special case of computational modelling of physical phenomena. In computational modelling, a very important and necessary step is to establish mathematical models, which create the abstract descriptions of the real-world entities of interest. In a classical paper in the area of geometric modelling, Requicha (1980) established a

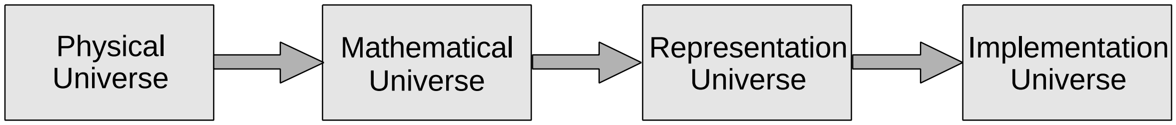


Figure 1 – Abstraction levels in Computer Modeling (from Gomes and Velho, 1995)

conceptual framework which distinguishes between the physical, mathematical and representational levels of abstraction. His work was further extended by Gomes and Velho (1995) which propose the "four universes paradigm" as a general modeling mechanism for applied computational mathematics in general, and not only for geometric modelling. The four abstraction levels are described as (See Figure 1):

1. The *physical* universe, which comprises the real-world entities that will be modelled in the computer.

2. The *mathematical* universe, which includes a formal definition of the entities which are included in the model.

3. The *representation* universe, which defines how the various continuous models are discretized.

4. The *implementation* universe, where the data structures are associated to the discretized objects of the representation universe.

These abstraction levels have been sucessfully used in different areas of computer graphics. An extensive use of the paradigm is found in (Gomes et alli. 1998), where it has enabled the formulation of a robust conceptual approach both for theoretical issues and implementation, related with the subject of warping and morphing.

Our experience has shown that the use of the four universes paradigm for geographic data, is also particularly suited: At the physical unverse level, we find elements such as *parcels, rivers and soil maps*; at the mathematical level, we define *geographical objects*, and specialise this notion into classes of geographical data which encompasses traditional concepts from GIS such as *fields* and *objects;* at this level, we distinguish between *raster* and *vector* representations, which may be further specialised, such as *grids, TINs, image* structures for raster and *arc-node* and *arc-node-polygon* structures; at the implementation level the actual coding takes place and we find data structures such as *R-trees* and *quad-trees.*

One of the important consequences of the multiple levels of abstraction paradigm is to make an explicit reference to design decisions which are very often implicitly and informally defined. For example, the use of raster data structures to store terrain models in a computer implies a mathematical formulation of what is a terrain. In this approach, we are compelled to define the elements of each universe and to establish the relation between the elements of each universe.

3 Geographical Objects

Using the abstraction paradigm introduced in the previous section, the most important issue is concerned in characterizing the mathematical entities which will describe the elements from the physical world. In the case of geographical data, this leads us to the concept of a *geographical object* (*geo-object*, or simply *go*) as the basic element of the mathematical universe. A geo-object is a triple $go = (S, A, f)$ where:

1. $S \subset \Re^2$ is a subset of the Euclidean plane, it is the *geometrical support*.
2. A is a set of *attribute domains* $A_1,..., A_n$,
3. $f: S \rightarrow A_1 x A_2 x ... x A_n$ is the *attribute function* of the geo-object, which associates, to each location in the support, a value on the set of attribute domains.

Note that this definition mimics that of a graphical object introduced in (Gomes et alli., 1996). It caters the different subtypes of geographical data. We should remark that this definition, in accordance to our paradigm, is completely generic and is not bound to any particular representation.

A geo-object is *geo-referenced* if there exists a parameterization g from the geometrical support S to the surface of the earh. Mathematically the map g can be described approximately by a parameterization from S to the surface of a sphere. Different parameterizations are possible related with the various map projections of the sphere.

3.1 Representation of Geographical Objects

Once we have a geo-object the first step twards implementation is to obtain its representation. The representation of a geo-object consists in discretising both its geometric support and the attribute function. The representation of the geometric support consists in representing its topology and geometry. This topic is extensively studied in the area of geometric modelling. In GIS it is used mainly the representation techniques based on decomposition. These techniques employ a top-down methodology where the geometric support is decomposed into simpler geometrical objects which are easier to represent.

A very important geographical data is a terrain. As a geo-object a terrain is a real valued function $f: S \rightarrow R$, where S is a subset of the euclidean plane. Two commonly used representations for terrains are the *TIN* (Triangulated Irregular Network) and the *grid*. A TIN representation subdivides the geometrical support S of the terrain into triangles in such a way to form a triangulation (intersecting triangles should share a vertex or edge). A grid representation subdivides the geometrical support S intro small rectangles so as to construct a lattice. Figures 2(a) and 2(b) show, respectively, a TIN and a grid representation of a terrain.

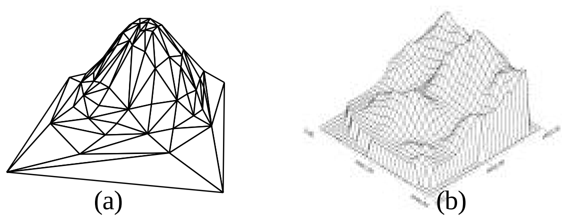


Figure 2 – TIN (a) and Grid representation (b) of

If we are interested only in the geometry and topology of the terrain it is enough to use a TIN or a grid representation. Nevertheless, in order to obtain a complete representation of the geo-object we must represent the attribute function on the TIN or on the grid. This is generally attained by sampling the function at each element of the representation. On a TIN samples are generally taken at the vertices of the triangles; on a grid samples can be taken either at the vertices of the grid or at the center of each rectangular cell.

Note that because the grid decomposition can be easily structured into a rectangular lattice, this representation is completely characterized by giving the number *mxn* of decomposition cells (rectangles) and the value of the attribute function in each cell. The structuring is given by the natural row/collumns ordering of the lattice. The grid representation of a geo-object is called a *matrix* or *raster* representation. The order *mxn* of the decomposition matrix is called the spatial resolution of the representation.

To conclude the representation of the geo-object the attribute function should assume only a finite number of attribute values (this corresponds to the use of a specified number of bits to represent the attribute values). The process of discretizing the attribute function is called *quantization* (this comes from the usage of the term in the area of image processing). Each of the values assumed by the function is called a *quantization level*. The number of quantization levels to be used depends on the nature of the geographic data being represented.

We should remark that if the attribute function is quantized to n levels, a_1, ..., a_n, then it determines a partition of the geometric support of the geo-object into a collection of disjoint sets S_i, 1=1, ..., n, such that the attribute f at each set S_i is constant and equal to a_i. This is illustrated in the Figure 3 below.

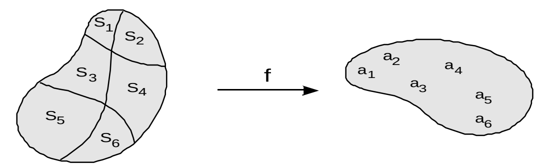


Figure 3 – Attribute function with n values and associated partition

An important issue when representing geo-objects consists in reconstructing the object from its discrete representation. We will not discuss this topic in this article.

3.2 Classification of Geographical Objects

Geo-objects are classified according to the topology of the support, and the values of the attribute function. Four different classes are possible: simple, composite, homogeneous and non-homogeneous geo-objects**.**

A geo-object is called *simple* if its support S is a connected region in $\Re^2$. Otherwise it is called a *composite* object. A geo-object is called *homogeneous* if its attribute function assumes a constant value $f(s) = (a_1,\ldots,a_n)$, $\forall\, s \in S$. Otherwise, the geo-object is called *non-homogeneous*. Some examples will be given below.

3.2.1 Simple Geo-Objects

Figure 4 shows an example of a simple geographic object. The object describes a country named "Brazil". The geometric support S is a connected region, and it has two constant attribute values: "name" and "population".

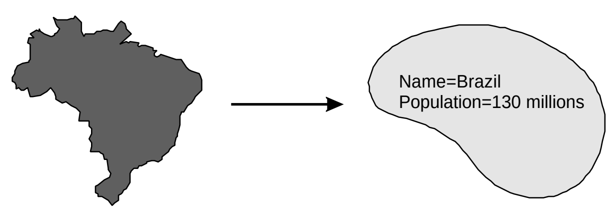
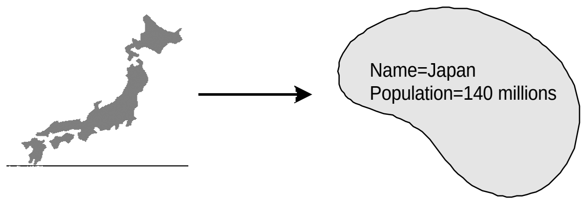


Figure 4 - A simple and homogeneous geo-object.

In practice, the notion of a *single, homogeneous geo-object* is too simplistic and we need to use the more complex classes of geo-objects to characterize the geographical entitities.

3.2.2 Composite homogeneous Geo-object

In this case, we are dealing with a geographical object where the geometric support S has several connected components and the attribute function assumes a constant value. Figure 5 shows an example, which describes the country named “Japan”, where S is a set with four connected components (representing the main islands of the japanese archipelago), and the attribute domains A_1 and A_2 are “name” and “population”



.Figure 5 - Example Of A Composite Homogeneous Object.

3.2.3 Simple Non-homogeneous Geo-object

For these object types, the support S is composed of a connected region, but the attribute function f varies for each point in the region. Depending on the number of levels used in the representation of the attribute function, the geometric support is partitioned into a finite number of sets. This concept corresponds to the notion of fields (Goodchild, 1992), such as vegetation maps and topography. Figure 6 shows an example of a simple non-homogeneous object (a vegetation map). Note the partitioning of the geometric support as we described in Figure 3.

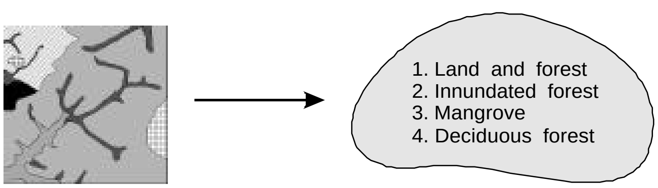


Figure 6 - Example of a simple and non-homogenous object.

3.2.4 Composite non-homogeneous Geo-object

In this case, the geometrical support S is composed of several connected components, and the attribute function is not constant over S. This definition, in its generality, has no direct counterpart in the traditional GIS elements. However, there is a very important special case, namely:

- The geometric support is a disjoint collection of connected regions, $S = \cup S_i$, $S_i \cap S_i = \varnothing$ $(i \neq j)$;
- The attribute function f is constant on each set S_i , and the constant values assumed are distinct, in general

This special type of composite non-homogeneous geo-object constitues a formal definition of a *coverage* in the ARC/INFO™ system (ESRI, 1994).

Figure 7 shows an example of an ARC/INFO™ coverage, which corresponds to a map of the South Asia region, composed of different islands. The map is dealt by the system as a single object, whose components (the geometrical supports associated to each island) are mapped to different values of the attribute set.

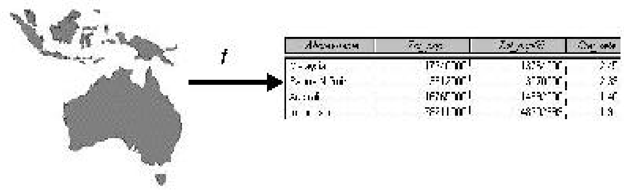


Figure 7 Example of a composite, non-homogeneous object.

4 Correspondence Of Definition To Gis Literature

In this section, we will attempt to relate the concepts introduced above with the traditional concepts in the GIS literature, especially those used by the OpenGIS® consortium (OpenGIS, 1998a; OpenGIS, 1998b). Since the latter definitions are mostly informal, an exact matching is not always possible, but the approximate correspondences are already illustrative of the hazards of semantic model conversion between systems.

The OpenGIS® model is based on an abstract class (*feature*) which has two specialisations: *feature with geometry* and *coverage.*

The definition of *feature with geometry* allows for complex geometrical representations to be associated to the same feature and for different features to share the same geometrical representation. The notion of *feature with geometry* in OpenGIS® (OpenGIS, 1998a) corresponds roughly to our definition of a composed, homogeneous object (Section 3.2.2) or a single non-homogeneous geo-object (Section 3.2.3).

An OpenGIS® *coverage* is an association between a geometric description of entitities and a set of attributes; this association is defined by a coverage function (or *c_function*) *f:(geometry)®(attribute set).* The OpenGIS® proponents chose not to use a representation-independent definition for *coverage*, and to define it in terms of specific geometrical representations, such as a *grid coverage*, *TIN coverage* or *geometry coverage* (OGC, 1998b).

Most of the OpenGIS® specialisations of coverages, such as *sample, grid, TIN and image* can be considered as computer representations of the concept of a *simple non-*

homogenous object (Section 3.2.3), using specific data structures. However, their definition *of geometry coverage* can be considered as a special case of a *composite non-homogeneous object* (section 3.2.4).

Thus, we can observe that the definitions of *coverage* on the OpenGIS are based on different formal concepts. In other words, a *grid coverage* in OpenGIS does not have the same formal basis as a *geometry coverage.* This situation may lead to potential problems in understanding and using this concept as a basis for interoperability. In fact, in a previous work (Câmara et al, 1999), we have argued against the choice of the OpenGIS consortium of using industry terminology, such as *feature* and *coverage*, which is already content-rich and are associated by the users with existing semantic concepts (OGC, 1998a; OGC, 1998b).

Table 1 provides a resume of the relation between our definitions and established industry notions.

TABLE 1 - CORRESPONDENCE OF DEFINITIONS

	Connected geometrical support	Non-Connected geometrical support
Homogeneous		*Simple feature (OpenGIS)*
Non-homogeneous	*Tin, Grid, Image and SurfaceCoverage (OpenGIS)*	*Coverage (ARC/INFO), Feature Collection Coverage (OpenGIS)*

5 OPERATIONS ON GEOGRAPHICAL DATA

One of the main purposes of a single formal definition for geographical objects is to provide a unfied perspective on the semantics of GIS operations. In current practice, the use of different models for fields and object usually leads to implementation of different subsystems on a GIS: raster map operations implementing Tomlin's Map Algebra (Tomlin, 1990) and vector spatial queries with languages similar to Spatial SQL (Engenhofer, 1994). This situation is not always desirable. For example, "overlap" is a well-known concept for denoting a topological configuration between two geographical entitities (Egenhofer et al., 1994). In many GIS, it is possible to inquire if two entitities represented by vector polygons "overlap", but it is not straightforward to inquire if the same property holds for two digital terrain models.

5.1 Atomic Operations on Geographical Objects

A general GIS should have basic operations which are generally applicable to different types of geographical entities, including both *field* and *object* models. Therefore, we have chosen to characterize what we propose to be a minimal set of atomic operations, applicable to different types of geographical objects (described in section 3). Upon this minimal set, more complex operations can be applied, eventually leading to more specific operation classes such as Map Algebra. In this section, we propose such a minimal set and in the next one, indicate how these atomic operation can be used as building blocks of more complex ones.

We consider three classes of operations with geo-objects: Attribute based, Spatial based and Creation-Delete operations. We will describe these operations below.

In keeping to our paradigm, which distinguishes between the mathematical and the representation universes, our definition of operations on geographical objects does not consider representation issues. These issues are considered to be implementation-dependent. For example, the pratical implementation of a mathematical operation between geo-objects may require their conversion from a vector to a raster representation. Ideally, this conversion should be performed automatically by the system (under certain rules). The fact that most commercial systems require the user to request an explicit vector-to-raster conversion only serves as indication that GIS technology would benefit from a formal basis, much as the database tehcnology was improved by Codd's relational model (Codd, 1962).

Using a formal definition as a basis, we can distinguish which operations are essential part of the properties of geographical objects and those which are constrained by representation-based issues.

5.1.1 Attribute Based Operations.

Two basic operations are proposed: VALUE (λ, A_i), which returns the value of attibute A_i at location l (this operation is actually the value of the attribute functuion for attribute A_i), and ASSIGN (λ, A_i, a), which assigns a value a for attibute A_i at location λ. For shorthand notation, we indicate these operations as $f(\lambda, A_i)$ and α (λ, A_i, a)¸ respectively.

5.1.2 Spatial based Operations

Spatial based operations can be futher classified based on a spatial predicate denoted by x. As an example we have the following cases:

5.1.2.1 Topological restrictions

Given a pair of geo-objects go_i and go_j, and a topological predicate θ, these operations return a boolean value ({true, false}) based on the result of the application of the topological predicate $\theta(go_i, go_j)$. The topological predicates proposed are EQUAL, DISJOINT, INTERSECT, TOUCH, CROSS, OVERLAP, CONTAINS, WITHIN, and

RELATE, as defined by the OpenGIS consortium (OGIS, 1998).

5.1.2.2 Distance-based operations

Given a pair of geo-objects go_i and go_j, the DISTANCE(go_i, go_j) operation returns the distance measure between these two objects. For shorthand notation, we indicate this operation as dist(go_i, go_j). We can also consider a boolean predicate $\delta(go_i, go_j, d)$ which returns true or false depending whether the distance between go_i and go_j is smaller than a given value d.

5.1.2.3 Direction-based operations

Direction relationships between objects can be qualitatively described using the notion ofcardinal directions (Frank, 1991). The object support is abstracted using its minimum bounding rectangle, and the direction relationships are expressed through a 3x3 matrix representing true or false values for NORTHWEST, NORTH, NORTHEAST, WEST, CENTER, EAST, SOUTHWEST, SOUTH, and SOUTHEAST relations. Given a pair of geo-objects goi and goj, the DIRECTION(goi, goj) operation returns the cardinal direction matrix between these two objects. We can also consider a boolean predicate PATH(goi, goj, dir_rel) which returns true or false depending on the direction relation dir_rel value between goi and goj

5.1.3 Combined Spatial and Attribute Based Operations

Given a pair of geo-objects goi and goj, this class of operation return a value which is calculated on the attribute values of goi, based on the geometrical support of goj. We call this types of operation REGION, and consider specialisations such as REGION_MAX (Ai,goi, goj) and REGION_AVE(Ai,goi, goj), which compute, respectively, the maximum and the average value of the atttibute Ai of the geo-object goi, constrained by the geometrical support of the geo-object goj. Figure 8 shows an example of a "region" operation, where the attribute of first geo-object is a numerical value, and we compute the maximum value of the region indicated by S2.

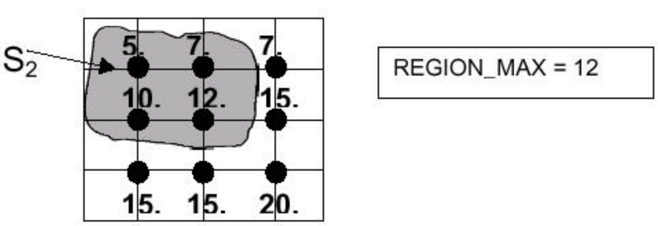


Figure 8 – Example of a region operation.

Again, it should be stressed that some operations need a conversion between representations of geo-objects (such as the conversion from a grid to a TIN). In this case the operation depends on the conversion technique used.

5.2 Composite Operations on Geographical Objects

In this section, we indicate how the usual operations of spatial queries and map algebra can be expressed in terms of the atomic operations proposed. We will consider two such types of operations: spatial selection and point operations (which usually are implemented in separate systems).

> *Definition 1. Spatial Selection.*
>
> The *spatial selection* operation can be defined as follows. Given a set of geo-objects $GO = \{go_1, .., go_n\}$, a reference geo-object go^*, the *spatial selection* operation $j : GO ® GO$, given a spatial predicate *x* which relates the geo-objects $go \ e \ GO$ to go^* is defined by:
>
> $j_x(GO) = \{ go \ e \ GO \mid x((go, go^*)) \}.$

The *spatial selection* operator is such that the output is a subset of the original set, composed of all geo-objects that satisfy the geometrical predicate, as in the example: "select all regions of France which are adjacent to the Midi-Pyrinees regions (which contains the city of Toulouse)", illustrated in Figure 9.

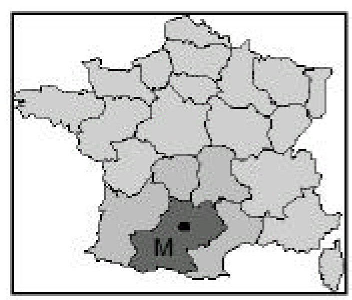


Figure 9 – Example of spatial selection operation

> *Definition 2. Spatial Join.*
>
> Let GO_1 and GO_2 be two sets of geo-objects. Let ξ be a spatial predicate computable for every pair of objects (go_1, go_2), where $go_1 \ \varepsilon \ GO_1$ and $go_2 \ \varepsilon \ GO_2$. The spatial join operation θ: $GO_1 \ x \ GO_2 \to GO_1 \ x \ GO_2$ is such that:
>
> $q_x (GO_1,GO_2) = \{ (go_1, go_2) \ e \ (GO_1, GO_2) \mid x (go_1, go_2) \}.$

Spatial join is an operation where a comparison between two sets of geo-objects GO1 and *GO2* takes place, based on a spatial predicate which is computed over the geometrical support of these sets. The name “spatial join” is employed by analogy to the join operation in relational algebra. The result of the spatial join operation is a *set of object-pairs*, which satisfy the spatial restriction. One example of spatial join would be: “Find all native reservations located closer than 50 km to the main roads in Amazonia”. The answer is a set of pairs of geo-objects (reservation, road). We can also consider operations where an ouput is generated, based on one or more inputs.

> *Definition 5. Point operations.*
>
> Let $go_1, go_2, .. go_n$ be geo-objects used as input, go_{n+1} be the output geo-object, $A^1, A^2, \ldots, A^{n+1}$ be the attribute sets associated to these geo-objects and S_o be the geometric support of the output. Let A_i^k be the i-th attribute of the k-th geo-object. The *point operation* Π: $S_o \times A_i^1 \times A_j^2 \times A_l^n \rightarrow A_i^{n+1}$ induces a function π such that:
>
> $f_{n+1}(p) = \pi (f_1(\lambda, A_i^1), ..., f_n(\lambda, A_l^n))$, " p e S.

For point operations, the value of the output attribute at each location is a function only of the input attribute values at the corresponding location. One example would be the boolean operations: “Calculate a soil aptitude map based on climate, soil, and slope maps, where the conditions are such that a soil is deemed “*good for agriculture*” if it rains more that 1000 m/yearand the soil has a *ph* between 6.5 and 7.5, and the slope is less than 15%”. This operation can be easily defined, based on the proposed atomic operations VALUE and ASSIGN (defined in section 5.1.1).

> *Definition 4. Zonal operations.*
>
> The zonal operation Z Z over two geo-objects $go1$, $go2$, whose geometrical supports are $S1$ and $S2$¸ given an attribute $Ai\ 1$ (the i-th attribute of $go1$), and a function υ defined over the range of $Ai\ 1$, is such that:
>
> $Z\ Z(go1, go2) = u\ (f1\ (p))$, " p e $S2$.

For the zonal operation, one geo-object is used as a spatial restriction on the operators on another. Examples which use zonal operations would be: “Given an slope map and a soils map, find the average slope for each soil area on the map” or "Given the counties of Brazil and a terrain map of the whole country, find the maximum altitude for each county". These operations can be implemented, given the REGION_MAX, VALUE, and ASSIGN operators given earlier.

Other operations on geo-objects can be defined in a similar fashion. Note that the above definitions make no explicit assumption on the specific type of geo-object considered. In current systems, the first two operations are which usually implemented in a separate sub-system than the third one. As we have argued, there is no compelling reason that this should be the case, and that the atomic operations proposed in section 5.1 should be general enough to serve as a basis for an interoperable language for geographical data.

6 CONCLUSIONS AND FUTURE WORK

In this paper, we have presented a generic definition for geographical objects. From this definition, we obtained different types of geographical entities and formulated a minimal set of general GIS operations. We have shown that this minimal set can be used to build more complex operations, equivalent to usual definitions of spatial selections and map algebra, but which are not constrained to any particular model of geographical data (such as the field or object models).

We point out that a unified formal definition of geographical objects does not, in any respect, diminish the relevance of the conceptual debate at the semantic level. The field and object views are, in fact, based on deeply rooted notions in human perception of his environment. Our primary aim was to indicate that the computerized modelling of the geographical data is necessarily a reductionist view, leading to formal notions which provides a basis for unified semantics of GIS operations.

We should point out that the concept of a geographical object introduced allows for new categories of geo-objects whose geometric support are not defined as a subset of the plane. This makes it very suitable to define, for instance, a concept of a volumetric geographical object which seems to be quite adequate to bring volumetric visualization techniques into the realm of GIS systems. More generally, GIS systems will incorporate more and more multimidia techniques. The concept of geographical object is ready to absorb this tendency. We intend to discuss these issues in our future work.

7 ACKNOWLEDGEMENTS

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