On 4–8 and Quasi 4–8 Meshes

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Extended Abstract

A four-directional mesh is one of the classical tessellation schemes. It provides a powerful structure for decomposition of two dimensional domains with desirable topological and geometric properties. This type of mesh is planar and regular. Vertices belong to two interleaved *quincunx lattices* and edges correspond the horizontal, vertical and the two diagonal directions. Such structure possess a rich set of internal symmetries. In fact, it is one of the 17 basic two-dimensional crystallographic groups, formed by reflections, two distinct sets of 4-fold rotations and one set of 2-fold rotations (indicated by *442 in the orbifold notation).

This mesh is closely related with the 4-direction box splines that are generated from the set of vectors $\{e_1, e_2, e_1 + e_2, e_1 - e_2\}$, where $e_1 = (1, 0)$ and $e_2 = (0, 1)$. As a consequence, four-directional meshes provide a natural triangulation for the parameter space of 4-direction box spline surfaces. The quincunx lattice can also be used to define other non-separable basis of $L^2(\mathbf{R}^2)$, such as scaling functions and wavelets. This makes four-directional meshes important for image processing and compression.

The four-directional mesh is particularly suited to hierarchical constructions based on simple templates. Mesh refinement and simplification are defined by split and merge operations along two alternating orthogonal directions. The result is an interleaved dependency graph that provides a topologically consistent variable resolution representation. The refinement of four-directional meshes combined with smoothing rules for box splines leads to a recursive scheme to construct regular subdivision surfaces. In addition, hierarchical four-directional meshes make possible to produce adapted triangulations. This capability has been exploited for progressive rendering as well as for visualization of terrain models with variable levels of detail. In this context, the sequence of refined meshes is known as *hierarchy of right triangles*.

Despite of all the nice properties listed above, four directional meshes have a significant limitations due to their regular structure. They can only be used on two dimensional domains that are topologically equivalent to the plane. In particular, they cannot be defined on general 2-manifolds, such as surfaces of arbitrary genus.

In order to generalize four directional meshes to 2-manifolds we define 4–8 meshes. A *4–8 mesh* is a triangular mesh which has only vertices of valence 4 and 8. A *regular 4–8 mesh* is a homogeneous simplicial complex in which the 1-neighborhood of every internal vertex of valence 4 has only neighbors of valence 8, and the 1-neighborhood of every internal vertex of valence 8 consists of a ring of vertices with alternating valences 4 and 8. A *quasi 4–8 mesh* is a simplicial complex which has mostly vertices of valence 4 and 8, except for isolated vertices with some other valence.

To construct hierarchical 4–8 meshes we have developed several schemes for refinement, simplification, extraction and queries. Refinement is based on sequences of triangle bisections along their longest edges. Simplification is based on edge swap and clustering. Extraction of variable resolution and progressive triangulations, as well as queries, are performed by recursive traversal of the cluster dependency graph.

In conclusion, general 4–8 mesh structures constitute the foundation of a valuable framework for representation of 2-manifolds. It can be exploited to build a flexible, effective and efficient computational environment with many applications in modeling and visualization.