Motion cyclification using time-frequency warping

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- Home page
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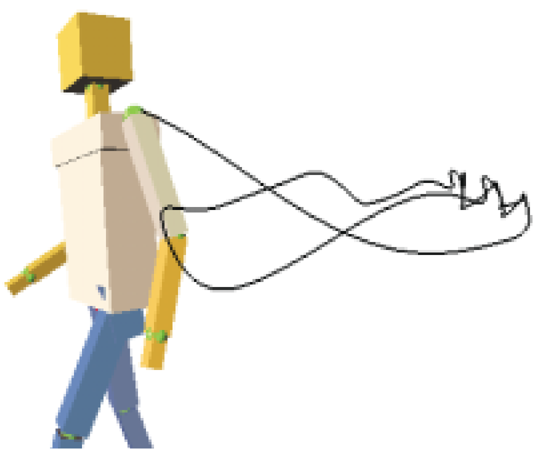
Motion editing

- Modification and reuse of animation parameters
- Examples
  1. Kinematic and dynamic parameters
  2. Motion capture data
- Our problem
  - Change the duration of an existing motion
  - Time warping

Motion processing

- Motion paths: positional and rotational values
- Motion capture: sampling at joints of a real subject
- Strategy
  - Signal processing techniques for vectorial signals $f\colon \mathbb{R} \to \mathbb{R}^n$



Motion processing

- Current techniques

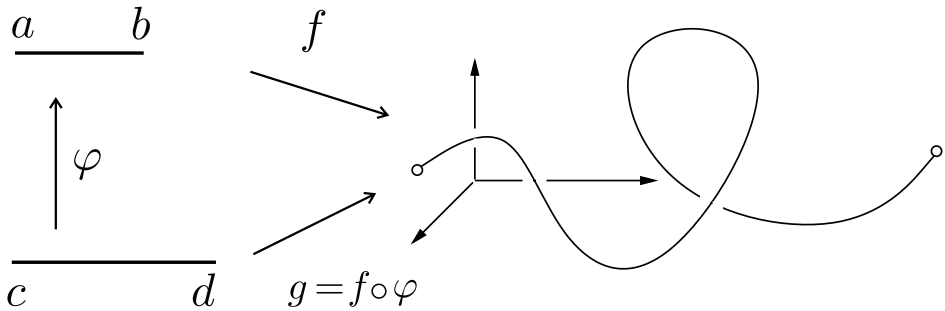
  1. Filtering
  2. Motion warping
  3. Motion morphing (blending)
  4. Time warping
     - Expand or contract the duration of the motion
     - Linear (affine) or non-linear warping.

- Our goal

  - Develop an automatic time warping technique for motion paths.

Time warping

- Reparameterization



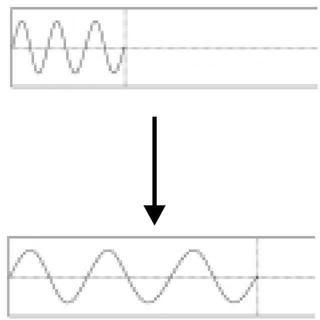
- Time dilation: $\varphi = t/\alpha$.

$$g(t) = f(\frac{t}{\alpha})$$

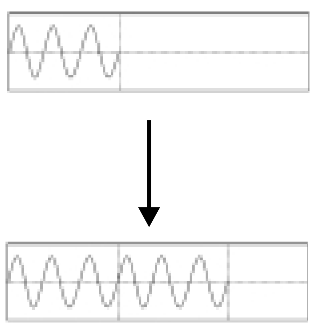
  - $\alpha > 1$ time is expanded
  - $\alpha < 1$ time is contracted

Time dilation by reparameterization

- Changes the frequency content of motion
- We obtain this

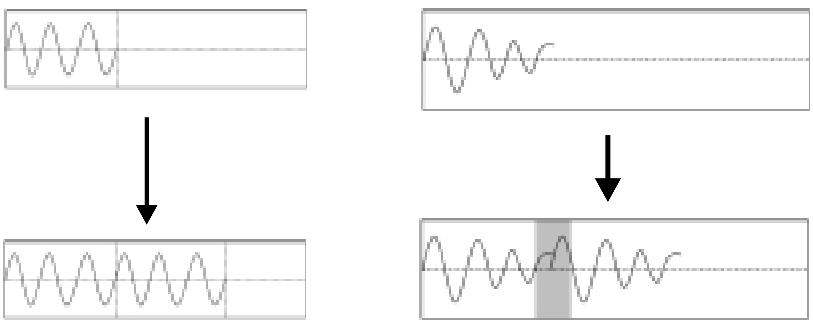


- We want this



Cyclification

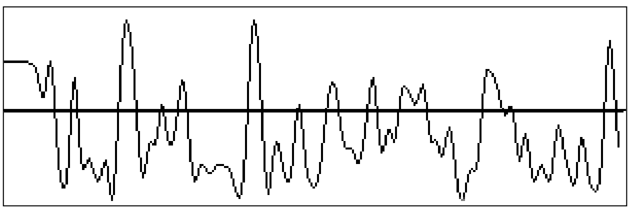
- We need *cyclification*
- Dilate the signal without changing its "harmonic contents": *amplitude, frequency and phase.*
- Easy for periodic signals



- Revising our goal
  - Develop an automatic cyclification technique for motion paths.

Motion paths characterization

- Motion paths are not periodic
- Biomechanic and external factors introduce a noise component, fundamental to natural-looking motion (Ken Perlin, 1995).

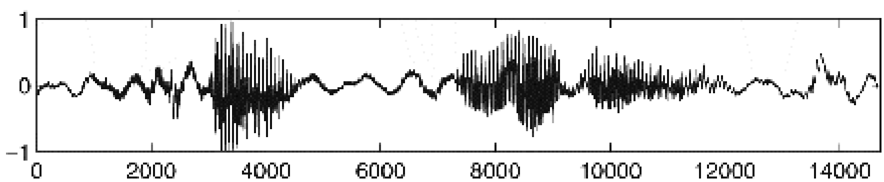


- "Motion path = periodic + noise"?
- Motion path = harmonic content + noise
- We need a mathematical model to characterize the *harmonic components*.

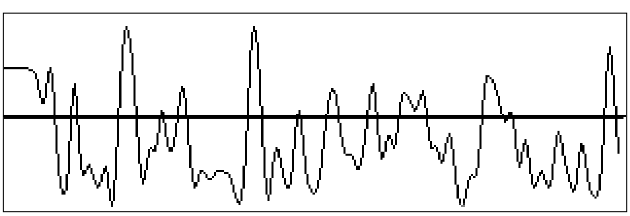
“Similar” signals

- Musical sound

- Speech



- Motion path



Characterization of Similarity

- Existence of harmonic contents
- Non-periodical: The frequency changes along the time
- Musical sound and voiced speech

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos \phi_k(t)$$

- $\phi'_k(t) \rightarrow$ *instantaneous frequency*

Harmonic components of similar signals

- Music signal or a voiced speech consists entirely of harmonic components

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos \phi_k(t)$$

- Speech in general

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos \phi_k(t) \; + \; \text{Noise}$$

- Motion path

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos \phi_k(t) \; + \; \text{Noise}$$

  - Harmonic component plus noise
  - Not "periodic + noise" model

Motion path

- Assumption: We will discard the noise part of a motion signal.
- Model for a motion path

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos \phi_k(t)$$

  - Variable amplitude $a_k(t)$
  - Variable frequency $\phi_k(t)$
  - Instantaneous frequency $\phi'_k(t)$

Linear time warping

- Dilation operator

$$f(t) \mapsto g(t) = \sum_{k=1}^{n} a_k(\alpha t) \cos\left(\frac{\phi_k(\alpha t)}{\alpha}\right)$$

  - $f\colon [a, b] \to \mathbb{R}$
  - $g\colon [\frac{a}{\alpha}, \frac{b}{\alpha}] \to \mathbb{R}$

- Instantaneous frequency

$$\frac{d}{dt}\left(\frac{\phi_k(\alpha t)}{\alpha}\right) = \phi_k'(\alpha t).$$

Dilation operator

$$g(t) = \sum_{k=1}^{n} a_k(\alpha t) \cos\left(\frac{\phi_k(\alpha t)}{\alpha}\right)$$

- Preserves harmonic components:
  1. **Amplitude**: The amplitude of f at t_0 is equal to the amplitude of g at αt_0.
  2. **Frequency content:** The instantaneous frequency of f at t_0 equals the instantaneous frequency of g at $t = \alpha t_0$.
- The dilation operation determines a motion cyclification
- **Problem:**
  - How to compute $\phi_k(t)$ and $a_k(t)$?

Computing the harmonic components

- Given $f\colon \mathbb{R} \to \mathbb{R}$,

$$f(t) = a(t)\cos\phi(t),$$

compute $a(t)$ and $\phi(t)$, a, $\phi\colon \mathbb{R} \to \mathbb{R}$.

- Time-frequency domain techniques
  - Windowed Fourier Transform
  - Wavelet transform
  - Wigner-Ville transform.
- Our method
  - Lapped Cosine Transform (LCT)

Windowed Fourier Transform (WFT)

- Use a variable, but fixed, scale on the window

$$g_s(t) = \frac{1}{\sqrt{s}} g(\frac{t}{s})$$

- Windowed Fourier atoms

$$g_{s,u,\omega}(t) = g_s(t-u)e^{i\omega t}$$

- Windowed Fourier transform

$$\begin{aligned} Sf(u,\omega) &= \langle f, g_{s,u,\omega} \rangle \\ &= \int_{-\infty}^{+\infty} f(t) g_s(t-u) e^{-i\omega t} dt \end{aligned}$$

- Choose window g with good properties:

1. Symmetric
2. Support of $g \subset [-1/2, 1/2]$
3. $\widehat{g}(\omega) \leq \widehat{g}(0)$
4. $\widehat{g}(0) = \int_{-1/2}^{1/2} g(t) dt \approx 1.$

WFT Computation of Harmonic components

Theorem (Delprat et alli.,[1]): If
$f(t) = a(t)\cos\phi(t)$ then

$$\langle f, g_{s,u,\omega}\rangle =$$

$$= \frac{\sqrt{s}}{2} a(u) e^{i(\phi(u)-\omega u)} \Big(\widehat{g}\left(s[\omega - \phi'(u)]\right) + \varepsilon(u,\omega) \Big),$$

The error term $\varepsilon(u,\omega)$ satisfies

$$|\varepsilon(u,\omega)| \leq \varepsilon_{a,1} + \varepsilon_{a,2} + \varepsilon_{\phi,2} + \operatorname{supp}_{|\omega|\geq s\phi'(u)} |\widehat{g}(\omega)|$$

with

$$\varepsilon_{a,1} \leq \frac{s|a'(u)|}{|a(u)|}, \quad \varepsilon_{a,2} \leq \operatorname*{supp}_{t\in[u-\frac{s}{2},u+\frac{s}{2}]} \frac{s^2|a''(u)|}{|a(u)|},$$

and if $\frac{s|a'(u)|}{|a(u)|} \leq 1$, then

$$\varepsilon_{\phi,2} \leq \operatorname*{supp}_{t\in[u-\frac{s}{2},u+\frac{s}{2}]} s^2|\phi''(t)|.$$

Moreover, if $\omega = \phi'(u)$ then

$$\varepsilon_{a,1} = \frac{s|a'(u)|}{|a(u)|} |\widehat{g}\,'(2s\phi'(u))|.$$

How to read the Theorem

- Compute the *spectogram*

$$|Sf(s,u,\omega)|^2 = |\langle f, g_{s,u,\omega}\rangle|^2$$

- Suppose that ε can be neglected

$$|Sf(s,u,\omega)|^2 = \frac{s}{4}a^2(u)\left|\widehat{g}(s[\omega - \phi'(u)])\right|^2$$

- $|\widehat{g}(\omega)|$ is maximum at 0

$$\Downarrow$$

$|Sf(s,u,\omega)|^2$ attains maxima at the points $\omega = \phi'(u)$.

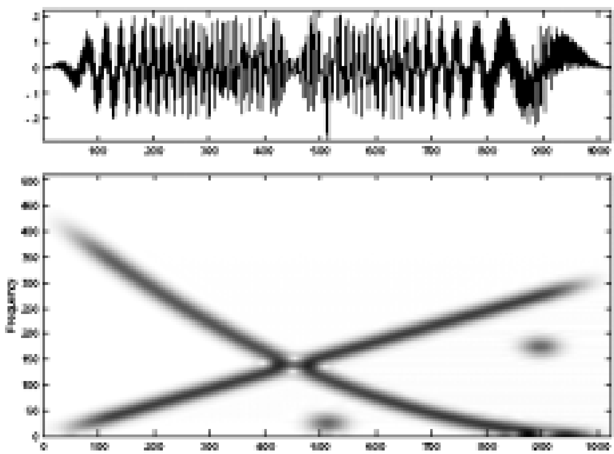
- *Ridge points* of the WFT localize the instantaneous frequency of f.

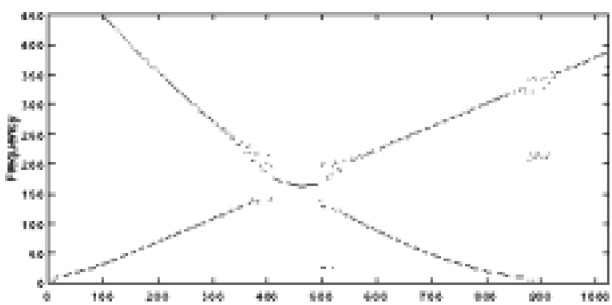
- Compute the amplitude

$$a(u) = \frac{2|Sf(s,u,\omega)|}{\sqrt{s}|\widehat{g}(0)|}$$

WFT ridges

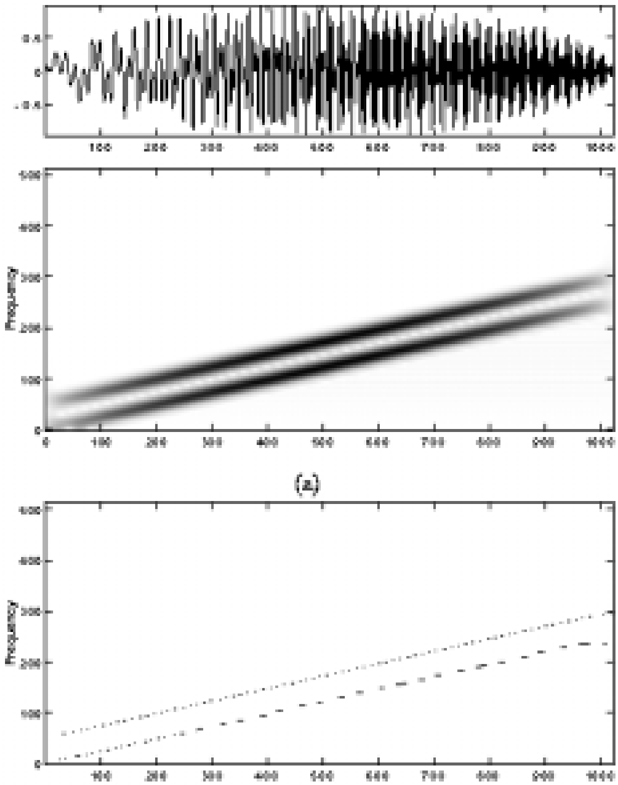
- Linear chirp, quadratic chirp and two modulated Gaussians.





WFT ridges

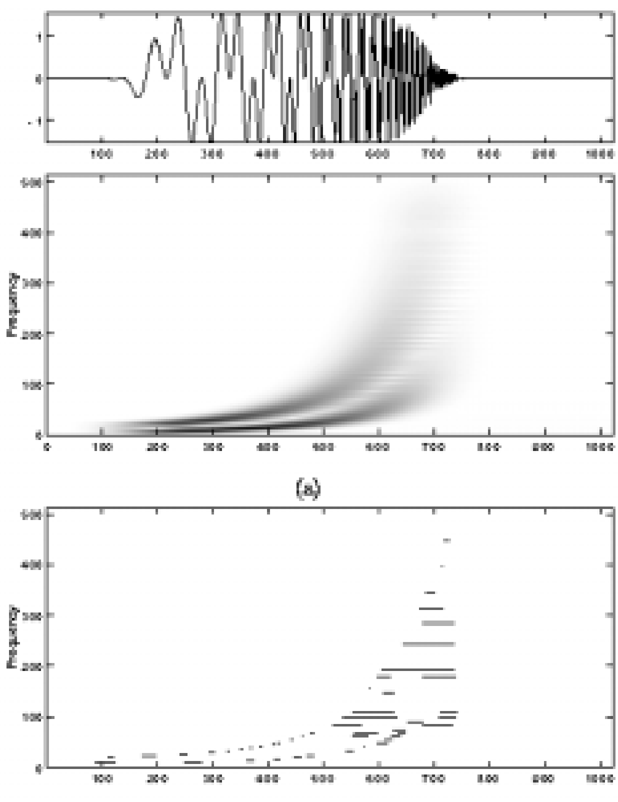
- Two parallels linear chirps



WFT ridges

- Two hyperbolic chirps

$$f(t) = a_1 \cos\left(\frac{\alpha_1}{700 - t}\right) + a_2 \cos\left(\frac{\alpha_1}{740 - t}\right)$$



Wavelet Computation of Harmonic components

- Mother wavelet ψ

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)$$

- Use a Gabor wavelet $\psi(t) = g(t)e^{i\eta t}$
- Wavelet atoms

$$\psi_{u,s}(t) = e^{-i\omega u} g_{s,u,\omega}(t),$$

with

  - $\omega = \eta/s$
  - $g_{s,u,\omega}(t) = \sqrt{s} g\left(\frac{t-u}{s}\right) e^{i\omega t}$

- $Wf(u,s) = \langle f, \psi_{u,s}\rangle = e^{i\omega u}\langle f, g_{s,u,\omega}\rangle$

Wavelet Computation of Harmonic components

Theorem (Mallat): If $f(t) = a(t)\cos\phi(t)$ then

$$Wf(u,s) = \frac{\sqrt{s}}{2} a(u) e^{i\phi(u)} \left(\hat{g}[s(\omega - \phi'(u))] \right) + \varepsilon(u,\omega)$$

- Normalized *scalogram*

$$\frac{\omega}{\eta} P_W f(u,\omega) = \frac{|Wf(u,s)|^2}{s}, \quad \omega = \eta/s$$

- If ε is negligeable

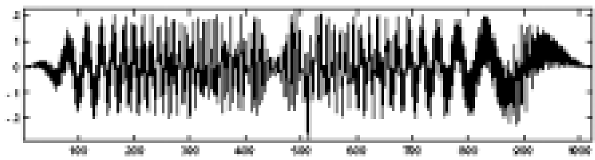
$$\frac{\omega}{\eta} P_W f(u,\omega) = \frac{1}{4} a^2(u) \left| \hat{g}\left(\eta(1 - \frac{\phi'(u)}{\omega}) \right) \right|^2$$

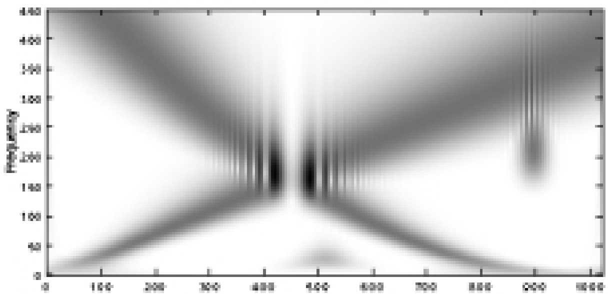
- Maxima of scalogram at points $\omega(u) = \phi'(u)$
- Instantaneous frequencies are computed from the *Wavelet ridges*
- Amplitude computation

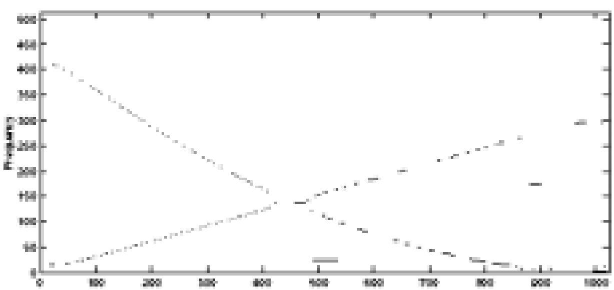
$$a(u) = \frac{2\sqrt{\frac{\omega}{\eta} P_W f(u,\omega)}}{|\hat{g}(0)|}$$

Wavelet ridges

- Linear chirp, quadratic chirp and two modulates

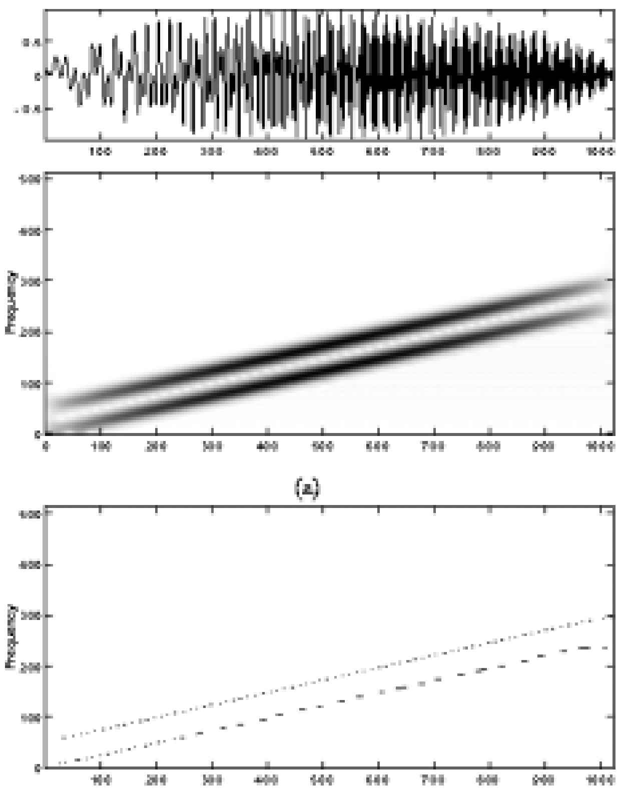






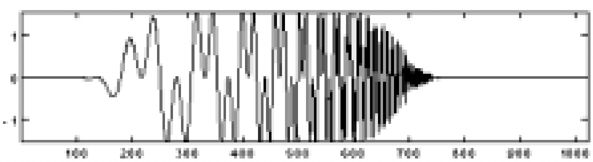
Wavelet ridges

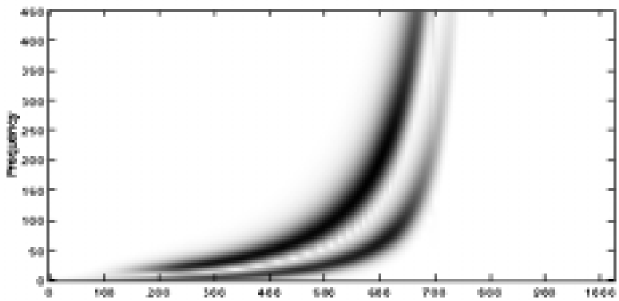
- Two linear parallel chirps



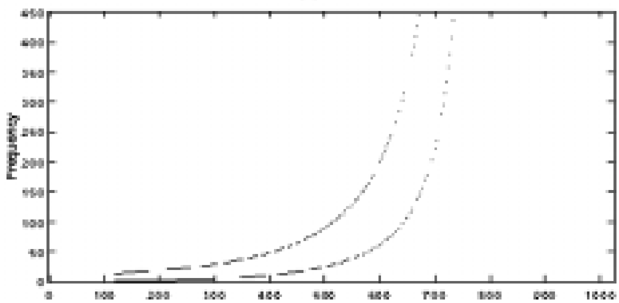
Wavelet ridges

- Two hyperbolic chirps





(a)



(b)

Wigner-Ville computation of harmonic components

- Quadratic transform

$$P_V f(u,\omega) = \int_{-\infty}^{+\infty} f\left(u+\frac{\tau}{2}\right) f^*\left(u-\frac{\tau}{2}\right) e^{-i\tau\omega} d\tau$$

  - J. Ville, 1948 - Instantaneous frequency
  - E. Wigner, 1932 - Quantum mechanics

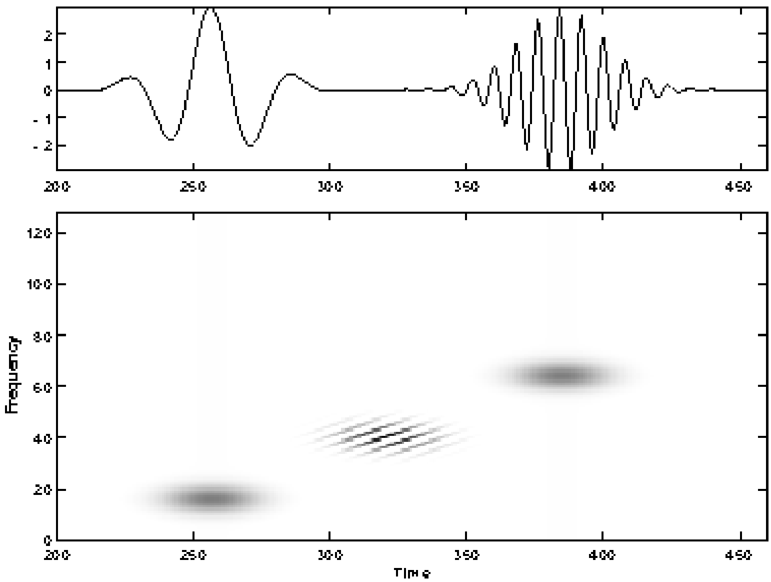
- **Theorem** (Ville): If $f(t) = a(t)e^{i\phi(t)}$ then

$$\phi'(u) = \frac{\int_{-\infty}^{+\infty} \omega P_V f_a(u,\omega) d\omega}{\int_{-\infty}^{+\infty} P_V f_a(u,\omega) d\omega}$$

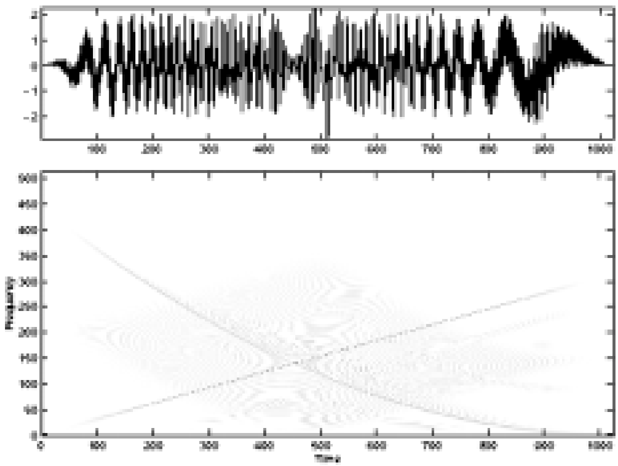
- The Wigner-Ville transform has interference problems

Wigner-Ville transform

- Interference problems
- Two modulated Gaussians



- Linear chirp, quadratic chirp and two modulated Gaussians.



- The modulated Gaussians disappear

Our approach

- Compute a time-frequency representation of the motion path model

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos \phi_k(t)$$

- Apply the dilation directly on the representation
- How to compute the representation?

Representation of harmonic components

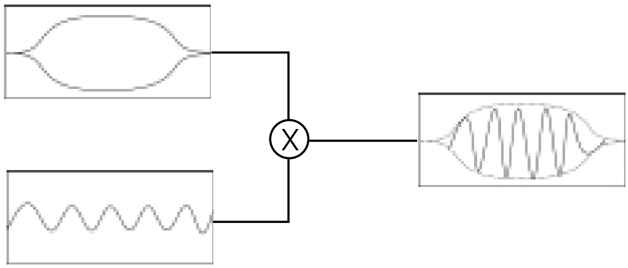
- Take a uniform partition $I_k = [c_k, c_{k+1}], k \in \mathbb{Z}$ of the time domain
- Choose a family of functions $\{e_{k,j}\}, j, k \in \mathbb{Z}$, such that:
  - Each $e_{jk}(t)$ is smooth
  - For each k, $\{e_{k,j}\}, j \in \mathbb{Z}$, is an orthogonal basis of $Ł^2(I_k)$
  - $\{e_{k,j}\}, k, j \in \mathbb{Z}$ is an orthogonal basis of $Ł^2(\mathbb{R})$
- Orthogonal transform

$$f = \sum_{j,k \in \mathbb{Z}} a_{jk} e_{jk}.$$

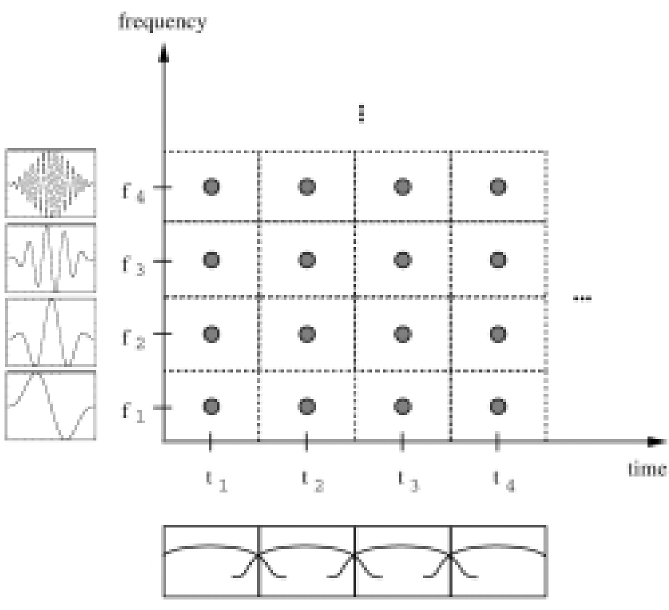
- For each k, a_{jk}, $j \in \mathbb{Z}$ measures the harmonic components on the time interval I_k.

Does this transform exists?

- Lapped Cosine Transform (LCT)
- Windowed cosines, dilated and translated

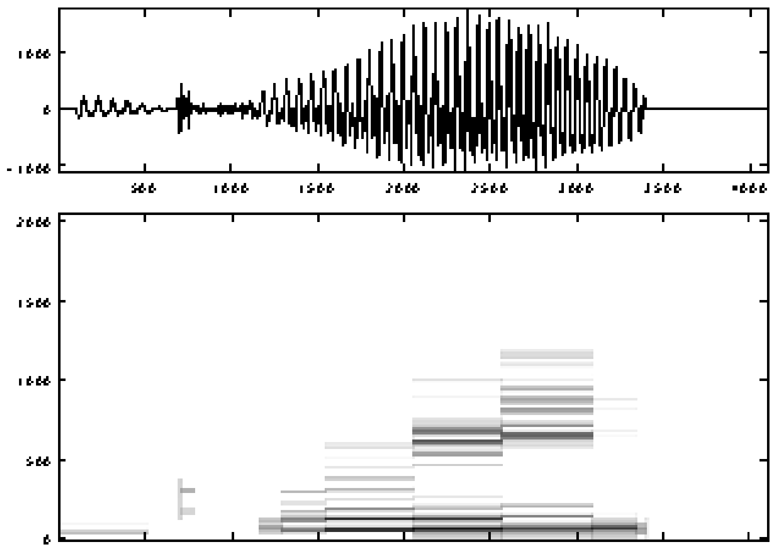


- LCT atoms



Lapped cosine transform

- Voiced speech signal decomposition
  - Two different time resolutions



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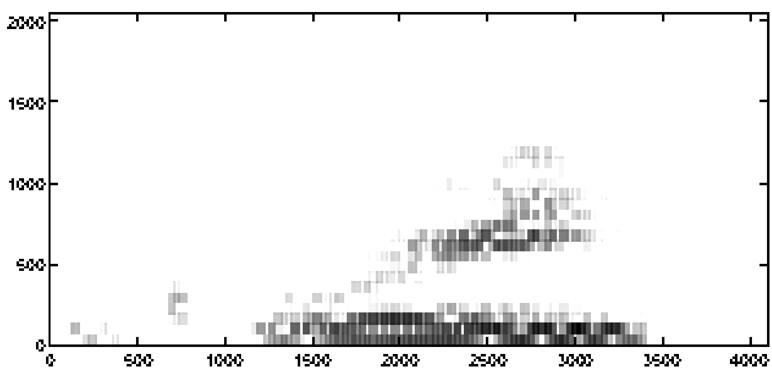


Cyclification using the LCT representation

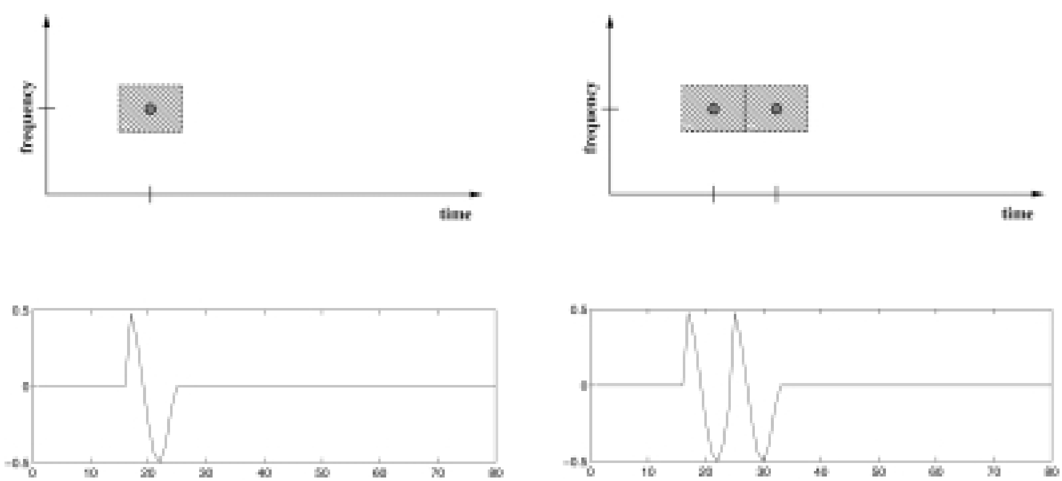
- Cyclification recipe

  1. Compute the LCT transform of the signal
  2. Apply the dilation operator on the time-frequency atoms (t_k, f_j) of the representation

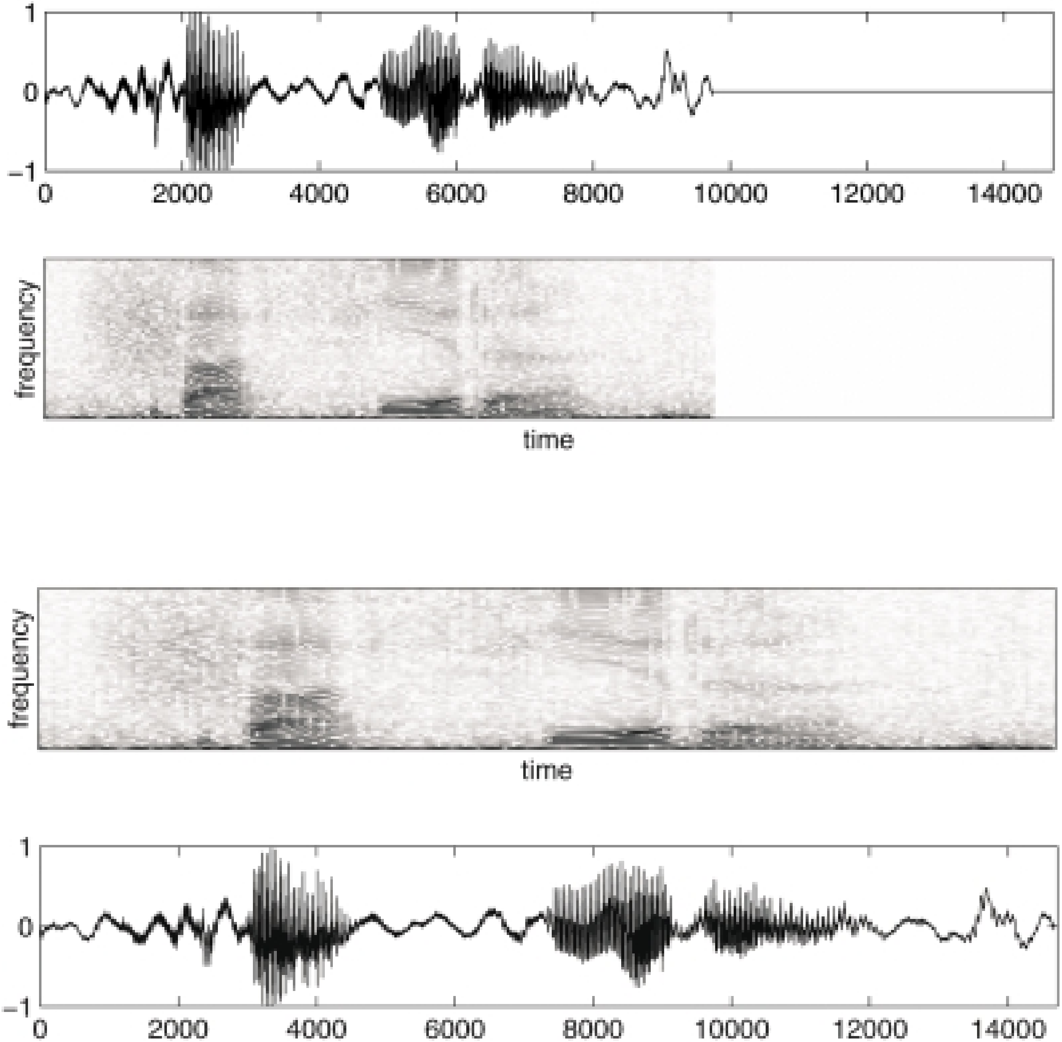
  $$(t_k, f_j) \mapsto (\alpha t_k, f_j)$$

  3. Compute the inverse transform

- Replication of atomic elements



Example with a voiced speech signal



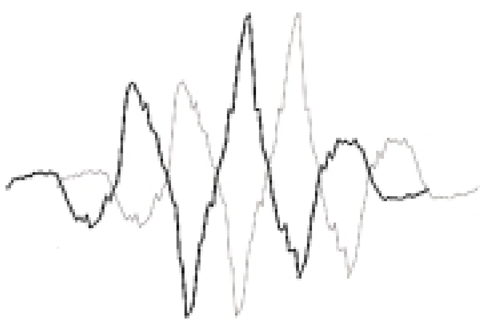
Cyclification of motion paths

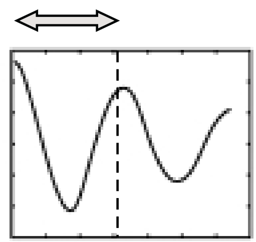
- Problem: what is the size of the partition interval?
- Very low frequency
- Use the fundamental harmonic cycle as the interval size

Computation of the fundamental cycle

- Use correlation

$$C(t) = \int_{-\infty}^{+\infty} f(u)f(u-t)dt$$





Cyclification of articulated bodies

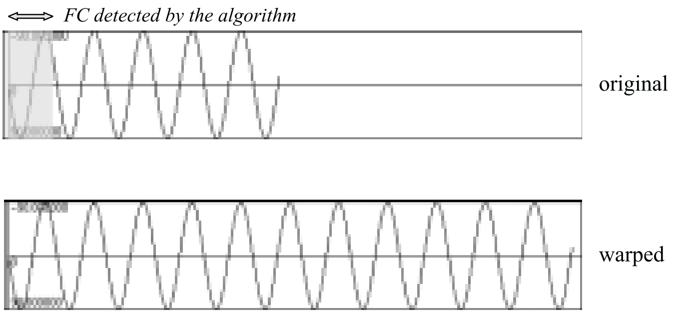
- Complex structure
  1. Multiple joints and DOF's
  2. Motion path $f\colon \mathbb{R} \to \mathbb{R}^n$
- Extend the LCT componentwise
- Maintain the correlation between path coordinates

Correlation between path coordinates

- Strong and weak phase dependence and correlation
- Strong
  - Direct structural relationship between joints
  - Example: Motion of knee and foot is influenced by upper leg joint motion
- Weak
  - Inderect structural relationship between joints (to balance stability and control)
  - Example: Motion of arms and legs

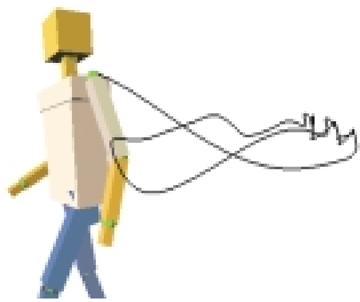
Motion path cyclification examples

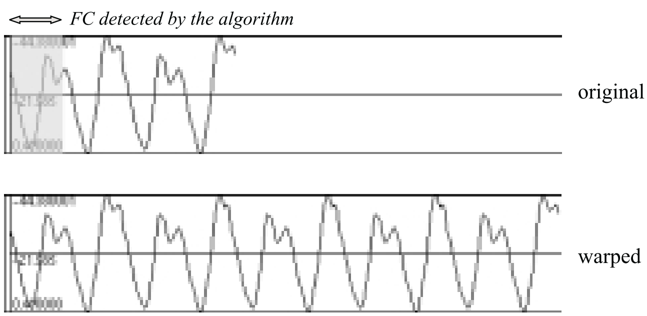
- Periodic path



Motion path cyclification examples

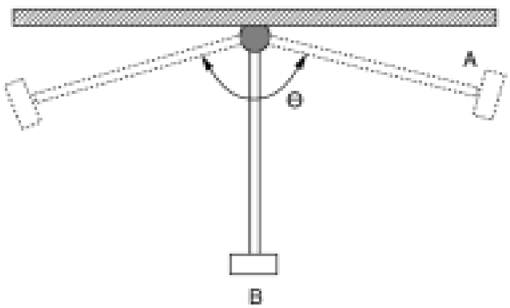
- Left upper arm joint motion curve

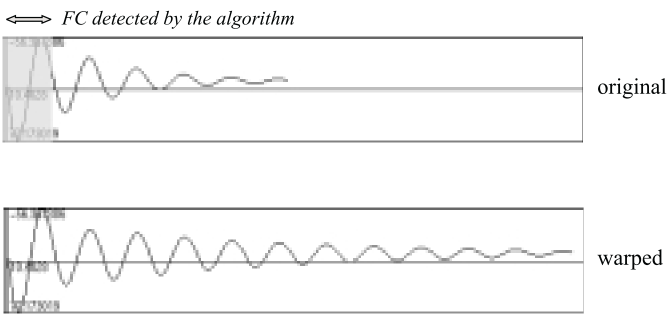




Motion path cyclification examples

- Pendulum with friction





Current research

- Integrate the time warping technique into full animation system, transforming simultaneously human motion and audio
- Specific use for facial animation with audio sincronization
- Apply our representation of the harmonic component to compute different motion filters
- Compute time warping using Windowed Fourier Transform and Wavelets to make comparisons
- Experiment with a non-uniform LCT (best basis)
- Use the model to obtain a better understanding of the noise content of motion (Ken Perlin).