Are Musical Networks Really Scale-free?

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Previous works[1, 2] suggest that musical networks often present the scale-free property found in fractals. From a musician's perspective, the most important aspect missing in those studies was harmony. In addition to that, the previous works made use of outdated statistical methods. Traditionally, least-squares linear regression is utilised to fit a power law to a given data set. However, according to Clauset et al.[3] such a traditional method can produce inaccurate estimates for the power law exponent. In this paper, we present an analysis on musical networks which consider the existence of chords, i.e., the essential elements of harmony. Here we show that only 52.5% of music in our database are fractal compatible. Both mentioned previous works argue that music is always fractal; consequently, it sounds appealing and coherent. In contrast, our results show that not all pieces of music present the scale-free property. In summary, this research is focused on the relationship between musical notes (Do, Re, Mi, Fa, Sol, La, Ti, and its #'s) and chords in classical music compositions.

The scale-free property is a significant finding in complex networks. When a network presents this property, its node degrees distribution follows a power law distribution[4]. It is important to highlight that the node degree consists in the number of edges connected to that node. This research aims to investigate whether or not classical music presents the scale-free property using complex network analysis. In order to find a consistent answer, forty pieces of classical music were selected and represented as musical networks. Then, we have applied Clauset's[3] statistical method on such musical networks. The scale-free property is particularly interesting in occidental music, because such type of music presents the sense of resolution according to a predefined tonal centre. Thus, the hypothesis that classical music presents the scale-free property due to successive returns to the tonal centre shall be investigated.

Perkins et al.[2] argue that most networks created by men present the scale-free property, for example: linguistics, physics, biology, and music. In general, those claims are based on least-squares linear regression method[5] or basic random walk analysis[6]. But, Clauset's[3] statistical method continuously refute many of those claims. Typically, the estimated power law exponent (a) of a network which presents the scale-free property is in the range $2 < a < 3$, because usually it has an ultra small diameter[7]. A power law distribution only has a well-defined mean over $x \in [\ 1\ ,\ \infty\ ]$, if $a > 2$. When $a > 3$, it has a finite variance that diverges with the upper integration limit x_{max} as $\langle x^2 \rangle = \int_{x_{max}}^{x_{min}} x^2 P(x) \sim x_{max}^{3-a}$. Ref.1 estimated power law exponents with the traditional least-squares linear regression method and reported musical networks with exponents in the range $1 < a < 2$. Ref.2 also utilised the same method to report an exponent in the range $1.05 < a < 1.28$ for their restricted musical network. The authors of ref.2 reported a log-log plot with a straight line on it, simplistically. Such a simple result is the first (and more basic) indication that a data set follows a power law distribution. However, according to Clauset et al.[3] such a basic idea is not enough to confirm if a particular data set follows a power law because alternative distributions must be tested as potential candidates as well.

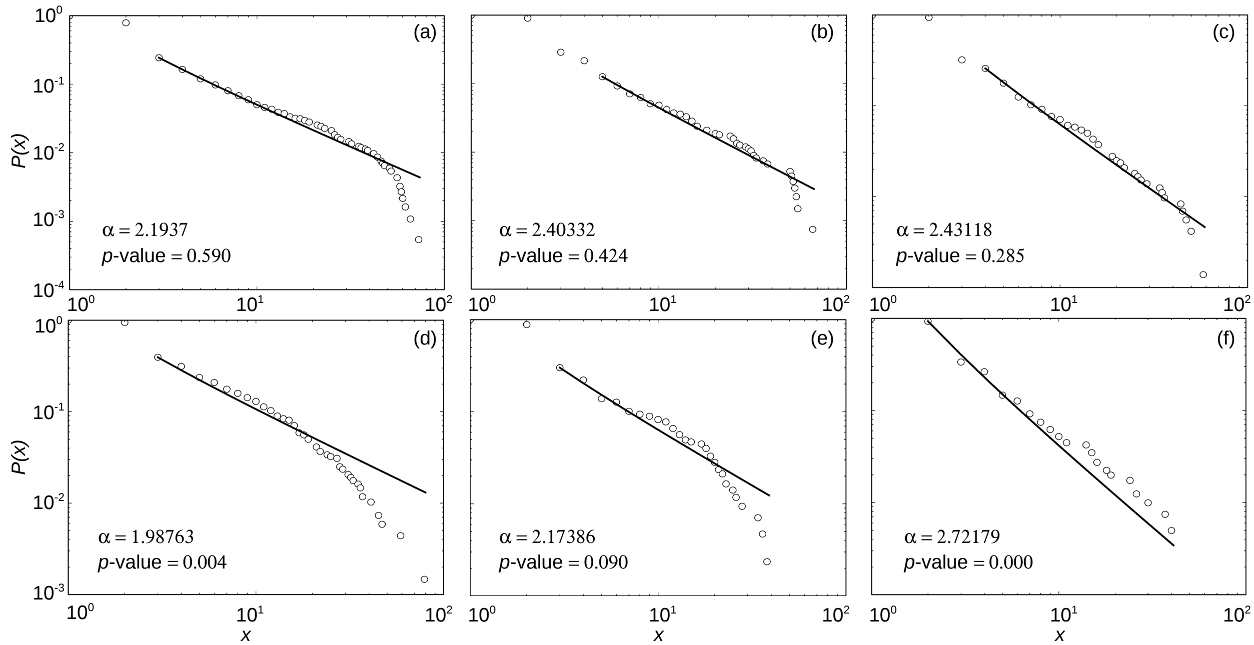

Figure 1: Log-log plot of node degree distribution $P(x)$ and their power law fitting process for six musical networks: (a) Sonata No. 23 in F minor (Appassionata) Opus 57 (1804) composed by Beethoven, (b) Sonata No. 12 in F major KV 332 (1783) composed by Mozart, (c) Piano Sonata in D major Hoboken XVI:33 (1778) composed by Haydn, (d) Violin partita No. 2 in D minor BWV 1004 (1720) composed by Bach, (e) Sonatina in F major Opus 36 No. 4 Opus 36 (1797) composed by Clementi, and (f) Sonatina in C major Opus 36 No. 3 Opus 36 (1797) also composed by Clementi.

An approach taken by authors of ref.1 and ref.2 was the combination of several songs into one musical network. Such an approach was made to provide a consistent numerical analysis in terms of network size (number of nodes and edges). Both authors disregarded the existence of chords. At least, both authors built musical networks with pieces within the same musical key. In particular, the authors of ref.2 transposed all songs to the C major key and built only one musical network. We have built musical networks taking chords into account. Furthermore, we have built musical networks that represent only one piece of music per network. For more information on the procedure adopted to build a musical network from a piece of music, please see Methods. Since we are taking harmony into account, our musical networks are considerably larger in terms of number of nodes and number of edges, when compared to the musical networks built by ref.1 and ref.2. A detailed discussion on previous works, fractals, and music is available in the Supplementary Information.

In summary, Clauset's[3] statistical method comprises three steps. The first step consists in estimating the power law exponent with maximum likelihood estimators[8]. The second step consists in applying the Kolmogorov-Smirnov (KS) test[9]. This test returns a p-value. If the p-value is greater than 0.1, it can be said that the power law distribution is a plausible hypothesis for the input data set in question (the node degrees distribution). If the p-value is less than 0.1, the distribution is rejected as power law distribution. Figure 1 shows the results after applying the first and second steps of Clauset's statistical method on six musical networks. It is evident that the prediction is consistently aligned with the data sets in Figures 1(a), 1(b), and 1(c). This is no longer true for Figures 1(d), 1(e), and 1(f). Such an issue is reflected by the p-values in Figures 1(d), 1(e), and

Likelihood Ratio Test								
Musical Network	Power law (KS)	Exponential		Log-normal		Stretched exponential		Fractal
	p-value	LR	p-value	LR	p-value	LR	p-value	
(a) Beethoven Opus 57	0.590	9.88	0.00	6.36	0.00	4.09	0.00	YES
(b) Mozart KV332	0.424	4.82	0.00	2.67	0.00	1.61	0.10	YES
(c) Haydn Hoboken XVI:33	0.285	5.54	0.00	3.78	0.00	2.87	0.00	YES
(d) Bach BWV 1004	0.004	3.16	0.00	-0.03	0.97	-1.15	0.24	NO
(e) Clementi No.4	0.090	-0.28	0.77	-0.42	0.67	-0.44	0.65	NO
(f) Clementi No.3	0.000	4.91	0.00	3.84	0.00	2.55	0.01	NO

Table 1: Likelihood Ratio (LR) test for the same six musical networks: power law vs. exponential, power law vs. log-normal, power law vs. stretched exponential.

1(f), all below 0.1. According to Janssen[10], due to the finite size of real-world networks the power law inevitably has a cutoff at some maximum degree. This cutoff is verified in Figures 1(a), 1(b), and 1(c). The exponents for all musical networks are close to the range $2 < a < 3$, which indicate a trend for ultra small diameters.

Finally, the third step of Clauset's statistical method compares the power law with alternative hypotheses through a Likelihood Ratio (LR) test[11]. For each alternative tested, if the calculated likelihood ratio is significantly different from zero, its sign indicates whether the alternative distribution is favoured over the power law model. A positive LR indicates power law preference, while a negative LR indicates the alternative hypothesis. Table 1 shows the results after applying the third step of Clauset's statistical method on the same six musical networks. The LR tests confirm the presence of scale-free property in musical networks (a), (b), and (c). This is not the case for musical networks (d), (e), and (f). The musical network (d) behaves

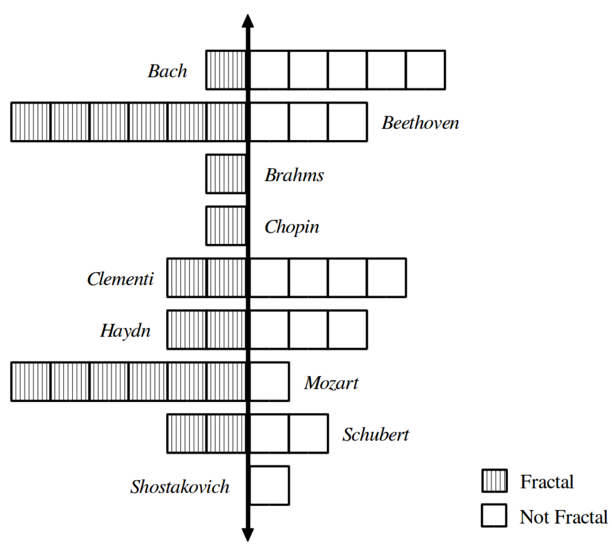

Figure 2: Forty pieces of music composed by nine classical composers. Twenty one musical networks are fractal compatible according to Clauset's statistical method.

more like a log-normal distribution than a power law, while the musical network (e) behaves more like an exponential distribution. The musical network (f) did not behave like any other distribution tested. The numerical results for the other thirty four musical networks are available at `https://eden.dei.uc.pt/~vitorgr/MS.html` and in the Supplementary Information. The source code and our musical database are also available at the same website.

Figure 2 represents a summary of our results. Previous works[1, 2] suggest that different genres (classical music inclusively) are fractal compatible because of the presence of scale-free property in their musical networks. But, the previous works disregarded three substantial aspects in their research: (i) the presence of chords; (ii) the combination of several songs into one musical network; and (iii) the use of outdated statistical methods. In contrast, our studies on classical music alone show that not every musical piece presents the scale-free property found in fractals. The hypothesis that classical music presents the scale-free property due to successive returns to the tonal centre is proven to be false, given that only 52.5% of classical music submitted to Clauset's test are fractal. Therefore, a tonal structure is not enough to create musical pieces which present the scale-free property. Mozart and Beethoven seemed to know better than others how to compose musical pieces with this property.

Future works include: (i) evaluate other music genres; (ii) investigate the edge weights distribution (see Methods); and (iii) evaluate fractal dimension of musical networks according to Song et al.[12] algorithm.

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Author contributions

Rolla and Kestenberg performed the experiments and analysed the data. Rolla, Kestenberg, and Velho wrote the paper.

Competing financial interests

The authors declare no competing financial interests.

Methods

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Music Database. The songs used to build our musical networks are available in Musical Instrument Digital Interface (MIDI) format[13]. They were downloaded from Bernd Krueger's website[14]. Before executing the procedure to create the musical network, it is necessary to convert the MIDI file into a text file and select the appropriate information: (i) time; (ii) Note-On and Note-Off events; and (iii) note number. In a MIDI file, the time is registered in pulses per quarter note (PPQN). The Note-On and Note-Off events determine whether the piano key is pressed or not. Finally, the MIDI note numbers specify 128 distinct pitches, i.e., there is a unique number to each piano key. Table 2 shows a representation of the first bar of Mozart's Sonata No. 16 (KV 545) after selecting the appropriate information from its MIDI file. Figure 3 shows other three representations of the same piece.

Time	Event	Note
0	Note-ON	72
0	Note-ON	60
240	Note-OFF	60
240	Note-ON	67
480	Note-OFF	67
480	Note-ON	64
720	Note-OFF	64
720	Note-ON	67
960	Note-OFF	67
960	Note-OFF	72
960	Note-ON	76
960	Note-ON	60
1200	Note-OFF	60
1200	Note-ON	67
1440	Note-OFF	67
1440	Note-OFF	76
1440	Note-ON	79
1440	Note-ON	64
1680	Note-OFF	64
1680	Note-ON	67
1920	Note-OFF	67

Table 2: Representation of the first bar of Mozart's Sonata No. 16 (KV 545) in a simplified MIDI format. The time is defined as 480 PPQN.

Building a Network from a Piece of Music. It is necessary to define nodes and edges to build a network from a piece of music. Nodes are defined as chords (or individual notes) matched by the duration (semi-breve, minim, crotchet, quaver, semiquaver...) of each particular note in the chord. So, each note number inside a network node is paired with its respective duration [please refer to Figure 3(c)]. It is important to remember that a chord is a set of pitches consisting of two or more (usually three) piano keys pressed simultaneously. In a MIDI file, two or more Note-ON events at the same time indicate a chord. Edges are defined chronologically through the connections between notes and/or chords as the music is played. When a new edge is created between two network nodes, its weight is equal to 1. Whenever an edge is re-used by the composer, the weight related to that particular edge is incremented by 1. Although the weight of an edge is not relevant to evaluate the scale-free property, it designates the most used transitions among network nodes within a musical network. The edge weights distribution can tell a lot about a piece of music. Thus, it deserves future investigation.

Source Code and Database. The source code used to create the forty musical networks, as well as our music database are available at `https://eden.dei.uc.pt/~vitorgr/MS.html`. The source code was written in Python/NetworkX[15] language.

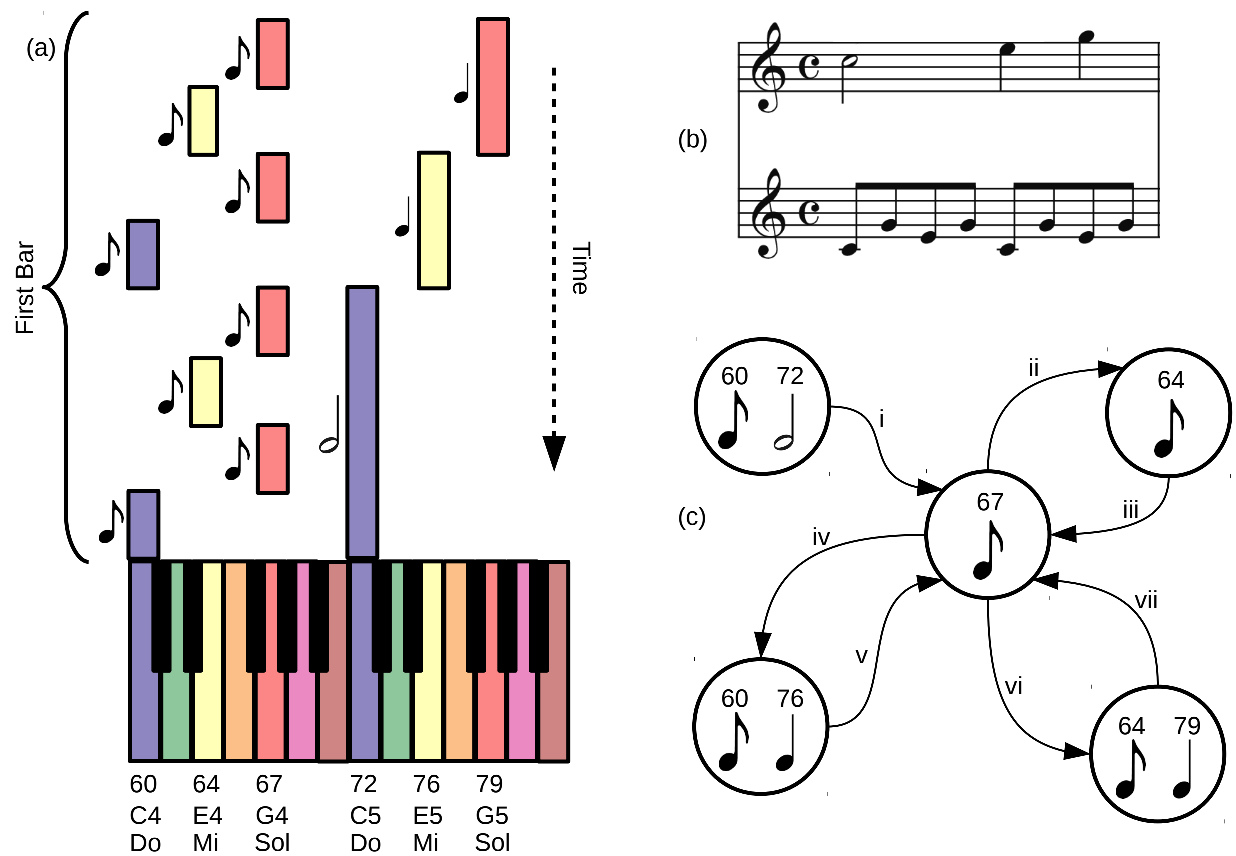

Figure 3: Three different representations of the first bar of Mozart's Sonata No. 16 (KV 545) in C major. **a**, A piece of music can be modelled as a dynamic system with a set of musical notes in time evolution. **b**, Traditional musical score. **c**, A musical network that takes chords (harmony) into account. The roman numbers represent the order in which each edge was created. In that particular case, all edges have weight equals 1, because none of them were used more than once.

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Supplementary Information

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This document is organised as follows. First, there is a discussion on the fractal nature of music. Afterwards, the numerical results for the forty musical networks (we generated) are presented in Tables S1 and S2. Finally, Table S3 shows detailed information about the musical pieces used in our research.

Discussion on the fractal nature of music

For almost thirty years researchers have stated that music has an inherent fractal nature[1]. Not exactly like Henderson-Sellers and Cooper[2], our work suggests that music may or may not present the scale-free property. By the way, Henderson-Sellers and Cooper argued against the assertion that music has a fractal nature.

Most statements[3, 4, 5, 6, 7] affirming that music has a fractal nature were based on fractal dimension methods, i.e., self-similarity. Usually, fractal dimension[8] is applied in the spectral analysis of music. Complex network analysis and fractal dimension are two topics that are related to scale in-variance and power laws in different ways. Song et al.[9] outline a very interesting connection between self-similarity in complex networks and in fractal dimension. A significant limitation of fractal dimension is that it does not necessarily prove that a pattern is fractal. According to Mandelbrot[10], at least a few other essential characteristics must be identified, for instance: different types of self-similarities[11] (exact, quasi, or qualitative) and/or multi-fractal scaling[12, 13]. In addition to that, the majority of papers based on fractal dimension admit that only segments of songs have scale in-variance and self-similarity.

Other statements[14, 15, 16] asserting that music has a fractal nature were based on traditional statistical methods. For example, Hsü[14, 15] analysed the variations in pitch interval between successive notes in twelve songs composed by Bach, Chopin, and Mozart. They show that the frequency of appearance of each pitch interval approximately follows a power-law relation. Liu et al.[16] also show evidence for scale in-variance over pitch fluctuations in songs composed by Bach, Mozart, Beethoven, Mendelssohn, and Chopin.

Liu et al.[17] and Perkins et al.[18] are the main references of our work, because they have made use of complex network analysis to show that several musical networks have fractal nature. These references[17, 18] are equivalent to ref.1 and ref.2 in the original manuscript, respectively. From a musician's perspective, it is richer and more interesting to represent a song within a network than simply count the variations in pitch or rhythm intervals. In fact, because nodes and edges can represent different musical attributes within a network, we believe that complex network analysis is ideal to evaluate the scale-free property in music, i.e., if music has a fractal nature.

	Network Size		**Clauset's First and Second Steps**	
Musical Network	nº nodes	nº edges	power law exponent (a)	KS p-value
Bach BWV 801	180	239	2.9336	0.906
Bach BWV 1001	413	1107	2.16431	0.112
Bach BWV 1003	549	1394	2.12844	0.214
Bach BWV 1004	681	1715	1.98763	0.004
Bach BWV 1005	620	1469	1.97255	0.184
Bach BWV 1041	1560	2598	2.2254	0.482
Beethoven Opus 10	1376	2151	2.54803	0.304
Beethoven Opus 13	1645	2395	2.64127	0.132
Beethoven Opus 22	1880	3175	2.31242	0.083
Beethoven Opus 27	1159	1835	2.30737	0.100
Beethoven Opus 53	2259	3707	2.25062	0.028
Beethoven Opus 57	1854	3206	2.1937	0.590
Beethoven Opus 81	1432	2317	2.09344	0.304
Beethoven Opus 90	1118	1720	2.37218	0.206
Beethoven Opus 106	4818	7907	2.18382	0.048
Brahms Opus 1	2903	4166	2.45447	0.504
Chopin Opus 35	1487	2319	2.60524	0.134
Clementi Nº1	297	466	4.55607	0.794
Clementi Nº2	308	494	2.38659	0.162
Clementi Nº3	402	724	2.72179	0.000
Clementi Nº4	425	787	2.17386	0.090
Clementi Nº5	478	900	2.27392	0.270
Clementi Nº6	477	777	2.35256	0.058
Haydn Hoboken XVI:9	290	464	2.74506	0.011
Haydn Hoboken XVI:33	721	1413	2.43118	0.285
Haydn Hoboken XVI:35	835	1560	2.38432	0.084
Haydn Hoboken XVI:40	642	1094	2.56768	0.132
Haydn Hoboken XVI:43	732	1440	2.26035	0.114
Mozart KV311	1234	2179	2.33052	0.222
Mozart KV330	947	1719	2.49826	0.223
Mozart KV331	1144	1977	2.64005	0.928
Mozart KV332	1340	2299	2.40332	0.424
Mozart KV333	1532	2815	2.08961	0.164
Mozart KV545	634	1107	2.6527	0.181
Mozart KV570	1052	1721	2.37106	0.392
Schubert D960	2768	4706	2.10174	0.008
Schubert D760	2493	3855	2.44852	0.070
Schubert D850	2948	4613	2.30162	0.332
Schubert D784	1349	1905	3.01568	0.342
Shostakovich Opus 57	1293	1842	3.06283	0.320

Table S1: Number of nodes, number of edges, a exponent, and p-value of musical networks.

Likelihood Ratio Test – Clauset's Third Step								
Musical Network	Power law (KS)	Exponential		Log-normal		Stretched exponential		Fractal
	p-value	LR	*p*-value	LR	*p*-value	LR	*p*-value	
Bach BWV 801	0.906	3.40	0.00	2.73	0.00	2.13	0.03	YES
Bach BWV 1001	0.112	3.02	0.00	0.98	0.32	0.07	0.94	No
Bach BWV 1003	0.214	3.96	0.00	1.56	0.11	0.54	0.58	No
Bach BWV 1004	0.004	3.16	0.00	-0.03	0.97	-1.15	0.24	No
Bach BWV 1005	0.184	5.27	0.00	2.18	0.02	0.85	0.39	No
Bach BWV 1041	0.482	1.92	0.05	-0.34	0.73	-0.69	0.48	No
Beethoven Opus 10	0.304	6.35	0.00	4.71	0.00	3.74	0.00	YES
Beethoven Opus 13	0.132	5.76	0.00	4.44	0.00	3.55	0.00	YES
Beethoven Opus 22	0.083	10.37	0.00	7.76	0.00	5.56	0.00	No
Beethoven Opus 27	0.100	7.67	0.00	5.44	0.00	3.96	0.00	YES
Beethoven Opus 53	0.028	2.49	0.01	1.66	0.09	1.44	0.14	No
Beethoven Opus 57	0.590	9.88	0.00	6.36	0.00	4.09	0.00	YES
Beethoven Opus 81	0.304	10.63	0.00	7.66	0.00	5.82	0.00	YES
Beethoven Opus 90	0.206	4.17	0.00	3.31	0.00	2.47	0.01	YES
Beethoven Opus 106	0.048	5.78	0.00	0.41	0.67	-1.05	0.29	No
Brahms Opus 1	0.504	4.87	0.00	3.75	0.00	2.82	0.00	YES
Chopin Opus 35	0.134	6.03	0.00	4.08	0.00	2.62	0.00	YES
Clementi No.1	0.794	1.38	0.16	1.00	0.31	0.78	0.43	No
Clementi No.2	0.162	3.52	0.00	2.34	0.01	1.48	0.13	YES
Clementi No.3	0.000	4.91	0.00	3.84	0.00	2.55	0.01	No
Clementi No.4	0.090	-0.28	0.77	-0.42	0.67	-0.44	0.65	No
Clementi No.5	0.270	5.57	0.00	3.67	0.00	2.64	0.00	YES
Clementi No.6	0.058	5.84	0.00	4.30	0.00	3.21	0.00	No
Haydn Hoboken XVI:9	0.011	0.64	0.51	0.15	0.87	-0.29	0.76	No
Haydn Hoboken XVI:33	0.285	5.54	0.00	3.78	0.00	2.87	0.00	YES
Haydn Hoboken XVI:35	0.084	8.71	0.00	6.62	0.00	5.14	0.00	No
Haydn Hoboken XVI:40	0.132	0.67	0.49	0.39	0.68	0.36	0.71	No
Haydn Hoboken XVI:43	0.114	5.99	0.00	3.92	0.00	2.68	0.00	YES
Mozart KV311	0.222	5.28	0.00	2.83	0.00	2.00	0.04	YES
Mozart KV330	0.223	6.15	0.00	4.82	0.00	4.05	0.00	YES
Mozart KV331	0.928	4.18	0.00	2.88	0.00	2.31	0.02	YES
Mozart KV332	0.424	4.82	0.00	2.67	0.00	1.61	0.10	YES
Mozart KV333	0.164	2.44	0.01	-0.39	0.69	-0.91	0.36	No
Mozart KV545	0.181	4.49	0.00	3.28	0.00	2.71	0.00	YES
Mozart KV570	0.392	6.25	0.00	4.85	0.00	3.61	0.00	YES
Schubert D960	0.008	-0.35	0.72	-1.05	0.28	-1.11	0.26	No
Schubert D760	0.070	1.26	0.20	0.77	0.44	0.62	0.53	No
Schubert D850	0.332	6.32	0.00	2.97	0.00	1.74	0.08	YES
Schubert D784	0.342	6.32	0.00	5.42	0.00	4.27	0.00	YES
Shostakovich Opus 57	0.320	2.00	0.04	0.99	0.31	0.58	0.55	No

Table S2: Likelihood Ratio (LR) test for forty musical networks: power law vs. exponential, power law vs. log-normal, power law vs. stretched exponential.

Composer	Information	Year
Bach	Sinfonia No. 15 in B minor, BWV 801	1723
Bach	Violin Sonata No.1 in G minor, BWV 1001	1720
Bach	Violin Sonata No.2 in A minor, BWV 1003	1720
Bach	Violin partita No. 2 in D minor, BWV 1004	1720
Bach	Violin Sonata No.3 in C major, BWV 1005	1720
Bach	Violin Concerto in A minor, BWV 1041	1717
Beethoven	Sonata No. 5 in C minor,Opus 10/1	1798
Beethoven	Sonata No. 8 in C minor (Pathetique), Opus 13	1799
Beethoven	Sonata No. 11 in Bb major, Opus 22	1800
Beethoven	Sonata No. 14 in C# minor (Moonlight), Opus 27-2	1801
Beethoven	Sonata No. 21 in C major (Waldstein), Opus 53	1804
Beethoven	Sonata No. 23 in F minor (Appassionata), Opus 57	1804
Beethoven	Sonata No. 26 in Eb major (Les Adieux), Opus 81a	1809
Beethoven	Sonata No. 27 in E minor, Opus 90	1814
Beethoven	Sonata No. 29 in Bb major (Hammerklavier), Opus 106	1818
Brahms	Sonata in C major,Opus 1	1853
Chopin	Piano sonata No. 2 in Bb minor, Opus 35	1839
Clementi	Sonatina Opus 36 No. 1 in C major, Opus 36	1797
Clementi	Sonatina Opus 36 No. 2 in G major, Opus 36	1797
Clementi	Sonatina Opus 36 No. 3 in C major, Opus 36	1797
Clementi	Sonatina Opus 36 No. 4 in F major, Opus 36	1797
Clementi	Sonatina Opus 36 No. 5 in C major, Opus 36	1797
Clementi	Sonatina Opus 36 No. 6 in D major, Opus 36	1797
Haydn	Piano Sonata in F major, Hoboken XVI:9	1766
Haydn	Piano Sonata in D major, Hoboken XVI:33	1778
Haydn	Piano Sonata in C major, Hoboken XVI:35	1780
Haydn	Piano Sonata in G major, Hoboken XVI:40	1784
Haydn	Piano Sonata in Ab major, Hoboken XVI:43	1783
Mozart	Sonata No. 8 in D major, KV 311	1777
Mozart	Sonata No. 10 in C major, KV 330	1783
Mozart	Sonata No. 11 in A major (Alla Turca), KV 331	1783
Mozart	Sonata No. 12 in F major, KV 332	1783
Mozart	Sonata No. 13 in Bb major, KV 333	1783
Mozart	Sonata No. 16 in C major (Sonata facile), KV 545	1788
Mozart	Sonata No. 17 in Bb major, KV 570	1789
Schubert	Piano Sonata in Bb major, D 960	1828
Schubert	Fantasia in C major (Wanderer), D 760, Opus 15	1822
Schubert	Piano Sonata in D major, D 850, Opus 53	1825
Schubert	Piano Sonata in A minor, D 784, Opus 143	1823
Shostakovich	Piano Quintet in G minor, Opus 57	1940

Table S3: Detailed information about the musical pieces studied in this research.

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