Oriented Bounding Boxes Based on Multi-resolution Contours

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Abstract

The determination of intersection or contact points between objects in interference is not a simple task. It could be time-consuming in a computer application. Generally, some real time applications use sophisticated algorithms based on hierarchical structure to isolate the segments of contours of objects in interference, to finally compute locally the contact points. Therefore, we are interested in search for the best way of constructing hierarchical structures bounding objects that allows us to quickly isolate the contour of segments in interference. To achieve that, the bounding must be adapted and fitted to the segments of the objects contours. In this work, we formulate a method to represent, in hierarchic structure, oriented rectangle boxes involving segments of object contours defined by closed cubical B-splines curves. Each oriented box is computed in adapted and fitted form to the segments of the contour by using the second order statistical indicator on some elements of the segments of object contour in multi-resolution representation.

Keywords: Interference Detection, Collision Detection, Bounding Box, Multi-resolution representation, Oriented Box.

1 Introduction

Many applications in Computer Graphics and Robotics demand real time analyses of interferences between objects. This type of analysis is used in animations, simulations and path planning to prevent interpenetration between objects in the virtual environment [12, 4]; it is also recommended for use in optimization of part cutting in two-dimensional space, by compacting techniques [9], where the contours of the part pieces are objects of irregular geometry.

The techniques of interference detection are of two types: *structured* and *direct verification*. The structured interference detection requires additional spaces to define hierarchical structures, but it optimizes the time consumed in the interference detection process. In the direct verification, the geometrical attributes of the object, such vertices, edges and faces, are main elements used to verify its neighborhoods and intersection. It does not use additional spaces, but in some cases the geometric comparisons are lengthy and make impossible its use in real time applications.

There are many efficient interferences detection algorithms for three-dimensional objects that could be used for testing two-dimensional objects. But they are not efficient with objects of arbitrary geometry contours, even if they have disturbances details[1] in their contours. For instance, the incremental algorithm [8] detects interferences using the Voronoi spaces defined by vertices, edges and faces of objects. The interferences based on the Clipping methods [5, 10, 7] use the projections of the polygonal attributes of the objects. The technique of the witness [2] uses separation planes defined by faces, edges and vertices of convex objects. The tree sphere method [6] and the oriented boxes tree method [4] define the hierarchical bounding sphere and rectangular box, respectively, from the vertices and edges of polygonal contours of objects.

A hierarchical structure of envelopes allows to quickly discard the parts of the objects that are not in interference. This structure can be built for use in the binary, quaternary or octal tree manner. In order to optimize the interference test time, the envelopes must bound in adapted and adjusted form the segments of the object contour and its details. This is the spheres, isothetic boxes[2] and ellipses do not necessary optimize the time consumed in the interference detection, because a minimum sphere covering a segment of contour would contain more empty space than an ellipse covering the same segment, they are not adapted to segments of the object contours. A oriented box might bound a segment with less empty space than the ellipse and the sphere (see Figure 1), because the orientation of the boxes is adapted to the segment of object contours. However, the test for intersection of two spheres is faster than the test for intersection of two oriented boxes or two ellipses, but we can decide if two objects are not in interference by testing, in hierarchical structure, less oriented boxes than spheres.

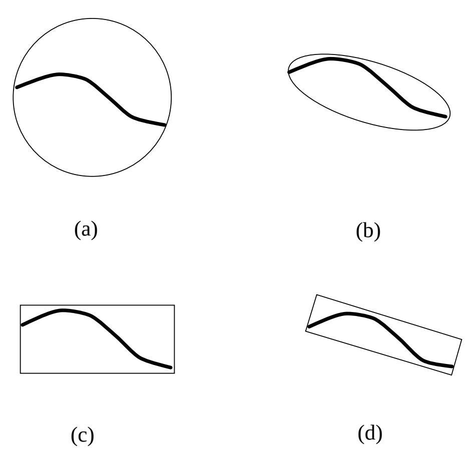


Figure 1: Types of envelopes: sphere, ellipse, isothetic box, oriented box.

The bounding boxes tree is used in ray-tracing and calculations of intersections and modelling [1, 17, 18]

[1]Irregularities of the detail that define the roughness of the contour
[2]Box of edges parallel bars to the coordinate system of the universe

and was extended by Gottschalk et al. [4] for interferences detection of three-dimensional objects of polygonal contours. In this work, we reformulate these ideas for interference detection in simulating rigid objects with roughness [12, 15], where the orientations of the boxes are computed by sampling the respective bounded contours segments [13]. We also present another way of computing the best adaptation of boxes from the coarse level of multi-resolution representation of object contours in two-dimensional space.

The rest of this article is organized as follows. In Section 2, we define the characteristics of the object used to test the idea formulated. In Section 3 we study hierarchical structures, in particular the generation of oriented bounded boxes. In Section 4, we formalize the interference detection of objects in animation, and in Section 5 we present the results of the implementation of the method. We conclude in Section 6 with some final remarks.

2 Object of arbitrary geometry contour

We consider, for the purpose of validation of the model formulated in this work, objects in 2D of contours defined for periodic and continuous cubical B-splines curves [16]. We define an object by m control points, $C = \{c_i\}_{i=0,\ldots,m}$, that generate m segments of the curve, $\{f_i\}_{i=0,\ldots m}$. The contour of the object is given by $\cup_{i=0}^{m} f_i$, possibly, with roughness defined for a random distribution normal $N(0, \sigma_i)$ of noise in each segment. The variances σ_i indicate the degree of irregularity of the details of the segment of the contour f_i. In the definition, each segment is defined geometrically and the roughness details might be reproduced probabilistically [11] using a tolerance interval, $\lambda_i = q\sigma_i$, for $0 < q < 1$. Figure 2 shows the segment f_j with roughness details inside of a oriented box.

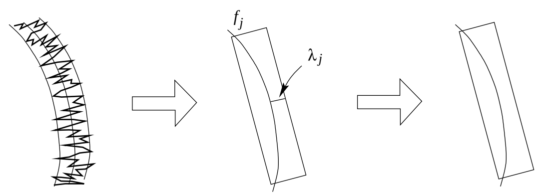


Figure 2: A segment of contour with details.

For the purpose of representation in multi-resolution, f is denoted by f^n where $n = \lg_2 m$, and its respective points of control C by C^n [3, 14]. Therefore, if f^n defines the fine contour, then f^{n-1} is the coarse representation of f^n, where the detail of this contour is $g^{n-1} = f^{n-1} - f^n$. In the multi-resolution representation, f^n can be expressed in a sequence of the coarse resolutions $f^{n-1}, f^{n-2}, ..., f^{min}$, where $f^j = f^{j-1} + g^{j-1}$, $min < j \leq n$ for any minimum value min. In this work, we use the bi-orthogonal wavelet transformation in the version of fast transformation [3] to represent the contours in multi-resolutions representation.

In practice, the transformations are given in level of control points of the contour. Thus, C^n is transformed into C^{n-1}, this into C^{n-2} and successively until C^{min}, using analysis filters A^j and B^j such $C^{j-1} = A^j C^j$ and $D^{j-1} = B^j C^j$, for $min < j \leq n$. The vector D^{j-1} is the detail-coefficients vector that is not used in this work, but the scale coefficients vector C^{j-1} is used to get the coarse version f^{j-1} such $f^{j-1} = C^{j-1}\Phi^{j-1}$, where Φ^{j-1} is the cubic B-splines basis function vector.

3 Hierarchical oriented boxes

If each segment f_i^n and its details are bounded by an oriented box b_i^n, all contour of the object will be covered by a set of boxes called *elementary boxes*. In order to construct a binary tree of oriented boxes, each pair of adjacent elementary boxes, b_{2i}^n and b_{2i+1}^n, is covered in adapted and adjusted way by one another box b_i^{n-1} called *super box*. The elementary boxes contained by each adjacent pair super boxes b_{2i}^{n-1} and b_{2i+1}^{n-1} are covered in adapted and adjusted form by one another super box b_i^{n-2}. Following this process, we construct a binary tree, where the root is a super box b_1^0 bounding adapted and adjusted all elementary boxes of the object. The inclusion of elementary boxes inside of the superior boxes allows the conservation of the segment and their respective roughness details. Figure 3 shows one segment of the oriented bounding box tree.

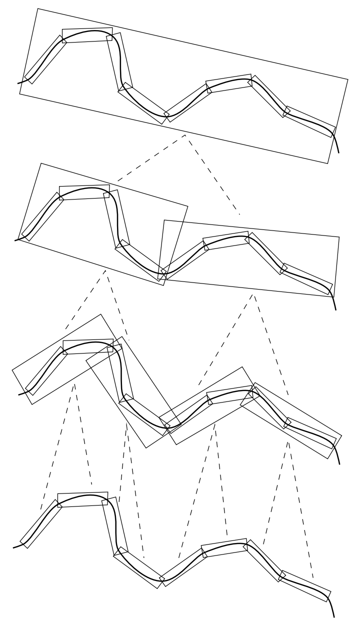


Figure 3: Hierarchy of oriented bounding boxes.

The orientation of the box, given for its main axis, is defined by the behavior of the segments bounded by them. The best form to determine the behavior of the segments by using the covariance matrix $\boldsymbol{\sigma}$ computed with the elements related to these segments, as detailed in the next subsection. The main axis $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, in two-dimensional case, are the unitary eigenvectors of $\boldsymbol{\sigma}$ [11].

In general, a oriented bounding box is defined in two phases: *adaptation* and *adjustment*. In the adaptation phase, we compute the orientation of the box; in the adjustment phase its dimensions are calculated. In the definition of elementary boxes, case of contours with details of roughness, we consider another third phase *increment* to included the details in the box.

The information that define a box, as the unitary axis, the dimensions, centroid, tolerance λ_i and the length of the segment f_i^n must be stored in the data structure that defines the oriented box. These information will be useful in the calculations of super boxes, and the test for interferences.

3.1 Elementary box

The elementary box b_i^n is generated from the elementary segment f_i^n of the object contour and its disturbances in the adaptation, Adjustment and increment phases.

Adaptation: the covariance matrix $\boldsymbol{\sigma}_i$ is computed respect to the simple average $\boldsymbol{\mu}$ of the r points $\boldsymbol{p}_i$ uniformly sampled on f_i^n. We considered $r = 5$ as appropriate to compute the orientation of the adapted axis to the tendency of the segment, since bigger concentration of the points in some part of f_i^n can not reflect in the axis the real tendency of the segment. Figure 4 shows two elementary boxes defined by uniform sampling of two types of curve segments. The elements of the covariance matrix, in this case, are of the form:

$$\sigma_{xy} = \frac{1}{r-1}\sum_{i=1}^{r}(p_i^x - \mu^x)(p_i^y - \mu^y)$$

$$\text{with} \quad \boldsymbol{\mu} = (\mu^x, \mu^y) = \frac{1}{r}\sum_{i=1}^{r}(p_i^x, p_i^y).$$

Adjustment: the segment f_i^n is projected on the axis with origin in $\boldsymbol{\mu}$. The sides of the box are defined by the segments of bigger dimension between the projections on each axis.

Increment: This phase is used when the contour of the object has some roughness defined by $N(0, \sigma_i)$, where $\sigma_i = q\lambda_i$, for $0 < q < 1$. The sides of the box are added with the tolerance λ_i that represents the quota, near to the superior, of the details associates to the segment f_i^n. After define the dimensions of the box b_i^n, we recompute their new centroid.

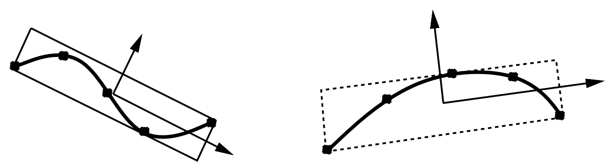


Figure 4: Elementary boxes adjustment with m sampling points.

3.2 Super boxes

The super boxes contain two or more elementary boxes; therefore, the adaptation and the adjustment are computed with other considerations that in the elementary boxes. We wish to construct a super box B bounding adapted and adjusted form the k adjacent elementary boxes.

3.2.1 Adaptation

The best adaptation of the box to the segment of object contour depends solely on the choice of its main axis, however on the covariance matrix that defines these axis. We formulated, in this work, one method to compute the *orientation defined by segment in multi-resolution*, for that is necessary to understand before method *orientation defined by elementary boxes* [13]. The new method generates more suitable orientation to the segment of the contour than the method based in elementary boxes, but requires previous decomposition process of the contour in lowers resolutions, that computationally is insignificant.

Adaptation based in elementary boxes

The covariance matrix is computed by using the centroids of the elementary boxes weighed with the length of the corresponding segment. So, it is not influenced by the concentration of small segments in any sector of the contour.

If $\boldsymbol{p}_i = (p_i^x, p_i^y)$ is the centroid of the box b_i^n, and l_i is the arc length of the elementary segment f_i, the average of k segments is given by

$$\boldsymbol{\mu} = \frac{1}{l}\sum_{i=1}^{k} l_i \, \boldsymbol{p}_i, \quad \text{with} \quad l = \sum_{i=1}^{k} l_i. \tag{1}$$

The element σ_{xy} of the covariance matrix $\boldsymbol{\sigma}$, is calculated, in this case, such

$$\sigma_{xy} = \frac{1}{l}\sum_{i=1}^{k} l^i (p_i^x - \mu^x)(p_i^y - \mu^y). \tag{2}$$

Using the wavelets transformation in multi-resolution representation of contours, the correspondence between the segments of two adjacent resolutions of the object contour is validated by the theorem follow formulated, whose proof meets in Rivera [12].

Theorem 1 *Given the adjacent inferior resolutions f^j and f^{j-1} of the contour f^n, for $j < n$, the segment f_i^{j-1} converges in $f_{2i}^j \bigcup f_{2i+1}^j$.*

These Theorem allows to relate each pair of segments and its respective boxes with a segment of superior immediate resolution. Moreover, the version f^j, for $min \leq j < n$, represents the average of the version f^{j+1}, according to theory of multi-resolution based on wavelet transformation [3]. Each segment of f^j defines, adapted to the segment, the orientation of the box of level j in the hierarchical structure. For example, the orientation of the box b_i^{n-1} is defined by the segment f_i^{n-1}, and its dimensions are defined by the elementary boxes b_{2i}^n and b_{2i+1}^n that bound segments f_{2i}^n and f_{2i+1}^n, respectively. These elementary segments oscillate around f_i^{n-1} [12]. Figure 5 illustrates one super box defined by a segment f_r^{n-1} and their two corresponding elementary boxes.

The main axis of the super box b_i^j is defined from the covariance matrix $\boldsymbol{\sigma}_i^j$ computed, of similar way that in the phase of adaptation of the elementary box, by uniform sampling the segment f_i^j. So, the orientations of the boxes of level j are defined by the contour segments f^j that is lower version of f^n in multi-resolution representation, for $min \leq j < n$. The others level of boxes, b_i^j for $0 \leq j < min$, are computed by using the adaptation based in elementary boxes, because we can only define lower resolutions of f^n until f^{min}.

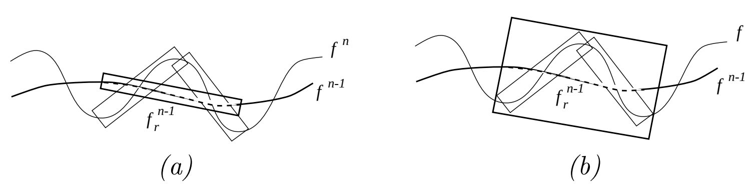
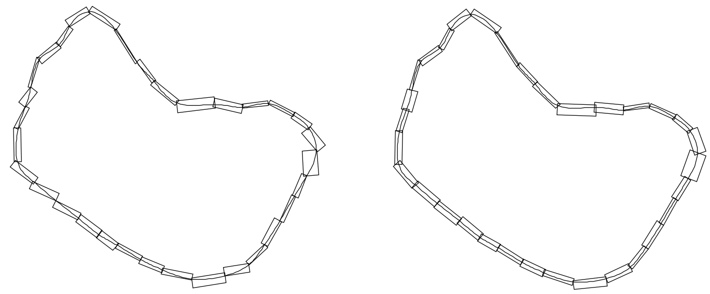


Figure 5: Generation of a super box based in multi-resolution segments: (a) adaptation; (b) adjustment.

3.2.2 adjustment

The point $\boldsymbol{\mu}$ is considered the coordinate origin for the axis $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, over that the vertices of the k elementary boxes are projected. The dimensions of the box are defined by the large projection segment on each axis; with these, we compute the vertices and the accurate centroid of B.

The method of adaptation in multi-resolution representation permit us to compute the main axis of super boxes more adapted form to the segment of corresponding resolution, in such way the boxes are adapted to the respective segments of f^n. In Figure 6 we can observe the differences between the super boxes defined by elementary boxes method *(a)* and the defined by multi-resolution representation *(b)*. The first method not generate boxes well held than the second method. Also, we can observe that the area of the boxes generated by multi-resolution representation are lesser than similar boxes gotten by the method of adaptation based on elementary boxes.



(a) boxes' area 5422.778 *(b) boxes' area 5302.826*

Figure 6: Generation of oriented boxes for level 4 of the tree: *(a)* adaptation defined for elementary boxes; *(b)* adaptation defined for segments in multi-resolution.

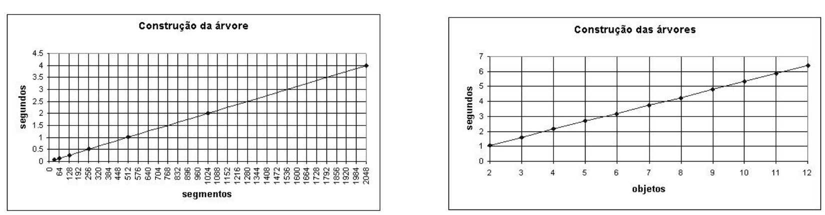
3.3 Tree construction evaluation

Each tree is associate to data structure that defines the object. It remains unalterable during the animation process. Number c of boxes of the tree for an object of $n = 2^k$ segments is given by

$$c = n + \frac{n}{2} + \frac{n}{4} + ... + \frac{n}{n} = 2n - 1.$$

The leves are composed by n boxes, the previous level for $n/2$. In general, the level j is formed by $n/2^j$ boxes. Each box is constructed in constant time, then the construction of a tree is made in linear time, $O(n)$.

Figure 7 shows the pictures of the behavior of the constructions of the trees respect to the time. The measurements had been made in a work station *Sun SPARCstation 20* with *Solaris*, being the taken time the minimum of the times of some repetitions in each case, due to politics system of work of station.



(a) *(b)*

Figure 7: Time of construction: (a) tree varying number of segments; (b) trees for objects of 256 segments each one.

4 Interference detection

If two objects are in interference, then some of its elementary boxes are intersecting. The analysis of interferences, formulated for Gottschalk et al. [4] and detailed in [12], is made recursively starting for the roots of the trees through the children while the intersection between boxes is detected. When finishing, in case that it has registered elementary boxes, it means that it is possible the interference of two objects, that will be verified analytically for its future treatment. Figure 8 shows two situations of two objects: separated or in interference.

(a) (b)

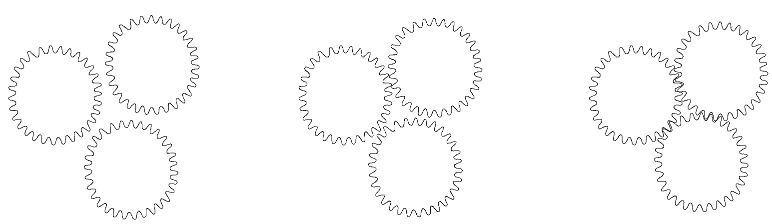
Figure 8: Situations: (a) separate; (b) in interference.

5 Results

The considered method was implemented in language C and was used, to visualize, graphical libraries IUP/LED and CD developed by the graphic technology group TecGraf-PUC-Rio. The method is efficient and robust for animations in real time.

Performance of the interferences detection

The efficiency of interferences detection in animations using the method of hierarchical bounding boxes for complex objects with disturbances inherits of OBBtree [4]. To illustrate the performance of the method, an example of three situations of three objects with 512 segments is formulated each one. The situations are shown in Figure 9: moved away objects, two objects in contact and, finally, a situation where the objects exist multiple contacts between all. The execution of comparison of the 3069 oriented boxes distributed in three trees was effected in the station Sun SPARCstation 20 with Solaris. The measured time is the minimum time in seconds of some experiments due to the multiprocessing system where the used machine works.



(a) *(b)* *(c)*

Figure 9: Objects with 512 segments each one: (a) moved away; (b) contact between two; (c) all in contacts.

Figure 10 shows a table with the gotten results. It is observed that, exactly when has interpenetration of objects, only one fraction of the boxes is compared (459 of the 3069) and the processing time is of the order of 0.01 seconds.

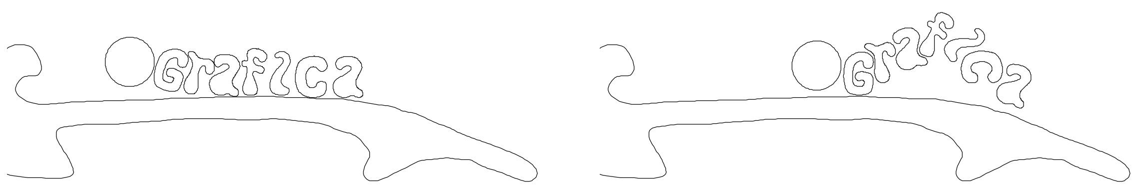
Examples of results in animations

They are presented as results some frames of animations of plain complex objects. In Figure 10 we can observe that the interference detection with the new method is similar or better than the elementary boxes based method. Figure 11 shows two frames of multiple objects animations, and Figure 12 shows their

Number of objets: 3 Total number of segments: 1536 Total number of boxes: 3069			
Situation	Time in seconds	Number of boxes tested	
		Elem. boxes	Multi-resolut.
(a)	0.0001	35	35
(b)	0.003	172	170
(c)	0.01	459	457

Figure 10: Numeric results of the situation of the Figure 9.

numerical result testing for interferences, where we can observe that any situation the boxes based in multi-resolution is better than the other method. Similar situation we observe in Figure 14 related to the Figure 13.



(a) *(b)*

Figure 11: Two frames of one sequence of an animation scene.

Number of trees: 9 Total number of segments: 615 Total number of boxes: 1225		
Situation	Number of boxes tested	
	Elem. boxes	Multi-resolut.
(a)	523	520
(b)	734	732

Figure 12: Numeric results of the situations in Figure 11.

6 Conclusions and future works

Hierarchic structures of bounding boxes permit us the quickly test for interference of complex objects in movement. So, we can isolate the pieces of the contour that possibly are in contact, that later the verification and the calculations of points of contacts is made by local procedures on the elementary segments of the object contour. In real times application, where the test for interferences is made in each time step, the quickly isolation of segments of object contours is very important for fast contact points detection. The isolation process must test a few pairs of bounding boxes, and by using few arithmetic operation in order to minimize the time consumed in contact detection process. So, we consider that our method is a best alternative for interference detection in animation and simulation that manipulates objects of complex geometry.

An application of the model formulated here is in the problem of part cutting in the manufactures. Each object represents a metallic part that we desire to cut of an entire material, that can be leather for shoes, blades, cards, etc. The objects compact following the physical laws, adding some heuristic to allow to put into motion objects until a static situation. Another application of fast interference detection is in

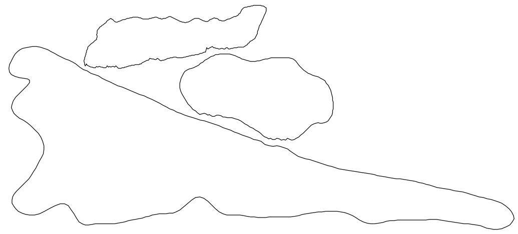


Figure 13: Irregular geometric objects in contact.

Number of trees: 3
Total number of segments: 1536
Total number of boxes: 3075
Boxes based in elementary boxes tested: 229
Boxes based in multi-resolution tested: 223

Figure 14: Numeric results for the situation in Figure 13.

the modelling of curves and surfaces for direct manipulation with interpenetration restrictions between the different segments of the same object.

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