# Point Cloud Denoising

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September 3, 2003

In order to denoise a point cloud our algorithm have the following steps:
i) Estimate an initial normal vector $n_p^{'}$ at each sample $p$.
ii) Determine a neighborhood set $N_p$ at $p$.
iii) Apply a robust smoothing operator $Q(.)$ at $p$ using the neighborhoot $N_p$.

We compute the normal vector $n_p^{'}$ at a point $p$ based in robust statistics theory, the normal vector is calculated as $n_p^{'} = \min_n \sum_{q \in K} g(n^t(q-p))w(\|p-q\|)$ subject to the restriction $\|n\| = 1$, where $K$ is the set of $k$-near neighbors of $p$ where $g(x) = \sigma - e^{\frac{-x^2}{2\sigma_s}}$, $w(x) = e^{\frac{-x^2}{2\sigma_c}}$ are the influence function and weight fuction respectively with $\Psi_s(x) = \frac{\sigma_s \cdot g_s^{'}(x)}{x}$, $\Psi_c(x) = w(x)$. This problem is solved using iterative method.

The neighborhood of the input data $p$ is computed constructiong an initial set $N_p$ with the k-near neightboors of $p$ and for each element $s \in N_p$ we add each k-near neighbor $s^{'}$ if the two condition are valid: i) $s^{'} \notin N_p$. ii) $\|s^{'} - p\| < 2\sigma_c$.

Once we have computed an initial normal $n_p^{'}$ and the neighborhood $N_p$, we apply the smoothing operator $Q(p) = p + t^* n^*$, where $n^*$ and $t^*$ satisfy the following minimization problem:

$$\{n^*, t^*\} = \min_{\{n,t\}} \sum_{q \in N_p} g(n^t(q-p-tn))w(\|q-p\|) \tag{1}$$

subject to $\|n\| = 1$. Minimizing the above equation is equivalent to solve the following two non-linear equations, the first is the partial derivative of (1) with respect to $t$

$$\sum_{q \in N_p} \Psi_c(\|q-p\|)\Psi_s(n^t(q-p-tn))(-(q-p)^t n + t) = 0 \tag{2}$$

and the second is the partial derivative of (1) with respect to $n$

$$\sum_{q \in N_p} \Psi_c(\|q-p\|)\Psi_s(n^t(q-p-tn))((q-p)(q-p)^t n - 2t(q-p)) = \lambda n \tag{3}$$

Given an initial normal $n_p^{'}$ we solve the equation (2) with respect to $t$ obtaining $t_{k+1} = k_{t_k}^{-1} \cdot \sum_{q \in N_p} \Psi_s(n^t(q-p-t_k n))\Psi_c(\|q-p\|)h_q$, where $h_q = n^t(q-p)$ is the height of the point $q$ with respect to the plane determined by $n$ and $p$ , the scalar $k_{t_k} = \sum_{q \in N_p} \Psi_s(n^t(q-p-t_k n))\Psi_c(\|q-p\|)$ is the sum of the weights. After few iterations of the above equation we obtain the vector $p^{'} = p + t_k n$ that smooth the noisy data in a better way than existing methods. Note that for the initial value $t_1 = 0$ we get $t_2 = k^{-1} \cdot \sum_{q \in N_p} \Psi_s(h_q)\Psi_c(\|p - q\|)h_q$ that is exactly the Fleishman et al method see [3]. When we have determined an optimal $t$ we minimize the equation (1) with respect to $n$ in a similar way to [4]. We iterate few steps interchanging the minimization with respect to $t$ and $n$ until we obtain an optimal value $Q(p) = p + t^* n^*$. The use of the influence function $g(.)$ make the smoothing method robust agains outliers and stop smoothing near of sharp features.

We also extend the methods in [2,3] to point clouds, we extend the Jones et al algorithm, computing the value $p^{'} = k^{-1} \cdot \sum_{q \in N_p^{\prime}} \Psi_s(\|\Pi(q) - p\|)\Psi_c(\|q - p\|)q$, where the predictor $\Pi(q)$ is computed using the normals $n_p^{'}$ and the neighborhood of $p$ is $N_p^{'} = \{q \in N_p \mid \|\Pi(q) - p\| < \sigma_s;\ \ n_q^{'} \cdot n_p^{'} > -\alpha\}$, the use of the neighborhood $N_p^{'}$ produce good results in thin regions where the existing method fails, it also enhances the features in the model. In oder to determine $N_p^{'}$ the normal has to be consistently oriented, we use EMST( extended minimum spanning tree ) in the same way as in [1] to orient the normals. In the case of Fleishman et al algorithm we compute $t = k^{-1} \cdot \sum_{q \in N_p^{\prime}} \Psi_s(\|\Pi(q) - p\|)\Psi_c(\|q - p\|)h$, where $h$ is height of the point $q$ and the smoothing value is $p^{'} = p + tn$. The use of the predictor $\Pi(q)$ and the neighborhood $N_p^{'}$ improve the original Fleishman algorithm.

# References


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