4–8 Factorization of Quadrilateral Subdivision

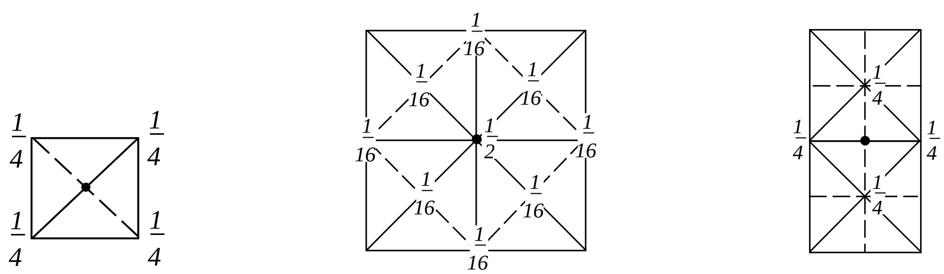
Luiz Velho
IMPA – Instituto de Matemática Pura e Aplicada
Estrada Dona Castorina 110, Rio de Janeiro, RJ, Brazil, 22460-320
lvelho@impa.br

Abstract

Mesh structures traditionally used for subdivision are derived from regular triangular or quadrilateral tilings. In contrast, the 4–8 mesh is based on the Laves tilings of type $[4.8^2]$, which is a *triangulated quadrangulation*. The semi-regular 4–8 mesh is a hierarchical structure for subdivision surfaces that has powerful adaptation capabilities. In this work, we show how to construct Catmull-Clark and Doo-Sabin surfaces, using 4–8 mesh refinement. We decompose the associated subdivision schemes into rules that are compatible with the underlying 4–8 mesh structure. Our motivation for developing such methods is to incorporate the power of 4–8 meshes into the above classical subdivision surfaces.

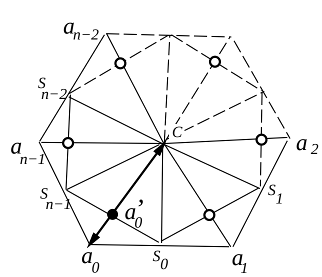
The refinement of 4–8 meshes is composed of two binary subdivision steps. In the first step, the mesh is refined in the horizontal and vertical directions, while in the second step, the mesh is refined in the two diagonal directions.

The principle for decomposing the Catmull-Clark subdivision scheme using 4–8 meshes is to distribute the rules at the appropriate steps of 4–8 refinement. According to this principle, face and corner rules are applied at even steps, while the edge rule is applied at odd steps. The associated masks are shown below:



(a) face (b) corner (c) edge

The principle to decompose the Doo-Sabin subdivision scheme is based on the observation that, in a 4–8 mesh, all dual blocks have a central vertex which is linked to each corner of the block, as shown below:



The decomposed 4–8 subdivision rule uses the old vertex a_0, and the midpoints, $\{s_i\}_{i=0,\ldots n-1}$, of the edges of the dual block. These midpoints are accessed through the center vertex c, because they are all in the star of c. From a_0 and s_i, $i = 1, \ldots, n-2$, we get the values of a_i, $i = 1, \ldots, n-1$. Since $s_i = \frac{a_i + a_{i+1}}{2}$, the value of $a_{i+1} = 2s_i - a_i$ Once that is done, we simply apply $a_0' = \sum_{i=0}^{n-1} \alpha_i a_i$, where α_i are coefficients of the Doo-Sabin rule: $\alpha_0 = \frac{1}{4} + \frac{5}{4n}$ and $\alpha_i = \frac{3+2\cos(2\pi i/n)}{4n}$, $i = 1, \ldots n-1$.

We remark that the above method is general, and allows the computation of other primal and dual subdivision schemes.