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I. Introduction

The study of 3-dimensional non-Euclidean manifolds brings up the question about wether it is possible or not to create a perceptively-friendly simulation of these mathematical objects. This natural curiosity has its root in the fact that we live in a 3-dimensional space, and it would be interesting to be perceptively immersed in other mathematical spaces different than ours. The conjectures on wether the universe has a possibly non-Euclidean, and maybe compact, topological structure when we consider cosmological distances, is another reason for our curiosity about these intriguing spaces.

To create this perceptive immersion, the problem of simulating the behavior of light in these spaces is of central importance but the immersive sensation is only partially achieved with light simulation. The simulation of sound in these spaces play a fundamental role in creating this immersive sensation, but we did not find any reference in literature about how is the sound behavior in non-Euclidean three-dimensional spaces.

We present here a ray-tracing based algorithm to simulate the sound propagation on these spaces. We also describe the implementation of this algorithm for three case studies, namely, the three dimensional torus, sphere and hyperbolic space. These three examples were part of the multi-media installation [Regards 3D], a french-brazilian collaboration that resulted in the [Exhibition] presented in April 2013 at the Forum of the Paris 13 campus.

We focus our attentions on the simulation of sound in certain closed spaces, where one source can be heard many times by a listener. This is due to the fact that several different paths of the same wavefront connect the listener to the same sound source. For that reason, our main concern in terms of implementation is on solving the problem of spatialization of several sources which will be discussed in the next section.

II. Related Work

The problem of simulating the light behavior in non-Euclidian spaces has first been faced by William Thuston at his Geometry Center ([Thurston W., 1994]). Recently [Berger et al.] presented a ray-tracing based algorithm that proved to be more efficient than the object based method used in the Geometry Center. An example of this efficiency is the possibility of real time drawing of up to 6^{100} copies of the same object instead of the 6^4 copies the other algorithm provided.

[Berger et al.] also presents the definition of a *3-manifold* and *orbifold*, develops the above mentioned visualization algorithm and its pipeline responsible for rendering the image of an object inside these *3-dimensional* spaces, and propose a method for building these spaces using Cell Complexes. The present work is a development of these results in the context of sound instead of light.

The first experiences on the spatialization of a sound source using several speakers dates back to the 1940's and has been the object of artistic and academic research since then. In [Malham, D. &] the author compares several techniques for creating virtual sound sources. Two of the more important ones are the ambisonic technique and the vector base amplitude panning. We use the last one to build our spatialization algorithm.

The problem of rendering complex audio scenes is well documented in literature (e.g. [Tsingos et al.], [Moeck et al.]) and typically a complex scene is reduced through clustering to a series of sound sources which are spatialized around the listener. In our case, we decided not to apply clustering algorithms and work with 125 different sources for technical reasons mentioned further on.

One of the most common applications for sound spatialization is audio rendering in the game industry. The above mentioned papers are examples of that, so as [Taylor et al.]. The last one uses the same approach as ours: ray-tracing of sound trajectory to define audio rendering, although its focus is on creating game-related applications.

III. Modeling sound in *3-orbifolds*

Briefly, what [Berger et al.] does is to approach the *3-orbifolds* as Cell Complexes, a finite collection $(D_\alpha)_{\alpha \in J}$ of disjoint closed sets that are "glued together" through an isometry g. The boundary of each D_α is a set of polygons $(P_{i,\alpha})$ and g takes each polygonal face $P_{i,\alpha}$ onto another $P_{j,\beta}$, for $\alpha, \beta \in J$.

A ray of light moving inside a *3-orbifold*, will then be seen as several geodesic segments moving inside the sets that form the respective Cell Complex. Whenever this ray crosses one side of the polygonal boundary $P_{i,\alpha}$, it will continue it's path in the other side of another boundary $P_{j,\beta}$ of the Complex $(D_\alpha)_{\alpha \in J}$.

Sound propagation can also be modeled using ray-tracing method (e.g. [Taylor et al.]), so if we consider a sound source and a listener inside a certain *3-orbifold* we can view them as if they were inside a certain Cell Complex. A ray that leaves the sound source can only be heard if it crosses the listener's ears. As above mentioned, this ray might cross several times the polygonal boundaries in its path. This means that not only the single ray which goes directly from the source to the listener's ear will be heard as is the case in the Euclidean space, and in general, the same sound source is heard several times and from different angles by a fixed listener.

In practical terms, apart from a non-linear change of variables, these rays can be calculated through the product of the position of the sound source in the fundamental domain by 4 dimensional matrixes that model the way each Cell of the Complex is connected to the other. By doing that, one source can be viewed as several ones, that can in its turn, be operated again by the same 4 dimensional matrixes. Each interaction will generate more and more sonic images of the same sound source and the quantity of these images is limited by the computer's computational power as we see in the next section.

We focused on the implementation of three different *3-orbifolds*, the sphere $\mathbb{S}^3$, the torus $\mathbb{R}^3/\mathbb{Z}^3$ and the hyperbolic space $\mathbb{H}^3$.

IV. Sound Spatialization

To create these sonic images, we have chosen to use a Mac Mini running Pure Data software, a MOTU Traveller sound card and four speakers attached to it. These speakers have been chosen to be placed in the same height, and in the vertices of a square around the listener.

The algorithm we have chosen to spatialize has been the vector base amplitude panning method(Vbap, see [**?**]) which has already an implementation for Pure Data. Practically, it assigns to each sonic image of the source a set of four coefficients, one to each speaker, that multiply the source's audio stream wave, creating a sensation of sound positioning for the listener in the middle of the speaker's square.

As a first experience on simulating sound propagation inside 3-dimensional manifolds, we decided to simulate up to 125 sonic images of the sound source. More than 125 has been proved to be inefficient using this hardware and software setup, which was the reason we limited this number.

V. Interactive Exhibition

The main goal of this implementation was to create a sound installation exhibition where the audience could move, through an interface, the position of a white sphere that acted as a sound source.

We used a Wii-mote inside an acrylic ball to serve as a steering wheel where the audience could guide the path of that sphere inside the manifold. Each time the sphere changed direction, a scratch sound was generated in its current position, and all its sonic images followed the original sound.

As we used the four speaker in the same height, for technical reasons, we had to project the sonic images on the plane parallel to the ground and crossing the listener's ears. Even though we did this projection, as the listener was positioned in the middle of the speaker's square, the immersive sensation was kept.

VI. Conclusion and Future Work

We presented the first simulation of sound propagation in 3-dimensional manifolds. Our goal was to develop the sonic aspects of the research presented in [Berger et al.] and to build a sound installation exhibition that has been presented in April 2013 at the Forum of the Paris 13 campus.

We plan to develop this research by creating virtual microphones and speakers that would simulate the sonic rays crossing the polygonal boundaries of the Cell Complex. This would use feedback technique that could create infinite sonic images instead of previously calculating a finite number of them.

References

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[Regards 3D] Regards dans les espaces de dimension 3 Web link: http://regards.math.univ-paris13.fr/

[Exhibition] Video of the exhibition Regards 3D Web link: "http://www.dailymotion.com/video/x10trbj_regards-dans-les-espaces-de-dimension-3_tech#.Ue_FmGRRhmn" Paris 13 University, April 2013.

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