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Image puzzle methods applied to the automatic reconstruction of ancient Portuguese tile panels

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Abstract. We explore the problem of automatically reconstructing an image from an unordered collection of rectangular non-overlapping tiles, an interesting and important formulation of the jigsaw puzzle problem. This is a challenging work, because the problem is classified as NP-complete. Here we describe and analyze two state-of-the-art image puzzle solvers and their application in the assembling of ancient Portuguese tile panels. We compare the obtained results in different formulations of the problem, depending on the prior knowledge of the puzzles: known or unknown puzzle dimension and tiles orientation.

1. Introduction

The traditional jigsaw puzzle is the problem of assembling several non-overlapping puzzle pieces that can be combined following a fitting and/or color pattern logic, with the final goal of obtaining a single plane or image.

Although this kind of puzzle is often considered a fun family activity, computational efforts for automatically solving it can generate solutions to related and very important scientific problems, such as reassembling of broken archaeological artifacts [1, 2], reconstruction of shredded documents [3, 4], speech recognition [5], DNA/RNA modeling [6], image editing [7], etc.

In this work, we analyze the problem of reconstructing images from rectangular non-overlapping puzzle pieces of identical shape and size, or tiles (Figure 1(b)). Unlike traditional jigsaw puzzle pieces (Figure 1(a)), tiles have regular boundaries and do not provide additional shape information, posing an even more challenging problem.

![Figure 1. Different formulations to the jigsaw puzzle problem.](image-url)
The described problem has been classified as NP-complete when the affinity between the tiles is uncertain [8]. Moreover any automatic puzzle solver has to deal with several difficulties: the combinatorial nature of the problem, in which the number of possible solutions increases super-exponentially according to the number of tiles; and the absence of a perfect, unambiguous and global compatibility between the tiles. To tackle these difficulties, solvers are generally divided into two main stages: computation of a compatibility value between the pieces and the puzzle assembling strategy (how the pieces are combined to yield the final result).

The first automatic solution for solving puzzles with traditional pieces was proposed in 1964 by Freeman and Gardner [9]. It considered apictorial puzzles, in which pieces are obtained by cutting a plane without chromatic variation, and was able to solve problems with 9 pieces. This method was never implemented, but it served as a basis for many subsequent work. Thirty years later, Kosiba et al. [10] were the first to take into consideration the chromatic information of the pieces, and was able to assemble 54 pieces with a greedy strategy.

The literature for the kind of puzzle considered here, with identical rectangular pieces, is somewhat recent and not extensive [11, 12, 13, 14, 15], although the problem is very important in practice. Moreover, not every work considers exactly the same a priori knowledge of the problem. Works by Cho et al. [11], Pomeranz et al. [12], and Andaló et al. [14] consider that the puzzle dimension and the orientation of each tile are known, opposed to the works by Gallagher [13] and Fonseca et al. [15]. All of them accept only square tiles, except by Andaló et al. [14] that can solve puzzles with arbitrary rectangular tiles, an useful characteristic when assembling shredded documents, for example.

Cho et al. [11] obtain an approximate reconstruction of the original image using graphical models and a probabilistic function maximized by Loopy Belief Propagation. However the method needs information about the layout of the original image, such as the correct location of some tiles informed by the user. Although being semi-automatic, this strategy allowed the assembling of puzzles up to 432 tiles.

Focusing in the disadvantage of the last work, Pomeranz et al. [12] presented a method that does not need user intervention. It is based in a greedy approach, in which a compatibility function is computed to measure the affinity between the tiles, and then the method solves three problems to assemble the pieces: positioning, segmentation, and translation. Given a single piece or a partial constructed puzzle, the positioning module places the remaining parts in a grid, following a predetermined logic; the segmentation module identifies the regions that are more likely to be assembled correctly using a region growing segmentation algorithm with random seeds; and the translation module reallocates the segmented parts and the tiles in the grid such that a better puzzle solution is constructed. With this strategy, they achieved the considerable improvement of solving puzzles with 3300 tiles.

Gallagher [13] proposed a new compatibility measure based on local gradients near the boundary of the tiles and a tree-based greedy assembling approach. The method is able to assemble puzzles with up to 9600 square pieces with unknown orientation and location (the image that forms the larger puzzle was not chosen naively, and contains certain properties that facilitates the assembling.)
The last two works [14, 15] are the state-of-the-art methods considering their respective problem formulation. The method *PSQP – Puzzle Solving by Quadratic Programming* – [14] solves the problem by reformulating it as a simple constrained quadratic programming problem and was tested in puzzles with up to 3300 tiles. The work by [15] presents a greedy method that minimizes distances between tiles at each iteration and can handle puzzles with 432 tiles. They will be described in the next section. We then evaluate both methods using a standard dataset of natural images, showing that they provide better results than the recent proposed methods [11, 12, 13].

We also consider a new application for image puzzle solvers in the art domain. Portuguese tiles, or *azulejos*, are a glazed ceramic tilework, and they have become an icon of Portugal’s culture. Because they have been used for decoration for almost five centuries, often the National Tile Museum (*Museu Nacional do Azulejo*, MNAz), in Lisbon, Portugal, receives donations of piles of tiles without assembling instructions. It is up to the museum restorers to assemble the tiles manually. In this work, we consider a database of Portuguese tile panels provided by MNAz and we apply both methods [14, 15] to automatically assemble them, showing promising results.

2. State-of-the-art methods

In this section, we describe two puzzle solvers [14, 15]. The solver by Andaló et al. [14] can be applied to puzzles with arbitrary rectangular tiles, with known dimension and orientation. The solver by Fonseca et al. [15] can be applied to puzzles with square tiles and unknown dimension and orientation. The following subsections briefly describe each solver. For a more detailed explanation, please refer to the original publications.

2.1. *PSQP – Puzzle Solving by Quadratic Programming*

The method proposed in [14], *PSQP – Puzzle Solving by Quadratic Programming* – is based in maximizing a global matching function which calculates the overall compatibility of a certain tile permutation.

Consider an image partitioned into a regular 2D grid, forming \( N \) tiles of identical dimensions; and an empty grid of the same size as the previous one with \( N \) locations. The problem is to determine a biunivocal correspondence between the \( N \) tiles and the \( N \) locations, optimal with respect to a global compatibility function \( \varepsilon(P) \). This function represents the sum of the compatibility values of the neighboring tiles, considering \( P \) as a permutation matrix that assigns tiles to locations (Figure 2).

The goal is to maximize \( \varepsilon(P) \) over all permutation matrices \( P \) of size \( N \times N \). Since this is a hard combinatorial optimization problem, it is necessary to extend the domain of the global compatibility function to the set of doubly stochastic matrices, and to reformulate the problem as a constrained continuous optimization problem, which can be solved by numerical methods.

The final global compatibility function to be maximized is

\[
\begin{align*}
\text{Maximize} & \quad f(p) = p^T A p, \\
\text{subject to} & \quad P1 = 1, \quad P^T 1 = 1, \quad \text{and} \quad p_{ij} \geq 0,
\end{align*}
\]
where $p$ is the column concatenation of $P$, $A$ is the Hessian of $\varepsilon(P)$, $\mathbf{1}$ is a column vector of size $N$ with all elements equal to one. A constrained gradient ascent algorithm, with gradient projection [16], is employed to search for local maxima of the problem.

2.2. Method by Fonseca et al.

The greedy method presented by Fonseca et al. [15] tries to minimize the distance between the tiles at each iteration of the algorithm, as tiles are connected to the final solution. It begins by computing a Global Distance Matrix $S$, of size $4N \times 4N$, that comprehends the distance between all tiles in every possible tile rotation. The lowest value is chosen and the corresponding tiles are put together in the final solution as neighbors.

At this point, there are six available borders in the solution so that a new tile can be connected, and $N - 2$ possible connections (tiles that have not been used yet). A $4N \times 4N$ mask is created and an element-wise product between the mask and $S$ provides the minimum value corresponding to the best tile connection. The purpose of this mask is to disallow new connections with tiles that have already been used in the final solution. This procedure is repeated until all tiles have been connected to the final solution.

To ensure a good quality result, an heuristic, called Lowe Scores, is also employed. When there are two tile candidates to be connected in the final solution, with close distance values according to a threshold, the connection is rejected. This heuristic was proposed by David G. Lowe [17] and suggests that a connection is not meaningful if the candidates have almost the same distance.
3. Experimental results

In this section we compare both solvers to the most recent methods in literature [11, 12, 13]. To do this, we first consider a standard database of twenty natural images provided by [11]. Each jigsaw puzzle consists of 432 tiles of size 28 × 28 pixels.

The accuracy of the solutions are measured according to three different metrics [11, 13]:

Direct comparison: the final solution is compared directly to the ground-truth image. This metric computes the ratio between the number of tiles that are assigned to the correct location and the total number of tiles.

Neighbor comparison: for each tile, this metric computes the fraction of its neighboring tiles that are also its neighbors in the correct solution. The accuracy is the mean fraction of correctly assigned neighbors.

Perfect reconstruction: binary indication of whether every tile is assigned to the correct location in a puzzle.

3.1. PSQP performance

We compare PSQP to the methods that consider the same puzzle formulation (known puzzle dimension and tile orientation) [11, 12].

The mean accuracy for PSQP was 96.0% under direct comparison, 95.6% under neighbor comparison, and 13 perfect reconstructions. Cho et al. [11], even by fixing some tiles in their right location, obtain 10% under direct comparison, 55% under neighbor comparison, and 0 perfect reconstructions. Pomeranz et al. [12] attained 91% under direct comparison, 94% under neighbor comparison, and 13 perfect reconstructions. Table 1 summarizes the performance values for each method and Figure 3 shows a reconstructed 540-tile puzzle using PSQP [14] and the method by Pomeranz et al. [12].

<table>
<thead>
<tr>
<th>Methods/Metrics</th>
<th>Direct (%)</th>
<th>Neighbor (%)</th>
<th># Perfect</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSQP 2012 [14]</td>
<td>96</td>
<td>95.6</td>
<td>13</td>
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</table>

3.2. Fonseca et al. [15] performance

We compare the method by Fonseca et al. [15] to the only method that also considers the same puzzle formulation (unknown puzzle dimension and tile orientation) [13]. Note that we are not reporting the accuracy under direct comparison in this case, because it is not provided in [15].

<table>
<thead>
<tr>
<th>Methods/Metrics</th>
<th>Neighbor (%)</th>
<th># Perfect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallagher 2012 [13]</td>
<td>90.4</td>
<td>9</td>
</tr>
</tbody>
</table>
Figure 3. Reconstructed image puzzle with 540 tiles, 28 x 28 pixels each. Left: PSQP [14] result with 100% accuracy under both metrics. Right: Pomeranz et al. [12], with 1% under direct comparison, and 64% under neighbor comparison.

4. Assembly of ancient Portuguese tiles

A tile, or azulejo in portuguese, is an important icon of Portugal’s culture. Painted tile panels are considered a decorative art and have been used throughout the last five centuries in both private interiors and façades of public buildings in Portugal [18]. The artists of azulejos were often inspired by or used to copy famous paintings or prints of those paintings [19]. As a result, thousands upon thousands of tiles were produced.

Several works have been devoted to the treatment and conservation of Portuguese tiles [18, 20], but when it comes to the assembling of panels made of tiles, the process is manual, which indicates a laborious and tedious work (Figure 4).

Often the National Tile Museum (Museu Nacional do Azulejo, MNAz), in Lisbon, Portugal, receives donations of unordered piles of tiles removed from panels of old façades. These tiles are donated without assembling instructions nor any other reference information. Historically they have been reconstructed manually, but here we analyze the application of image puzzle solvers to their automatic reconstruction.
Despite the previously discussed difficulties inherent to image puzzles, the assembling of Portuguese tiles poses a new set of challenges: unknown panel dimensions and tile orientation, the advanced degradation state of some tiles, and also panels missing some tiles. Other factors interfere in the process of comparing the tiles: variations in colors, cracks along the borders, non-continuous strokes from tile to tile, texture created by typically thorough strokes, among others.

To test the two puzzle solvers [14, 15], we considered twelve Portuguese tile panels with 25 tiles each provided by MNAz. Some examples are shown in Figure 5. The acquisition of the tiles was done by [15], in which they described a tool to automatically correct homography and the size of each tile.

Considering that only the panel dimensions are unknown, both methods can be tested [14, 15]. Note that, because the tested panels are small (25 tiles each), PSQP can be applied to the possible three configurations and the configuration that yields the highest global compatibility is chosen as the solution.

PSQP was able to reconstruct all the panels with 100% accuracy under all metrics, while the method by Fonseca et al. obtained 57.8% under neighbor comparison, and 0 perfect reconstructions.

When both panel dimensions and tile orientations are unknown, only the method by Fonseca et al. can be applied. The accuracy of the method is 35.9% under neighbor comparison, and 0 perfect reconstructions.

5. Conclusion

In this paper, we have studied two state-of-the-art image puzzle solvers when applied to the reconstruction of ancient Portuguese tile panels. The NMAz museum receives piles of tiles each year and the manual work to reconstruct them is very laborious. It is important to have a tool that can aid museum restorers to automatically assemble the panels or at least part of them.

Preliminary experimental results showed that the PSQP [14] is promising in reconstructing panels when the tile orientations are known. Nevertheless, it is important to extend its application to panels with unknown dimensions, because as more tiles are considered, it is impossible to test every configuration.
Although the accuracy of the method by Fonseca et al. [15] is not high when no a priori information about the panel is available, its application can aid restorers reconstruct several parts of the panels, and then visualize the big picture.

References


