
Publicações Matemáticas do IMPA.

The objective of this book is to provide a brief and clear presentation of a novel idea in data acquisition consists of sparse or compressible signals reconstruction from a very limited number of measurements, possibly contaminated with noise. This technique known as “compressed sensing” or “compressive sampling” (CS). CS rethink DSP fundamentals based on Nyquist rate theory and appears to become more and more popular in engineering communities nowadays.

CS can be formulated as a classical recovery problem in signal/image processing: One is interested in reconstructing a vector $u \in \mathbb{R}^m$ from given linear functionals $(u, \phi_j)$, $j = 1, 2, \ldots, n$, with some known values $\phi_1, \ldots, \phi_n \in \mathbb{R}^m$. In the most typical and demanding applications, $n$ is substantially smaller than $m$. In order to obtain an efficient algorithm it is particularly important to choose the $\phi_j$ carefully.

This book starts from classical methods for image compression based on DCT, PCA and Wavelets and apply the sample-then-compress framework. In the next chapter authors provide a brief introduction to signal processing eventually approaching the idea that it is possible to sample the original signal in a rate smaller than the Nyquist limit and reconstruct it by means of optimization procedure. Finally, the authors verify CS theory by means of examples in image acquisition and discuss new applications in Computer Graphics and Vision. It is to be noted that CS theory is related to concepts of Kolmogorov widths and Gelfand widths which are well known in approximation theory. The connections between the CS problem and the problem of Kolmogorov widths estimation were studied by B. S. Kashin and V. N. Temlyakov [Math. Notes 82, No. 5, 748–755 (2007); translation from Mat. Zametki 82, No. 6, 829–837 (2007; Zbl 1155.94014)]. Covering most of the recent publications on CS, this book can be used as a substantial bibliographic guide on compressed sensing.