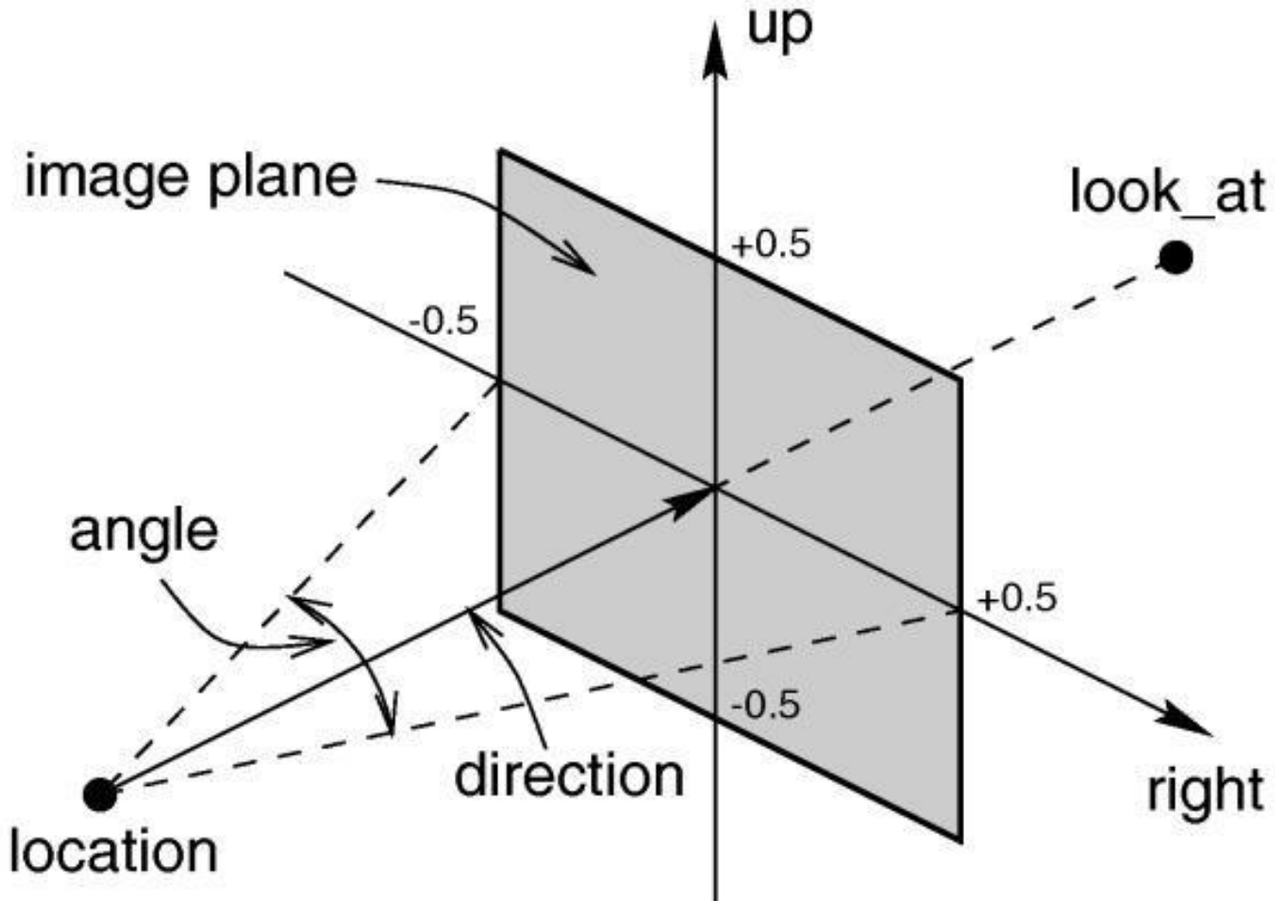
Rays in curved spaces

Tiago Novello
IMPA

Ray tracing

- n-dimensional space
- Points
- Vectors
- Metric
- Inner product



Turner Whitted

Ray tracing

- n-dimensional space
- Points
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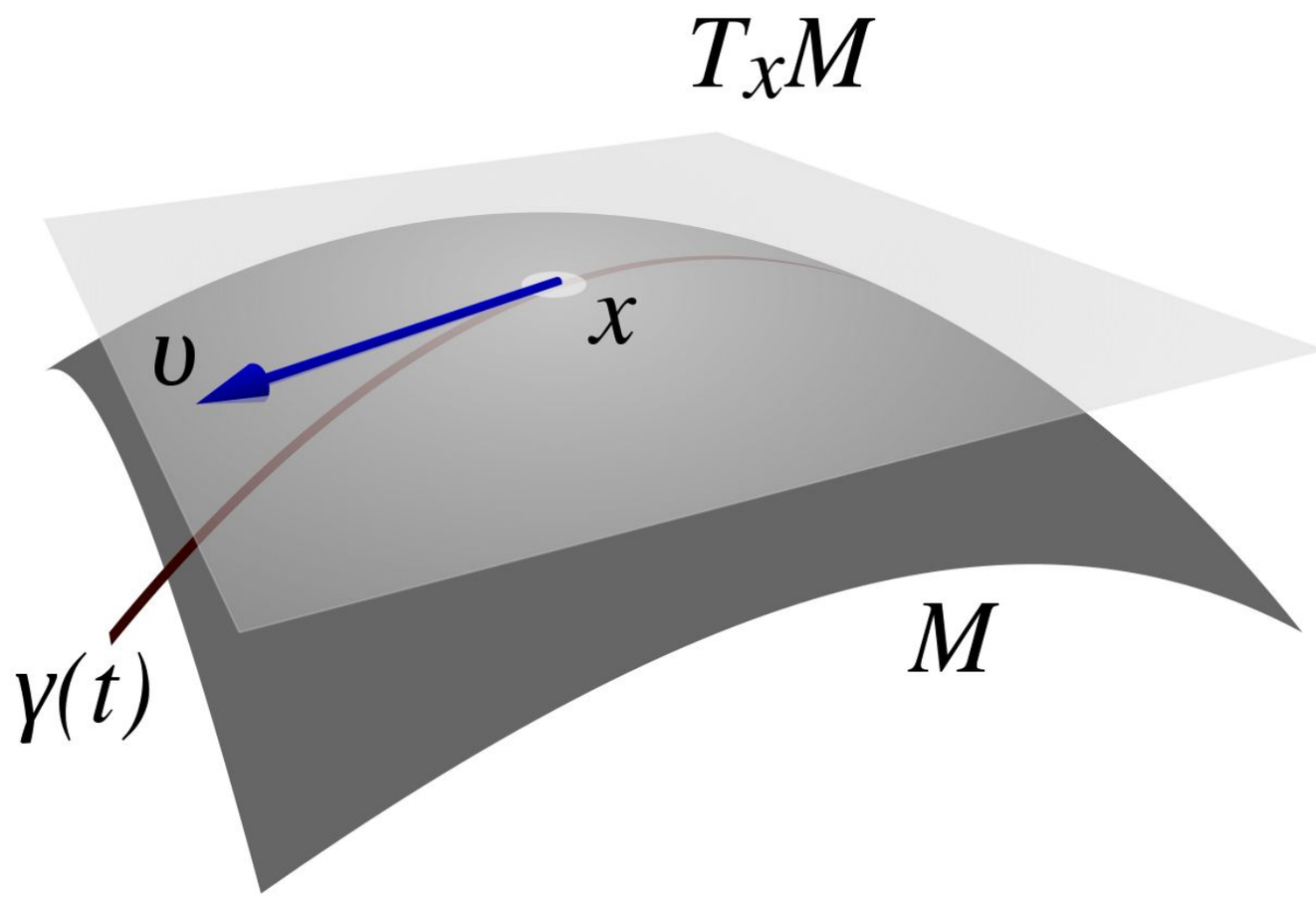
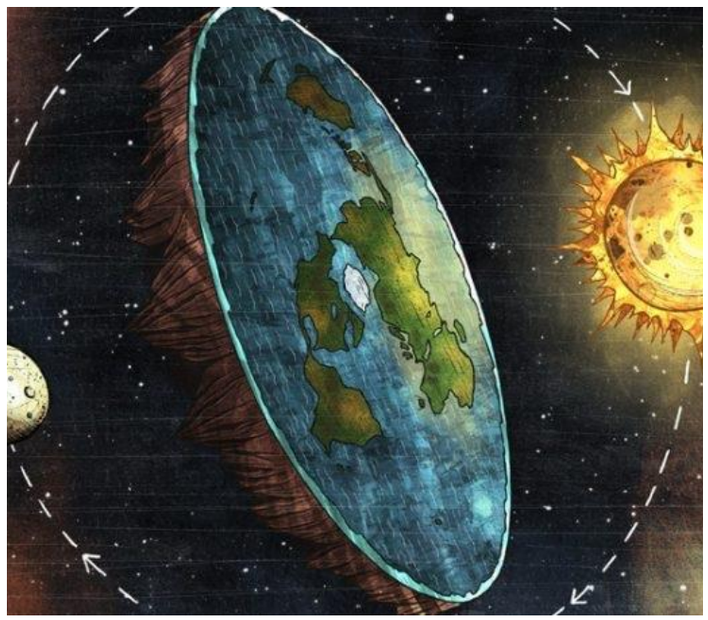
- Inner product

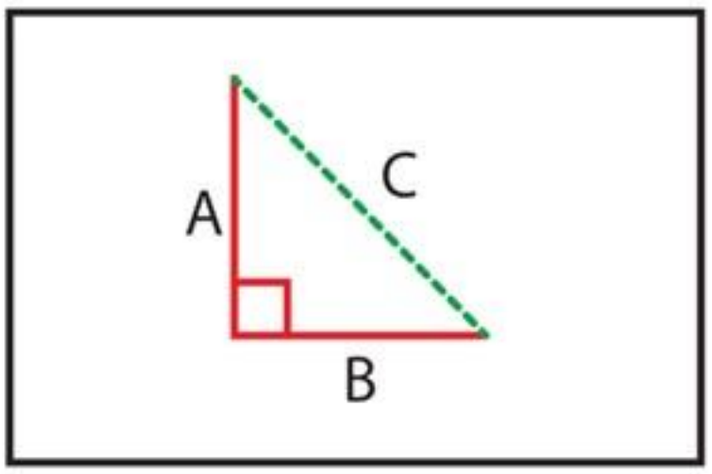


Bernhard Riemann

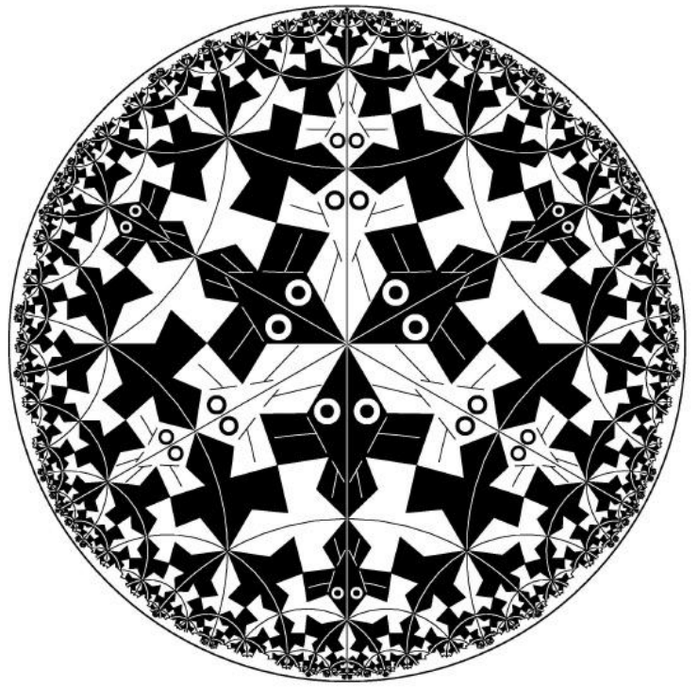
Models

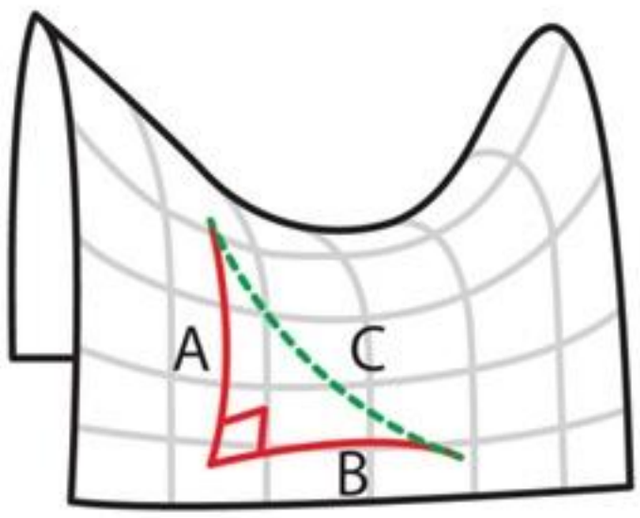
Euclidean



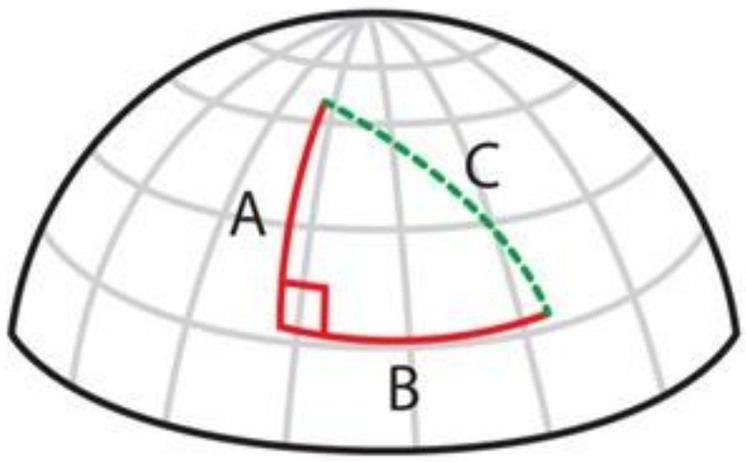


Hyperbolic





Spherical



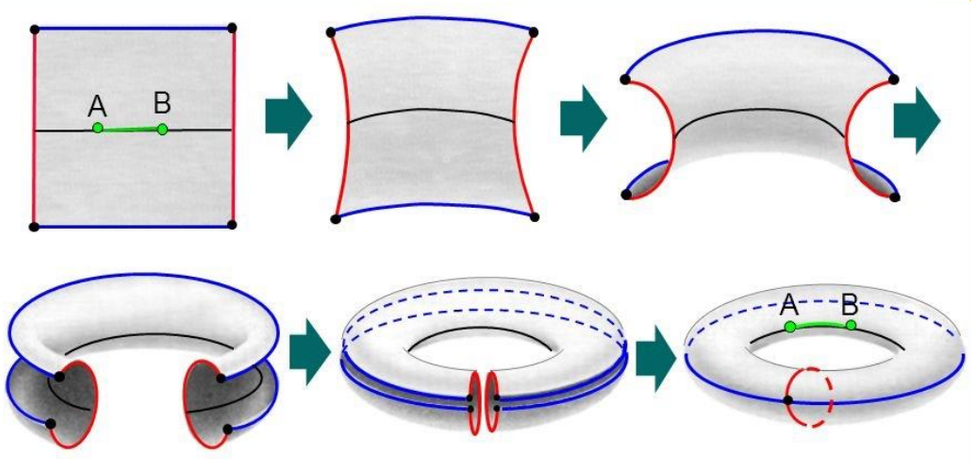
Models

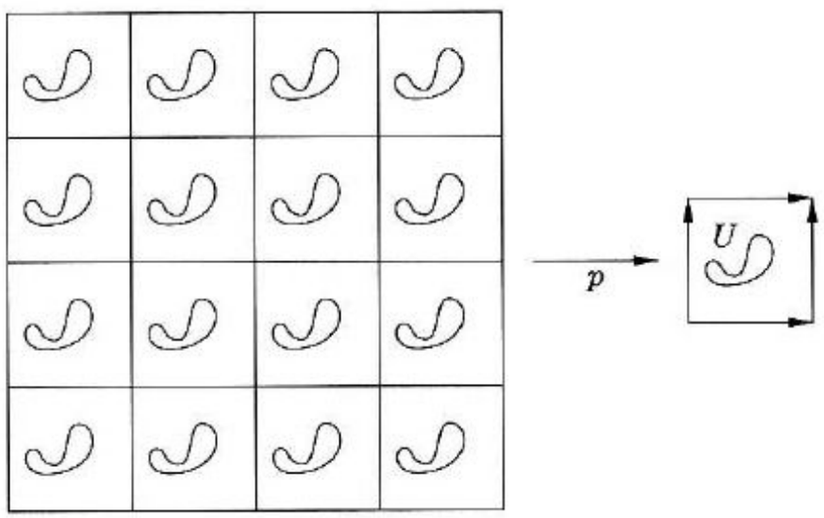
Elements	Euclidean	Hyperbolic	Spherical
Space	n-dimensional vector space	Points with “norm” -1	Points with norm 1
Points	n-tuples	(n+1)-tuples	(n+1)-tuples
Inner product	Euclidean	Lorentz	Euclidean
Norm	Euclidean	Lorentz	Euclidean
Distance	Euclidean	“Angle”	Angle
Vectors	n-tuples	tangents (n+1)-tuples	tangents (n+1)-tuples
Rays	Straight lines	Hyperboles	Big circles
Isometries	Translation + rotation	“Rotations”	Rotations

2D geometrization

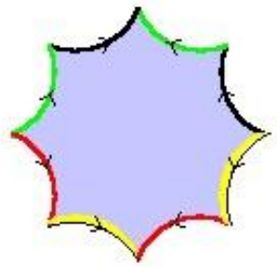
Klein--Poincaré conjectured and Poincaré--Koebe proved

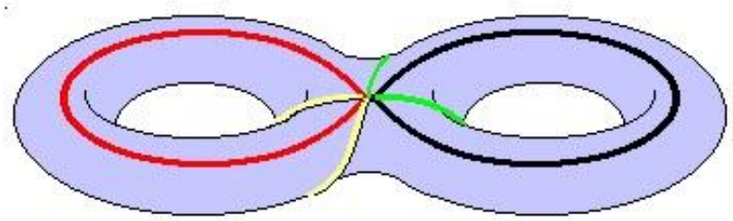
Flat torus

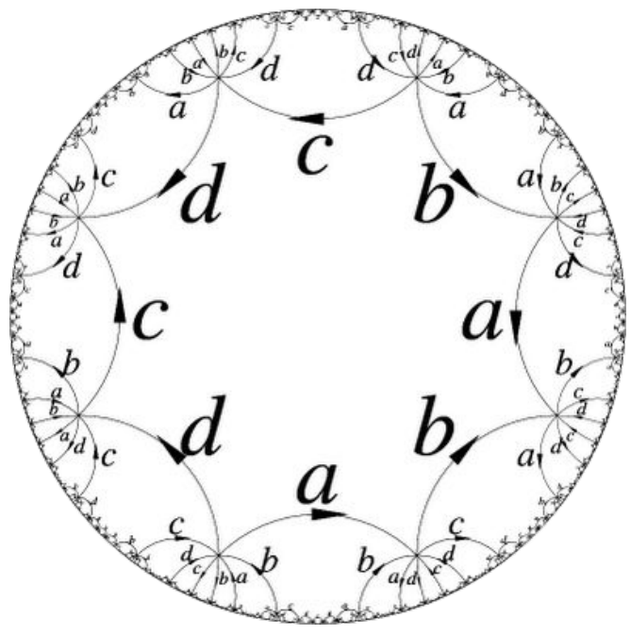




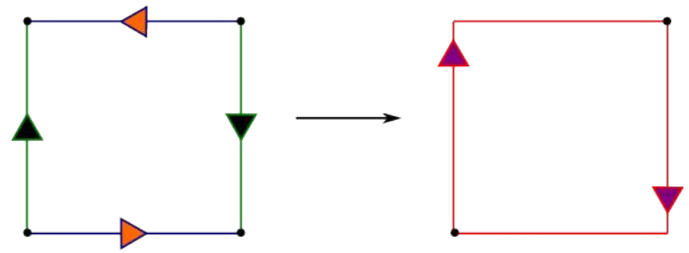
Bitorus

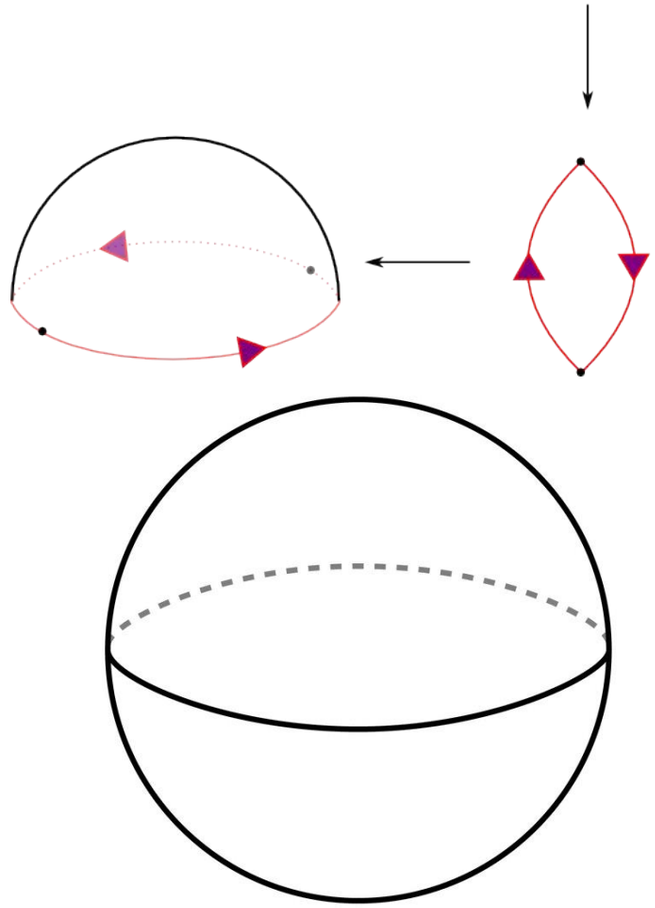






Projective space

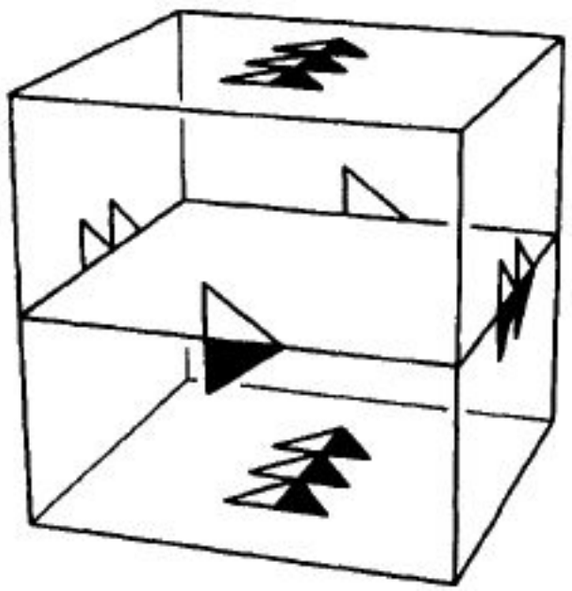




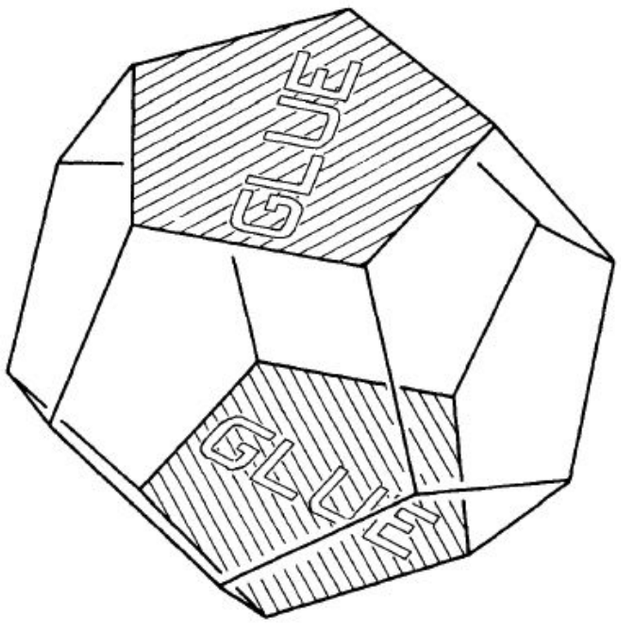
3D geometrization

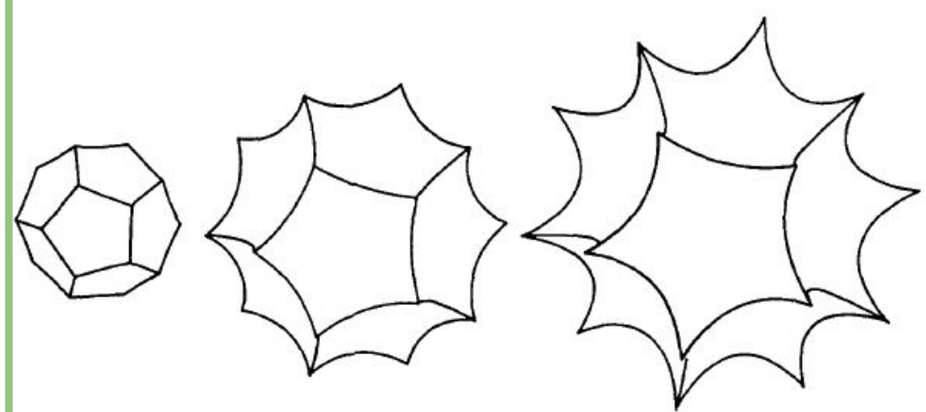
Thurston conjectured and Perelman proved

Flat torus

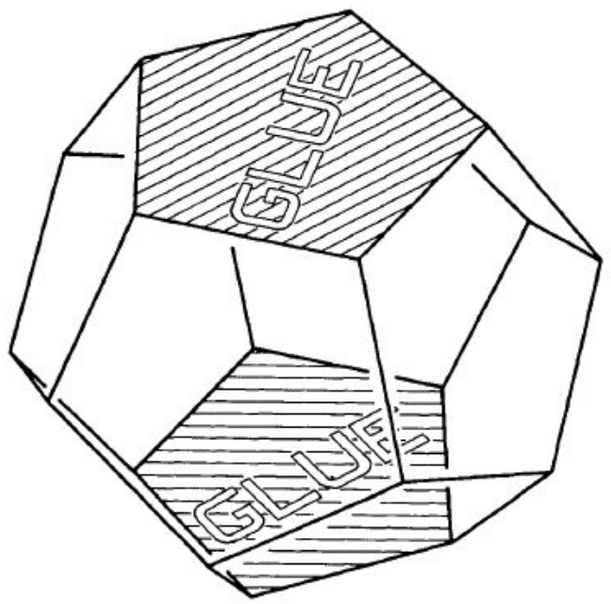


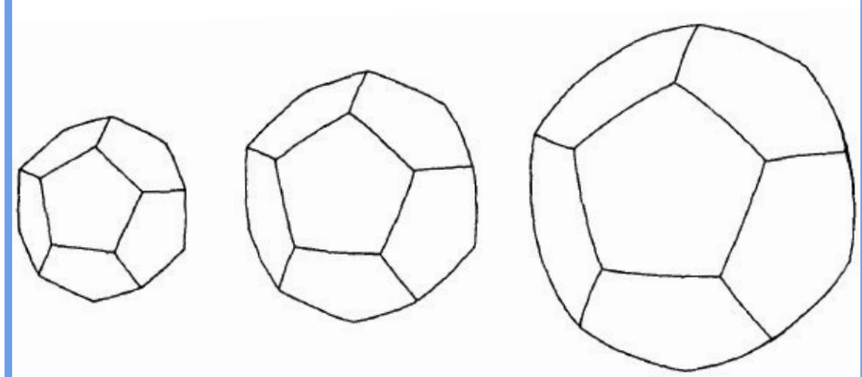
Seifert-Weber dodecahedron space



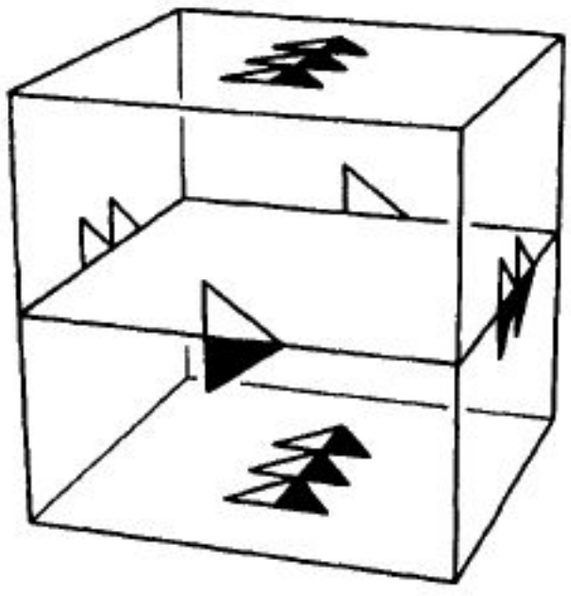


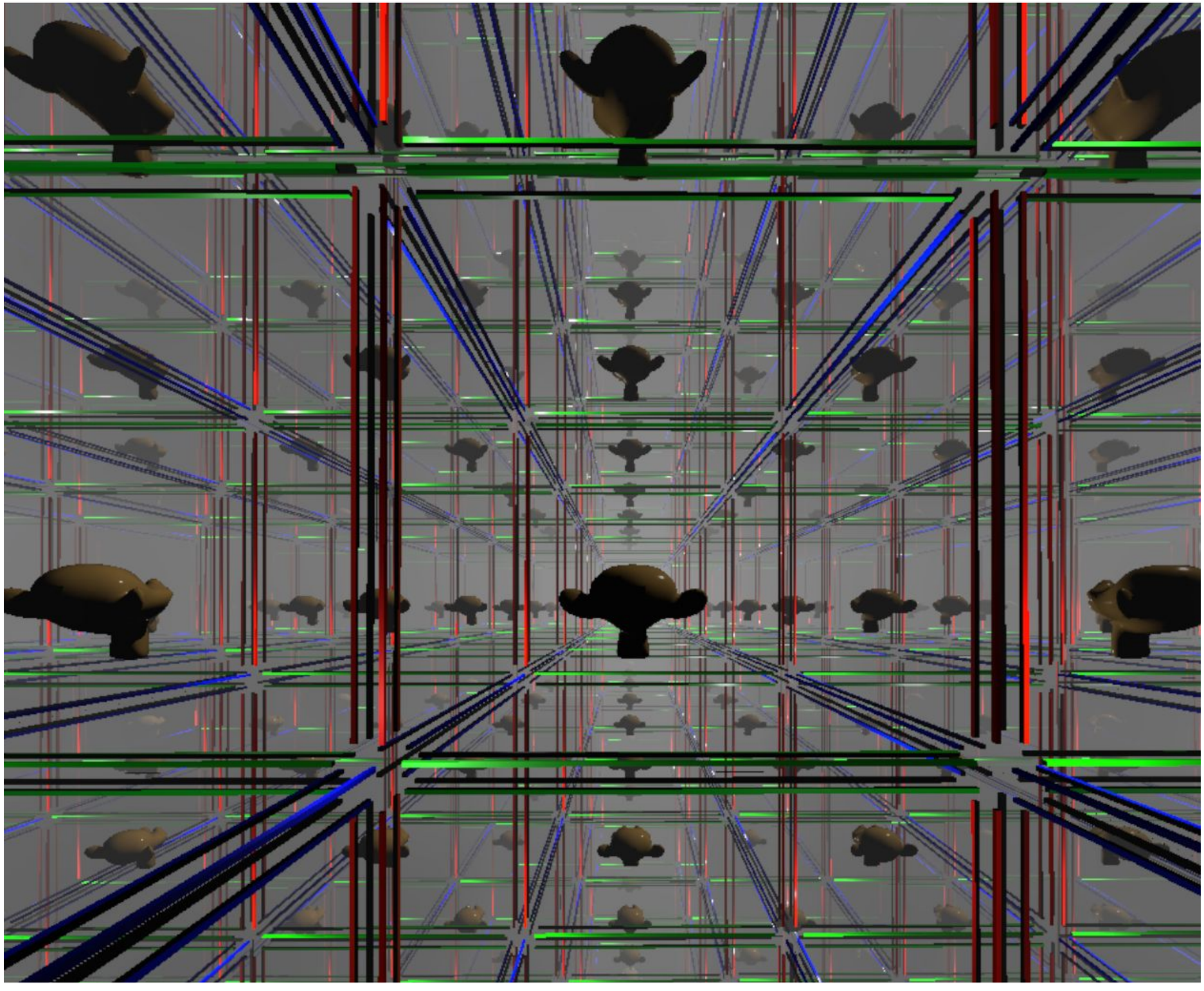
Poincaré dodecahedron space



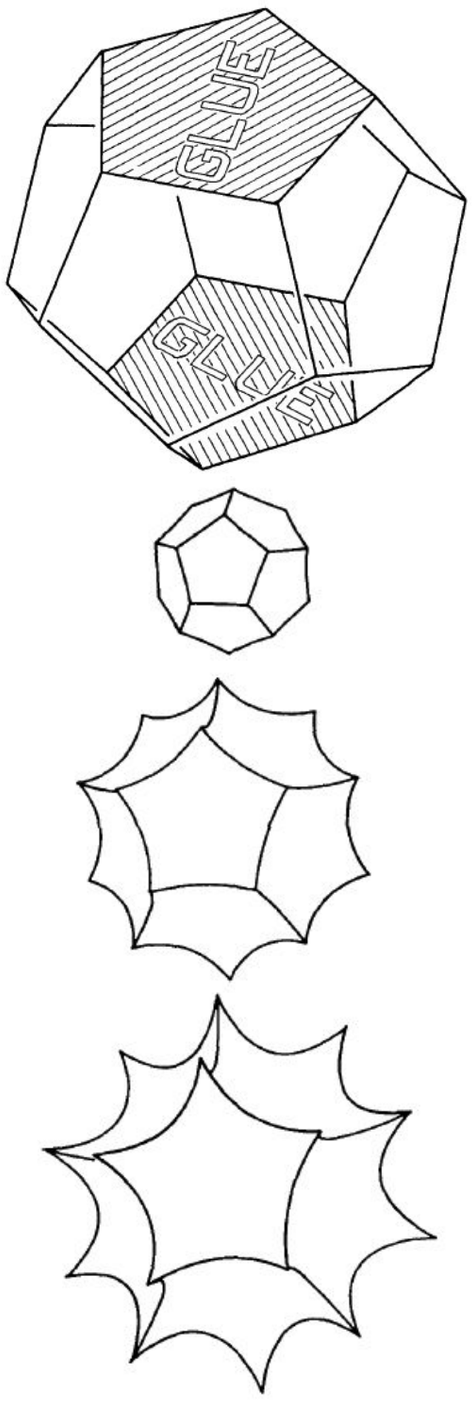


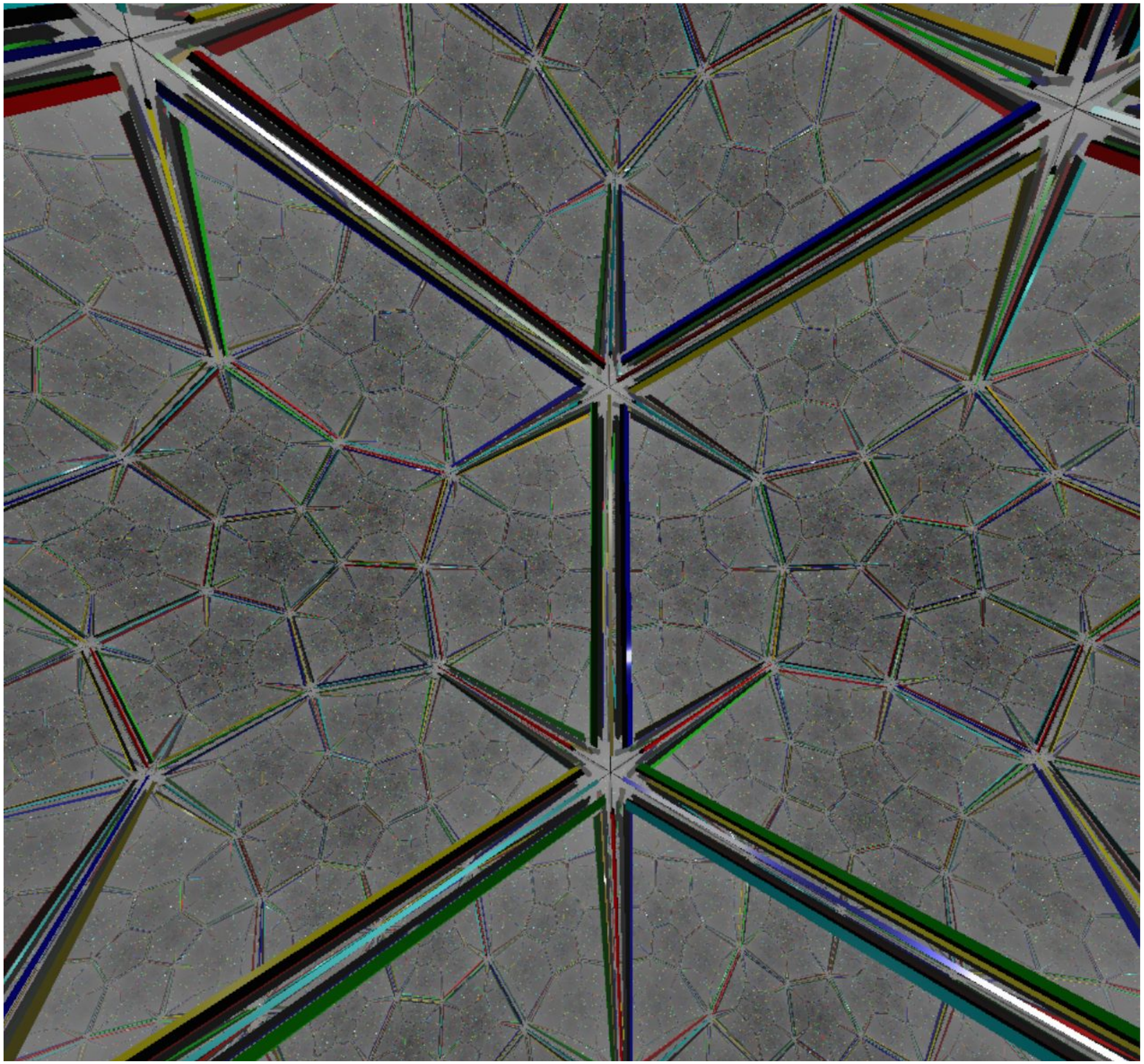
Flat torus



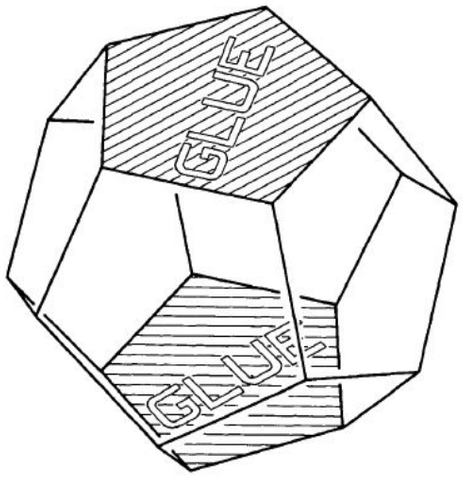


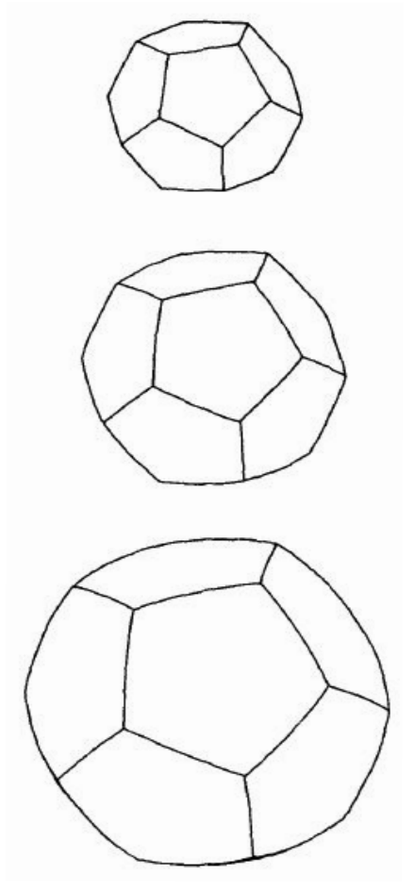
Seifert-Weber dodecahedron space

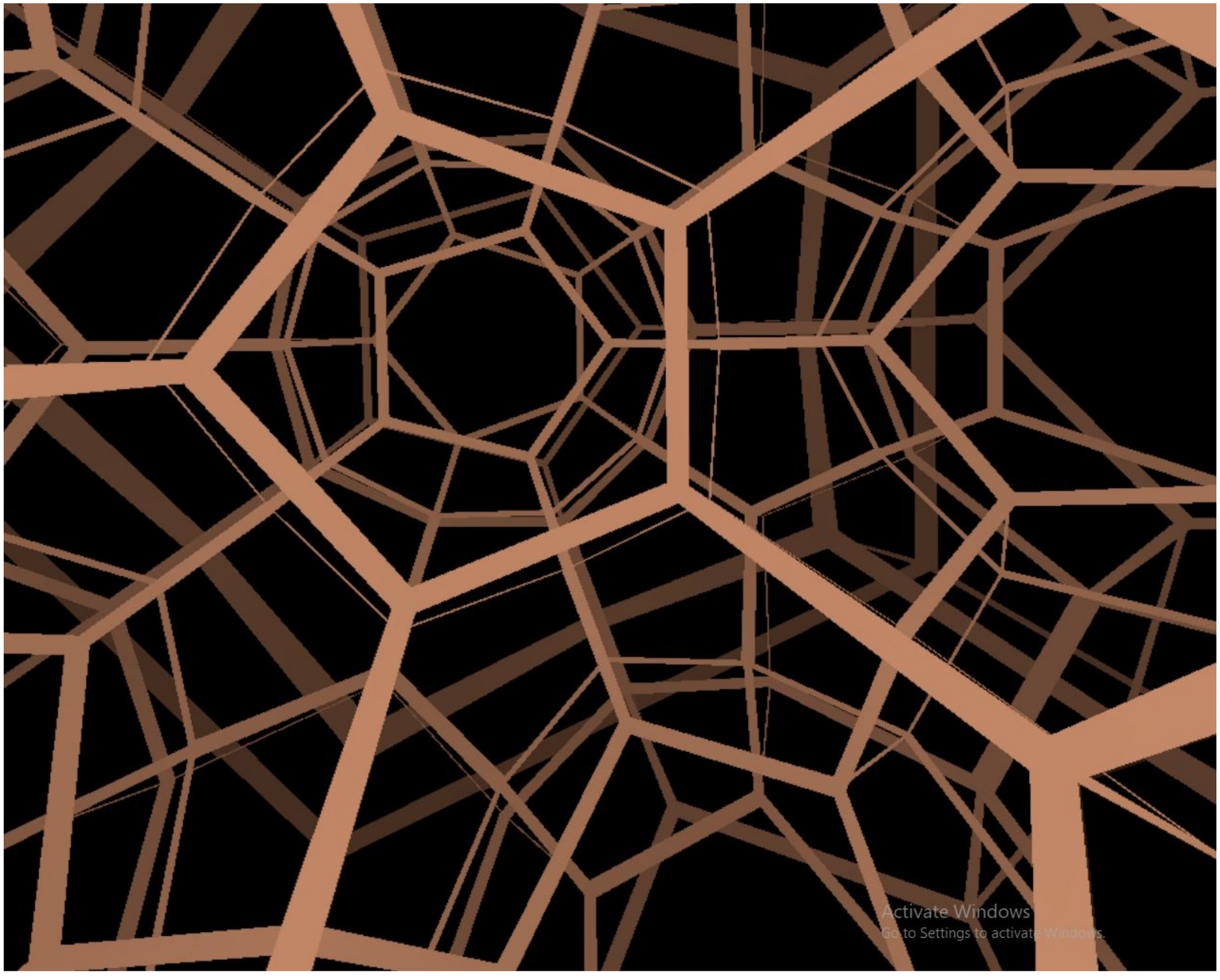




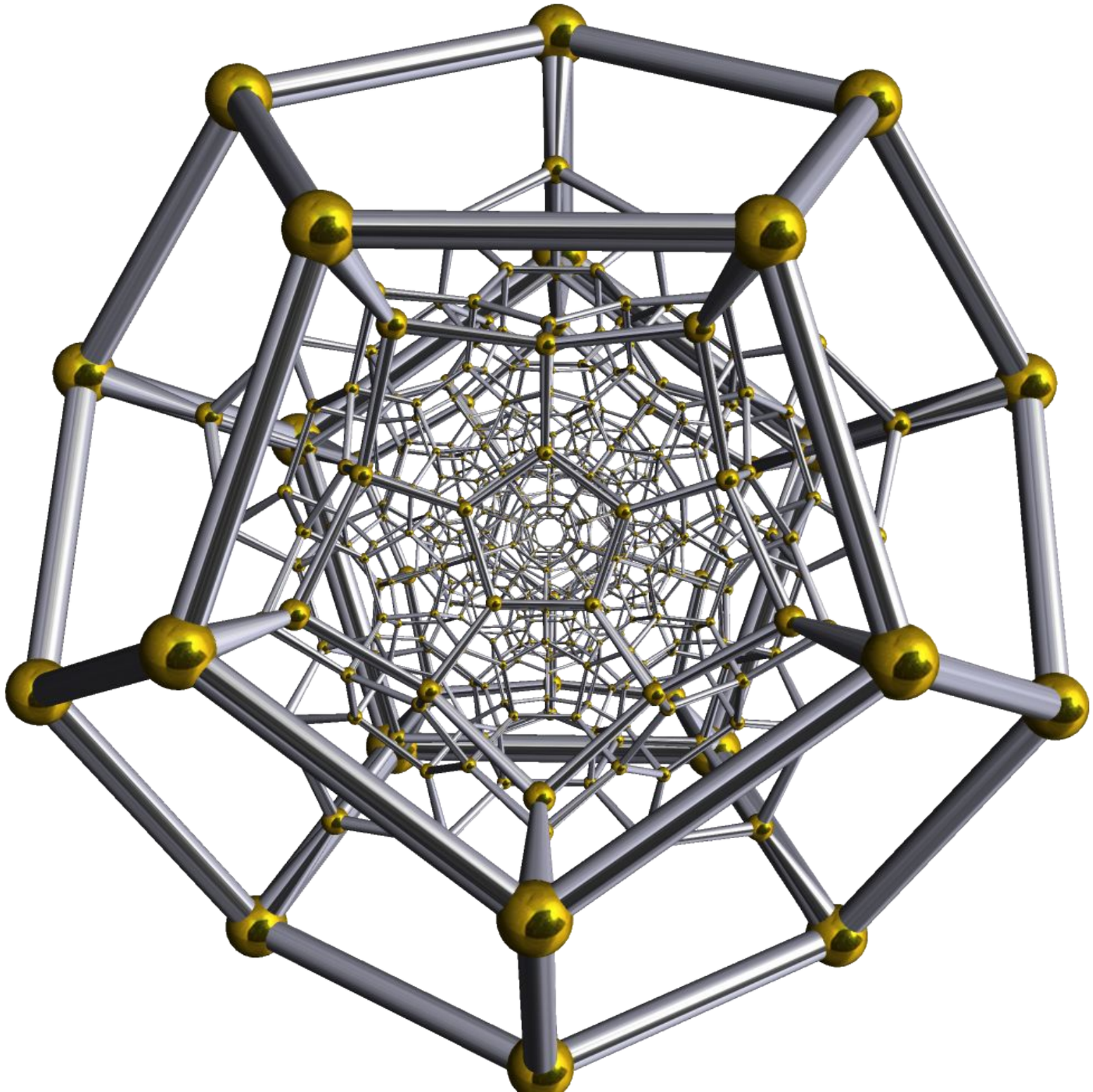
Poincaré dodecahedron space



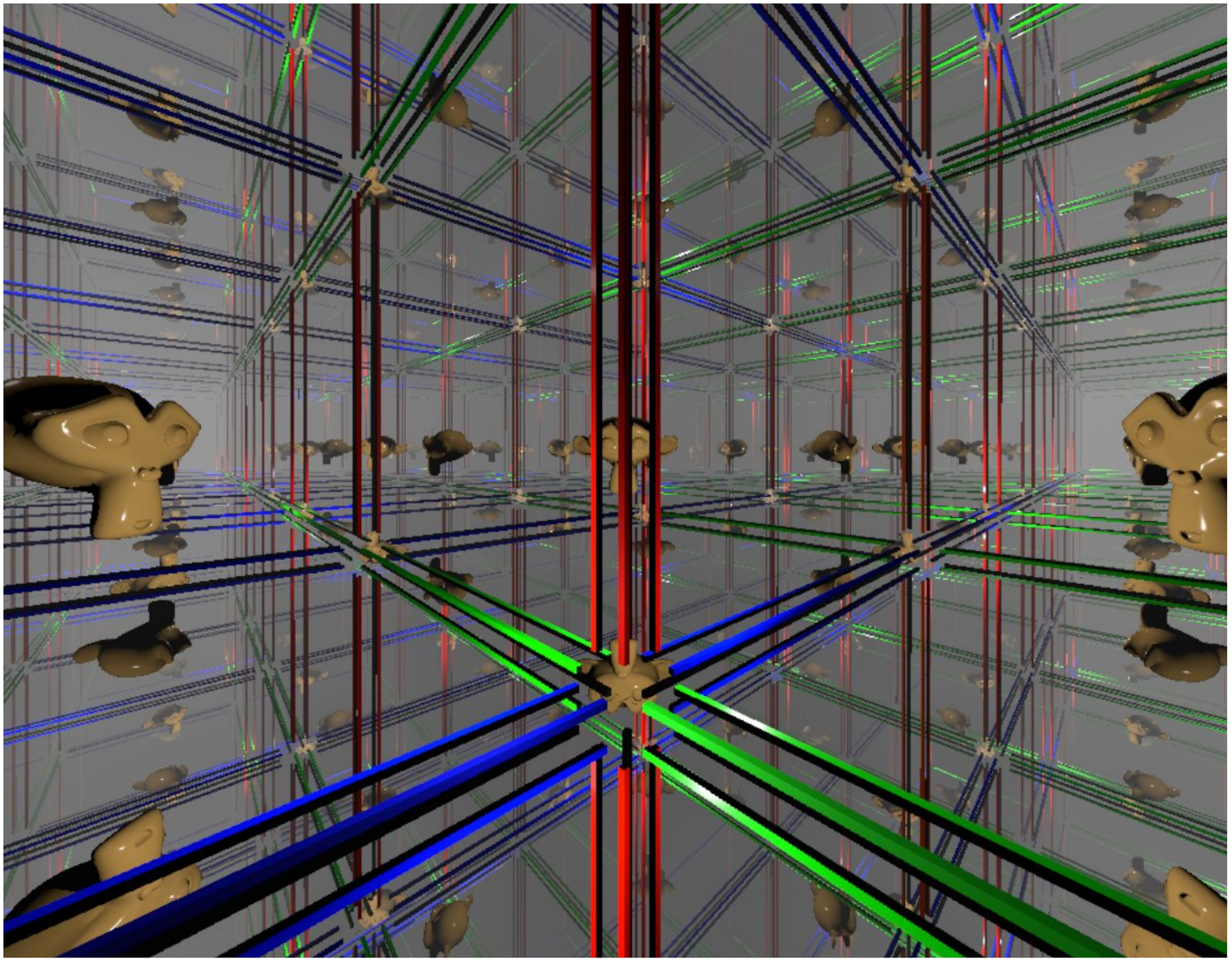




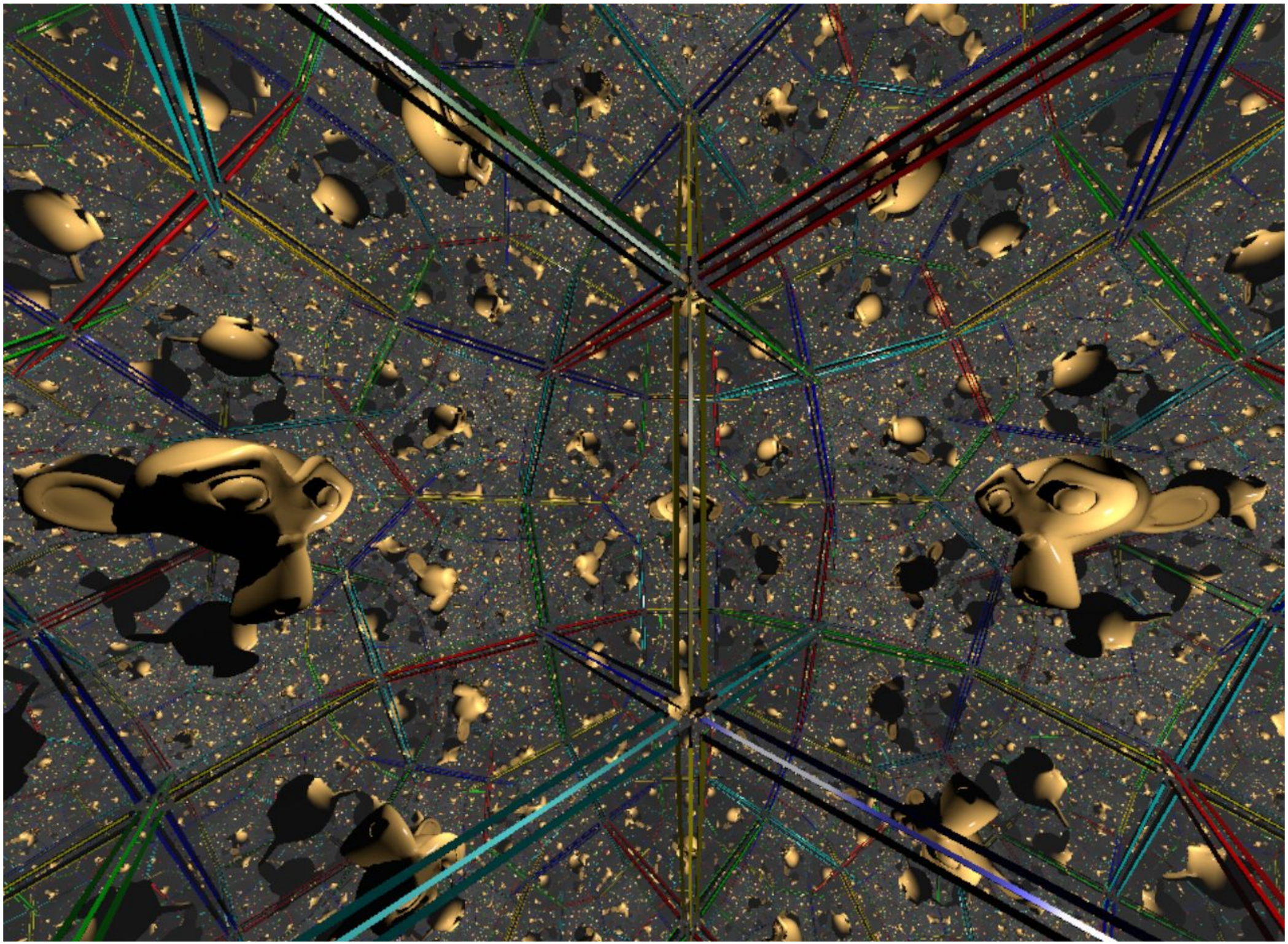
120-cell



Mirrored cube



Mirrored dodecahedron



Implementation

- Blender
- MeshLab
- Falcor / DirectX

Obrigado!