The complexity of classical music networks

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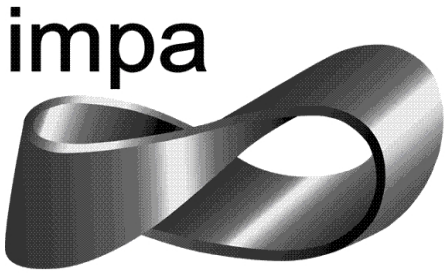

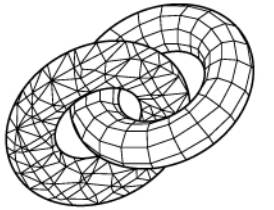

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Summary

Introduction

Related Work

Musical Networks

Scale-free

Small-world

Results

Fractal Nature of Music

Conclusions and Future Work

Introduction

Introduction

40 pieces of classical music → MIDI format

Bach (6), Beethoven (9), Brahms (1), Chopin (1), Clementi (6), Haydn (5), Mozart (7), Schubert (4), and Shostakovitch (1)

Built a network from each piece of music

Perform scale-free and small-world tests

Related Work → Music

Related Work → Music

- Liu et al.

  “Complex network structure of musical compositions: Algorithmic generation of appealing music”

  Physica A: Statistical Mechanics and its Applications (2010)

  I.F. 2,243
  63 citations

- Perkins et al.

  “A scaling law for random walks on networks”

  Nature Communications (2014)

  I.F. 12,124
  18 citations

- Ferretti

  "On the Complex Network Structure of Musical Pieces: Analysis of Some Use Cases from Different Music Genres"

  Multimedia Tools and Applications (2017)

  I.F. 1,530
  1 citation

Related Work → Music

- Liu et al.

  “Complex network structure of musical compositions: Algorithmic generation of appealing music”

  Physica A: Statistical Mechanics and its Applications (2010)

  Scale-free: Yes Small-world: Yes

  I.F. 2,243
  63 citations

- Perkins et al.

  “A scaling law for random walks on networks”

  Nature Communications (2014)

  Scale-free: Yes Small-world: No report

  I.F. 12,124
  18 citations

- Ferretti

  "On the Complex Network Structure of Musical Pieces: Analysis of Some Use Cases from Different Music Genres"

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  Scale-free: Yes Small-world: Yes

  I.F. 1,530
  1 citation

Related Work → Music

- Liu et al.

  “Complex network structure of musical compositions: Algorithmic generation of appealing music”

  Physica A: Statistical Mechanics and its Applications (2010)

  202 pieces → Classic & Chinese Pop

  I.F. 2,243
  63 citations

- Perkins et al.

  “A scaling law for random walks on networks”

  Nature Communications (2014)

  8473 pieces → Folk from Europe & China

  I.F. 12,124
  18 citations

- Ferretti

  "On the Complex Network Structure of Musical Pieces: Analysis of Some Use Cases from Different Music Genres"

  Multimedia Tools and Applications (2017)

  8 pieces → Rock, Blues, Jazz...

  I.F. 1,530
  1 citation

Related Work → Math Tests

Related Work → Math Tests

- Clauset et al.

  “Power-law distributions in empirical data”

  Siam Review (2010)

  I.F. 4,897
  5947 citations

- Watts & Strogatz

  “Collective dynamics of ‘small-world’ networks”

  Nature (1998)

  I.F. 40,137
  35731 citations

- Newman & Watts

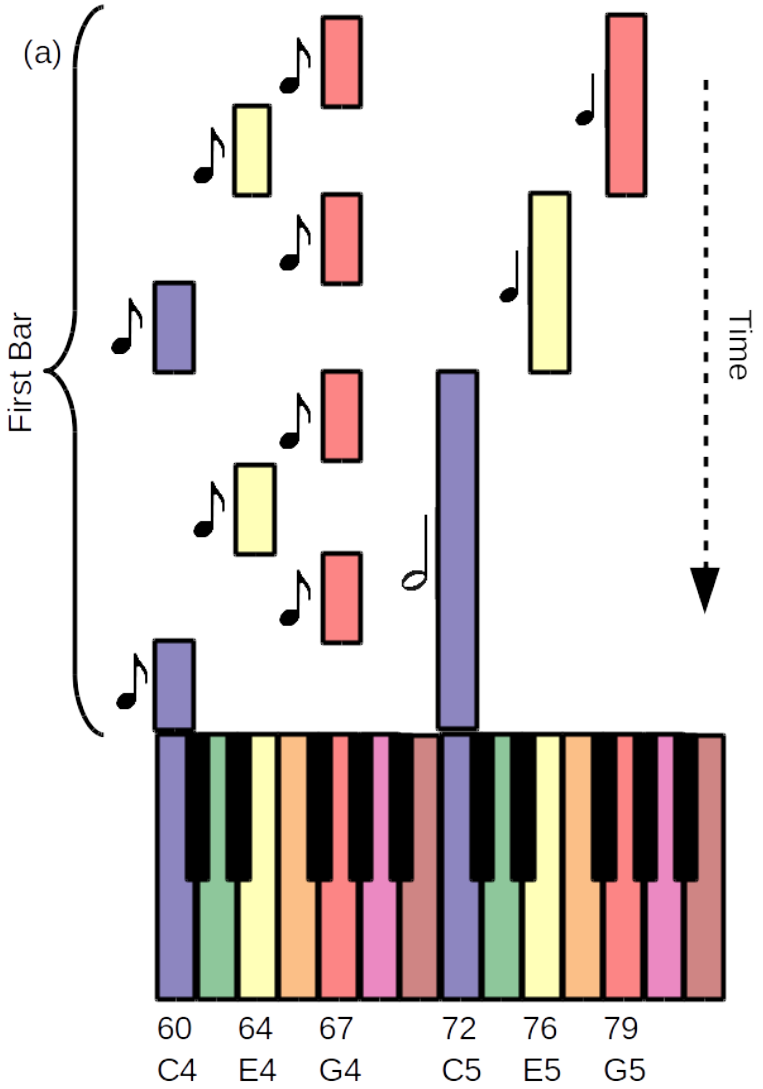
  "Renormalization group analysis of the small-world network model"

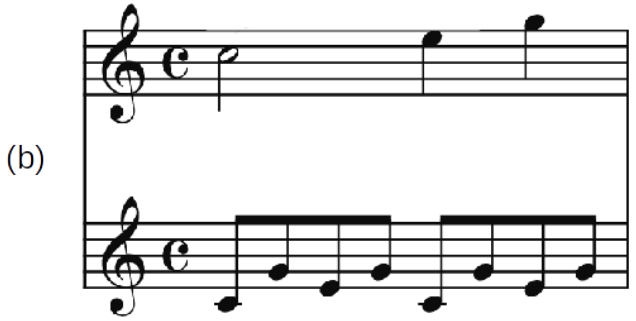
  Physics Letters A - Elsevier (1999)

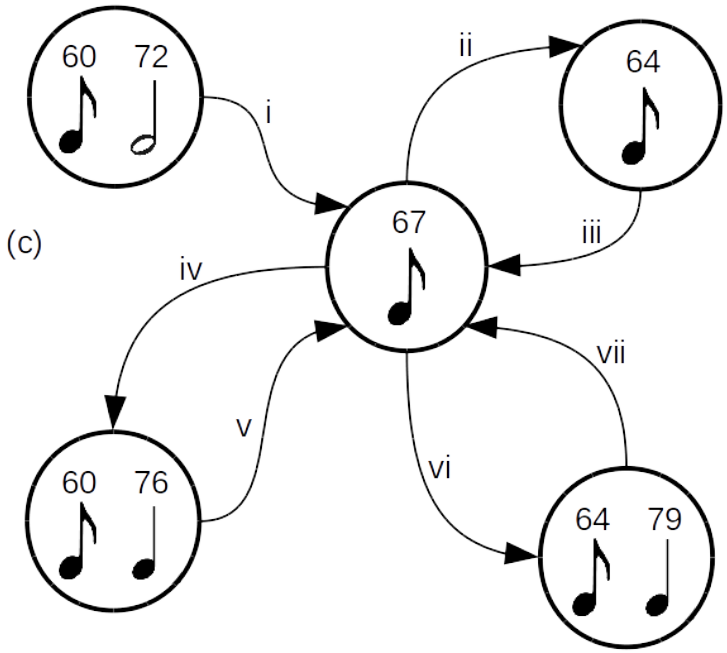
  I.F. 1,772
  1364 citations

Musical Networks

Musical Networks







(d)

Time	Event	Note
0	Note-ON	72
0	Note-ON	60
240	Note-OFF	60
240	Note-ON	67
480	Note-OFF	67
480	Note-ON	64
720	Note-OFF	64
720	Note-ON	67
960	Note-OFF	67
960	Note-OFF	72
960	Note-ON	76
960	Note-ON	60
1200	Note-OFF	60
1200	Note-ON	67
1440	Note-OFF	67
1440	Note-OFF	76
1440	Note-ON	79
1440	Note-ON	64
1680	Note-OFF	64
1680	Note-ON	67
1920	Note-OFF	67

Mozart's Sonata No. 16 (KV 545) first bar

Musical Networks

Project's website:

http://w3.impa.br/~vitorgr/CNA/index.html

Python/NetworkX

Software for complex networks

https://networkx.github.io/

Scale-free Property

Scale-free Property

Node degree distribution → Power law estimation

Least squares method (Old) → used by Liu and Perkins

Clauset's test
- i. Maximum likelihood estimation (α)
- ii. Kolmogorov-Smirnov (*p*-value > 0.1)
- iii. Likelihood Ratio (LR)
  - → power law vs. alternative hypotheses: log-normal, exponential, stretched exp

- Cohen & Havlin

  "Scale-free networks are ultrasmall" → $2 < \alpha < 3$

  Physical Review Letters (2003) I.F. 8,462 801 citations

Related Work → Music

- Liu et al.

  “Complex network structure of musical compositions: Algorithmic generation of appealing music” → $1 < \alpha < 2$

  Physica A: Statistical Mechanics and its Applications (2010)

- Perkins et al.

  “A scaling law for random walks on networks” → $1{,}05 < \alpha < 1{,}28$

  Nature Communications (2014)

- Ferretti

  "On the Complex Network Structure of Musical Pieces: Analysis of Some Use Cases from Different Music Genres" → No report

  Multimedia Tools and Applications (2017)

Small-world Property

Small-world Property

Mean Shortest Path Length (MSPL)

→ Six degrees of separation – Myth

Average Cluster Coefficient (ACC)

→ Fraction of triangles

Musical Networks vs

- Random Networks → Newman, Watts Strogatz
- &
- Small-world Networks

(near equivalent)

Related Work → Music

- Liu et al.

  “Complex network structure of musical compositions: Algorithmic generation of appealing music” → No report

  Physica A: Statistical Mechanics and its Applications (2010)

- Perkins et al.

  “A scaling law for random walks on networks” → No report

  Nature Communications (2014)

- Ferretti

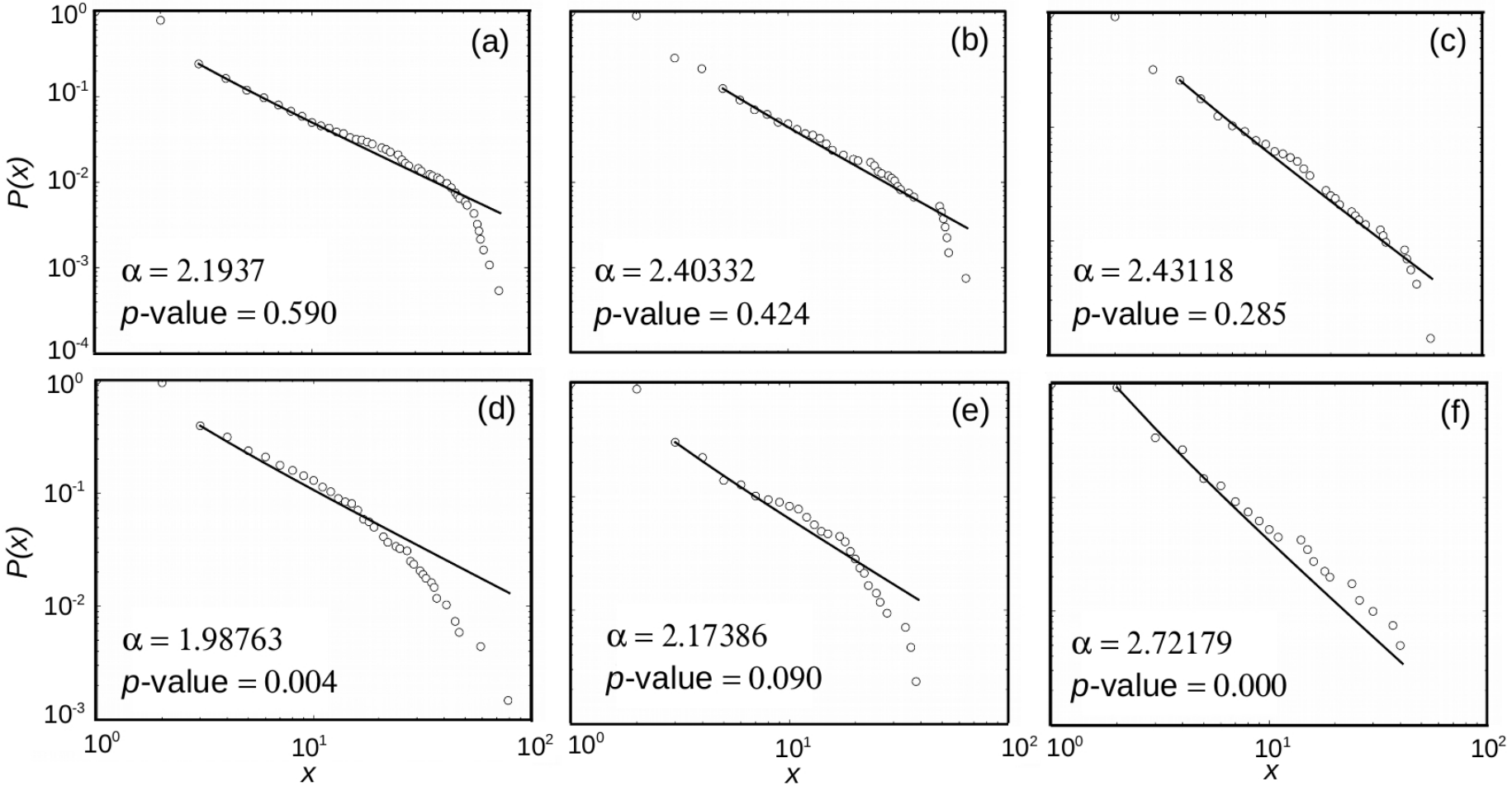
  "On the Complex Network Structure of Musical Pieces: Analysis of Some Use Cases from Different Music Genres" → Small-world !!!

  Multimedia Tools and Applications (2017)

Results → Scale-free

Results → Scale-free

Clauset's test – i & ii steps:



(a) Sonata No. 23 in F minor (Appassionata) Opus 57 (1804) composed by Beethoven, (b) Sonata No. 12 in F major KV 332 (1783) composed by Mozart, (c) Piano Sonata in D major Hoboken XVI:33 (1778) composed by Haydn, (d) Violin partita No. 2 in D minor BWV 1004 (1720) composed by Bach, (e) Sonatina in F major Opus 36 No. 4 Opus 36 (1797) composed by Clementi, and (f) Sonatina in C major Opus 36 No. 3 Opus 36 (1797) also composed by Clementi.

Results → Scale-free

Clauset's test – iii step:

Likelihood Ratio Test								
Musical Network	Power law (KS) p-value	Exponential		Log-normal		Stretched exponential		**Scafe-free?**
		LR	p-value	LR	p-value	LR	p-value	
(a)	0.590	9.88	0.00	6.36	0.00	4.09	0.00	YES
(b)	0.424	4.82	0.00	2.67	0.00	1.61	0.10	YES
(c)	0.285	5.54	0.00	3.78	0.00	2.87	0.00	YES
(d)	0.004	3.16	0.00	-0.03	0.97	-1.15	0.24	No
(e)	0.090	-0.28	0.77	-0.42	0.67	-0.44	0.65	No
(f)	0.000	4.91	0.00	3.84	0.00	2.55	0.01	No

(a), (b), and (c) present the scale-free property.

(d) behaves more like a log-normal

(e) behaves like an exponential distribution

(f) did not behave like any distribution tested.

Results → Small-world

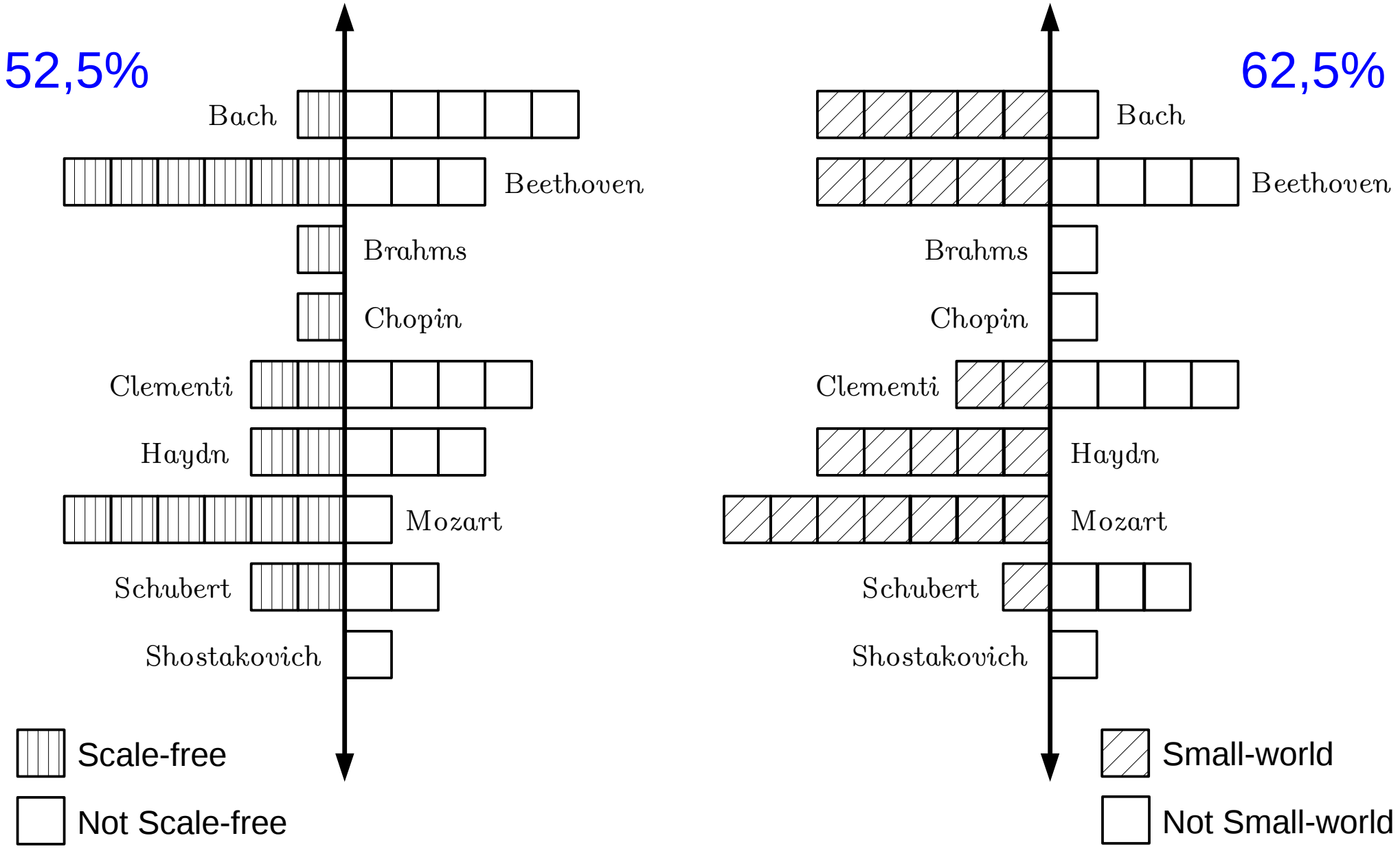
Results → Small-world

Musical Networks			**Random Networks**		**Small-world Networks**		
	MSPL	ACC	MSPL	ACC	MSPL	ACC	**Small-world?**
Beethoven Opus 81	6.02	0.15	6.24	0.00	5.91	0.07	YES
Brahms Opus 1	9.33	0.07	7.55	0.00	6.53	0.07	No
Chopin Opus 35	12.64	0.09	6.50	0.00	5.95	0.08	No
Clementi No.1	6.37	0.14	5.14	0.01	4.51	0.07	No
Mozart KV330	4.89	0.09	5.42	0.00	5.47	0.06	YES
Mozart KV331	5.24	0.11	5.78	0.00	5.74	0.08	YES
Mozart KV332	5.51	0.11	5.96	0.00	5.84	0.07	YES
Mozart KV333	5.02	0.18	5.77	0.00	5.96	0.06	YES
Schubert D784	13.67	0.06	7.10	0.00	5.91	0.08	No
Shostakovich Opus 57	9.68	0.05	6.93	0.00	5.86	0.08	No

MSPL and ACC for musical networks, random networks, and small-world networks.

Final → Results

Results


52,5%
Bach
Beethoven
Brahms
Chopin
Clementi
Haydn
Mozart
Schubert
Shostakovich
Scale-free
Not Scale-free
62,5%
Bach
Beethoven
Brahms
Chopin
Clementi
Haydn
Mozart
Schubert
Shostakovich
Small-world
Not Small-world

Fractal Nature of Music

Fractal Nature of Music

- Schroeder

  “Is there such a thing as fractal music?”

  Nature (1987)

I.F. 40,137
19 citations

- Henderson-Sellers & Cooper

  “Has classical music a fractal nature?—A reanalysis”

  Computers and the Humanities (1993)

I.F. 0,738
10 citations

Fractal Nature of Music

Fractal Dimensioning vs. Complex Network Analysis

Self-similarity — Scale-free property

Mandelbrot — Newman

- Song et al.

  “Self-similarity of complex networks”

  Nature (2005)

  I.F. 40,137
  1102 citations

- Song et al.

  “Origins of fractality in the growth of complex networks”

  Nature Physics (2006)

  I.F. 22,806
  424 citations

Conclusions & Future Work

Conclusions

Previous work (Liu et al., Perkins et al., Ferreti) disregarded:

- Harmony
- One piece per network
- Updated statistical methods → Clauset et. al.

Our work suggests that classical music may or may not present the scale-free and the small-world properties

Future Work

Evaluation of other music genres

Investigation of edge weight distribution

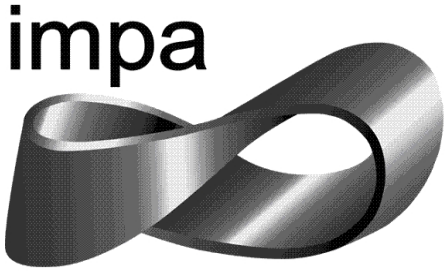
Evaluation of fractal dimension according to Song et al.

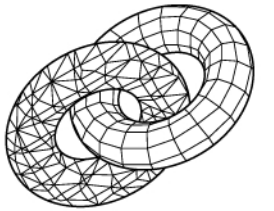
algorithms

Understanding the community structure of our musical networks.

Computer Music @ VISGRAF

Thank you!





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Extra – Hubs

Although we provide a precise evaluation of the power law, our musical networks did not present a long tail as many scale-free networks, i.e., we could not identify a small number of nodes with very high degree. On the other hand, according to Janssen due to the finite size of real-world networks the power law inevitably has a cut-off at some maximum degree. Such a cut-off can be clearly verified in Figures 2(a), 2(b), and 2(c).

- Janssen

"Giant component sizes in scale-free networks with power-law degrees and cutoffs"

Europhysics Letters (2016)

I.F. 1,957
3 citations

Extra – ACC

Local clustering coefficient for undirected graphs:

$$C_i = \frac{2|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}$$

Average cluster coefficient:

$$\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i.$$

Extra – Cohen & Havlin

- Cohen & Havlin

"Scale-free networks are ultrasmall"

Physical Review Letters (2003)

$\longrightarrow$ $2 < \alpha < 3$

I.F. 8,462
801 citations

A power law distribution only has a well-defined mean over $x \in [\,1\,,\,\infty\,]$, if $a > 2$.

When $a > 3$, it has a finite variance that diverges with the upper integration limit

$$x_{max} \text{ as } \langle x^2 \rangle = \int_{X_{max}}^{X_{min}} x^2 \; P(x) \sim x_{\max}^{3-a}$$