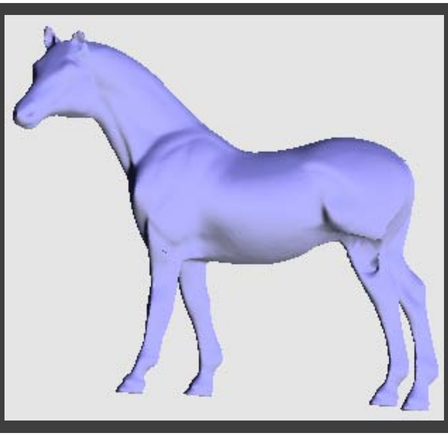
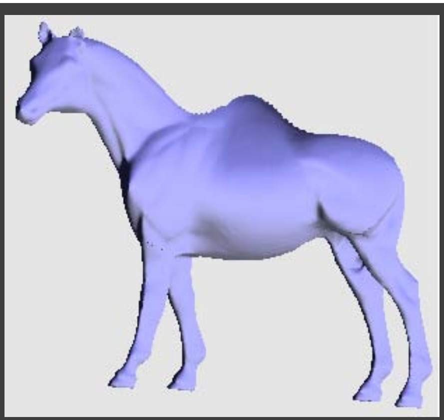
Editing triangulated surfaces





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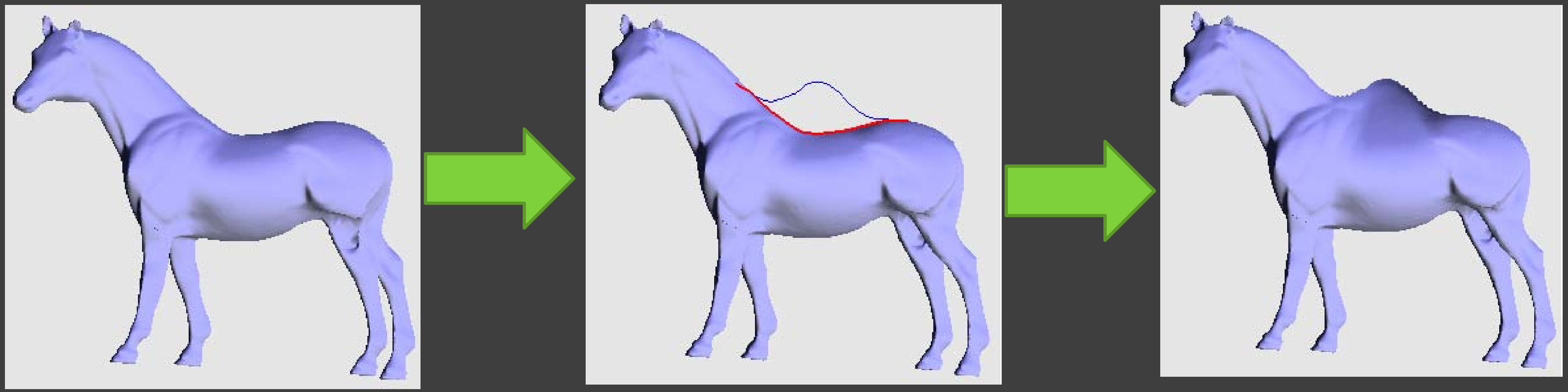
1-Motivation

How to modify interactively a triangulated surface?



Previous Work

2004

Laplacian Surface Editing

O. Sorkine[1], D. Cohen-Or[1], Y. Lipman[1], M. Alexa[2], C. Rössl[3] and H.-P. Seidel[3]

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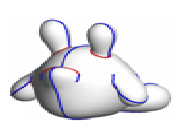
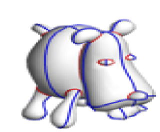
2007

FiberMesh: Designing Freeform Surfaces with 3D Curves

SIGGRAPH2007

Andrew Nealen | TU Berlin
Takeo Igarashi | The University of Tokyo
Olga Sorkine | TU Berlin
Marc Alexa | TU Berlin

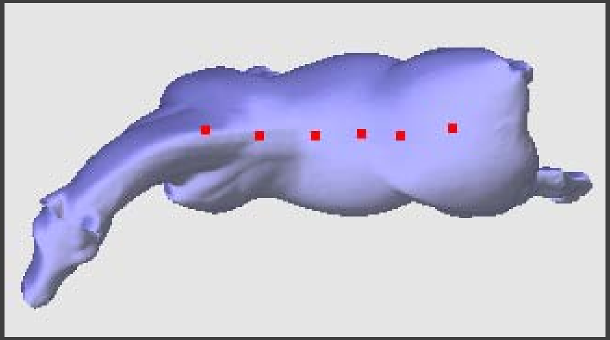




General strategy: *given a triangulated surface S*

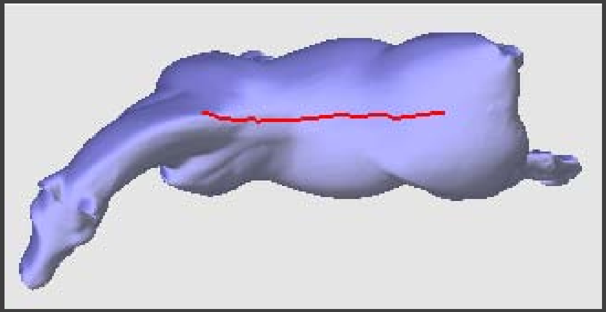
1

- Select a set of control points on S



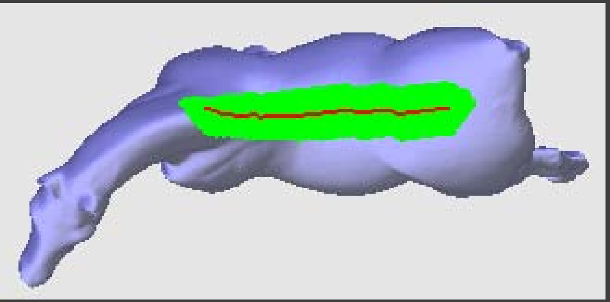
2

- Construct a control curve on S interpolating the control points.



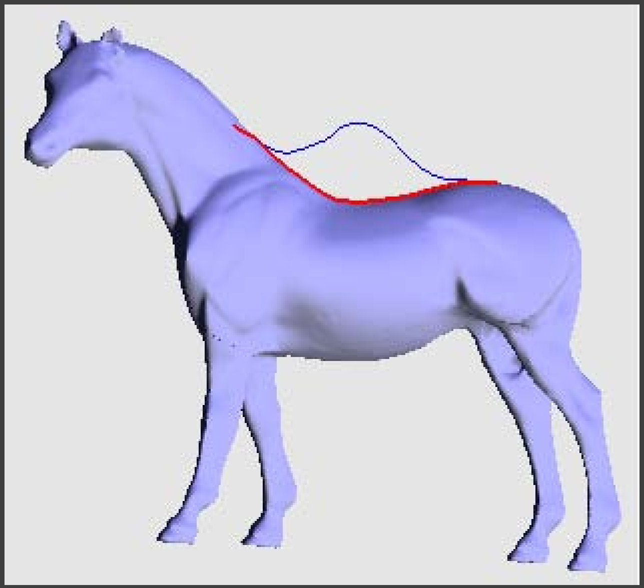
3

- Define a control region.



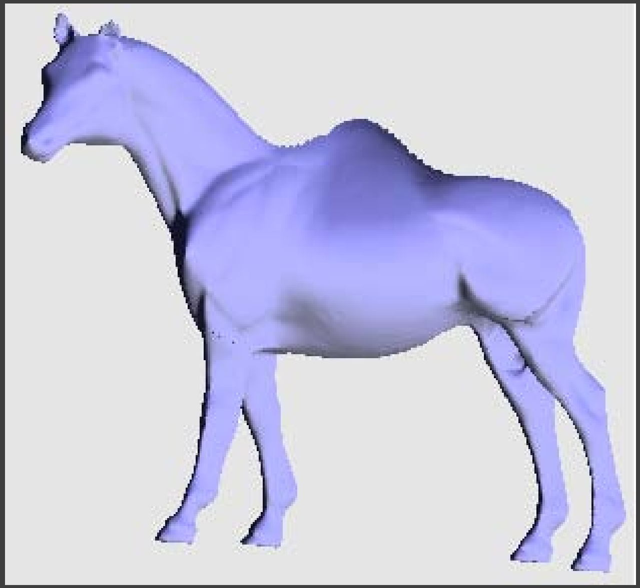
4

- Edit the control curve.



5

- Compute the deformation of the control region.



2-Geodesic curve- Geodesic distance

Geodesic curve CG(A, B)

CG(A, B): Shortest curve on the surface passing through A and B.

Geodesic distance Dg(S; AB)

Dg(S; AB): length between A y B of the geodesic curve CG(A,B)

Computing Geodesic distance

DIFFERENCIABLE SURFACE

Eikonal equation: nonlinear PDE

$$\|\nabla D_g(P)\| = 1$$

NON DIFFERENCIABLE TRIANGULATED SURFACE

Good approximation to the gradient function restricted to a triangle

$$\nabla D_g(P) = \left(\frac{\partial D_g}{\partial x}, \frac{\partial D_g}{\partial y}\right)\Big|_P$$

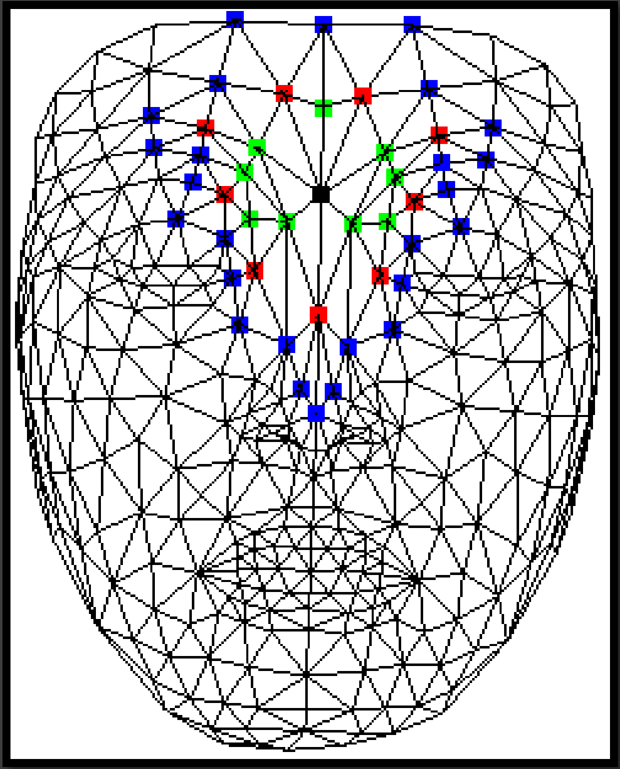
Geodesic curve- Geodesic distance

Theoretically
Geodesic distance is computed as the length of the geodesic curve

Practically
Solving the Eikonal equation it is possible to compute approximately the Geodesic Distance from any point on a triangulated surfaced to a prescribed boundary. Geodesic distances can be used to compute the geodesic curve passing through two points.

3-Fast Marching Method

Idea	**Kimmel & Sethian, 1998**
• To sweep the front ahead in an upwind fashion by considering a set of points in a narrow band around the existing front	



- Boundary vertices
- Vertices where the geodesic distance is already known
- Vertices in the front
- Vertices in the narrow band of the front

$d=0 < d_1 < d_2 < d_3$

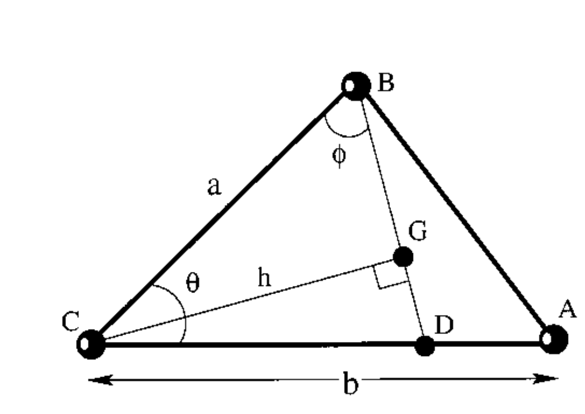
Fast Marching Method

Geodesic distance function restricted to a triangle

- Dg(C) is computed from all triangles ABC having C as a vertex if the geodesic distance is already known in the other vertices A,B.
- The procedure for computing Dg(C) is different for acute and obtuse triangles.

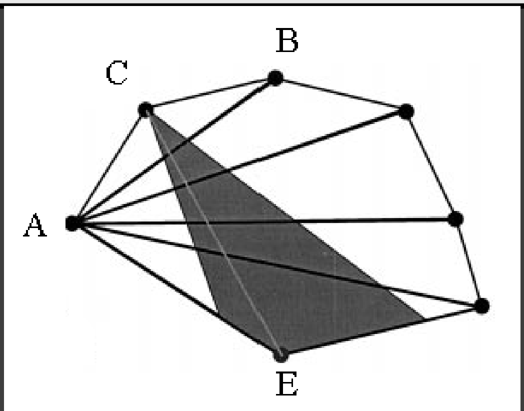
Acute triangles

Geodesic distance function in ABC is approximated by a linear function interpolating Dg(A) and Dg(B) and satisfying the Eikonal Equation. Dg(C) is computed as solution of a quadratic equation.



Obtuse triangles

Splits the obtuse angle into two acute ones, joining C with any point in the sector between the lines perpendicular to CB and CA. Extend this sector recursively unfolding the adjacent triangles, until a new vertex E is included in the sector. Compute Dg(C) as the distance between C and E on the unfolded triangles plane.



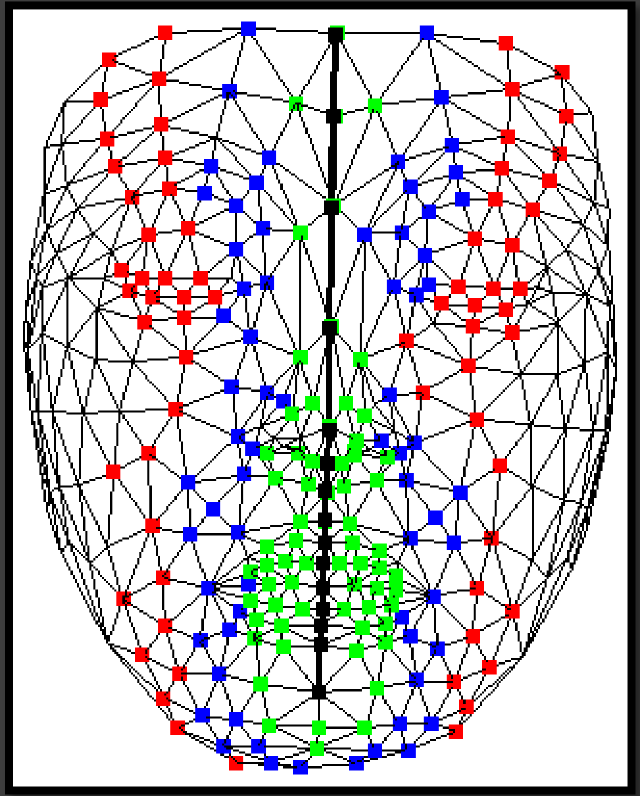
Extensions of FMM

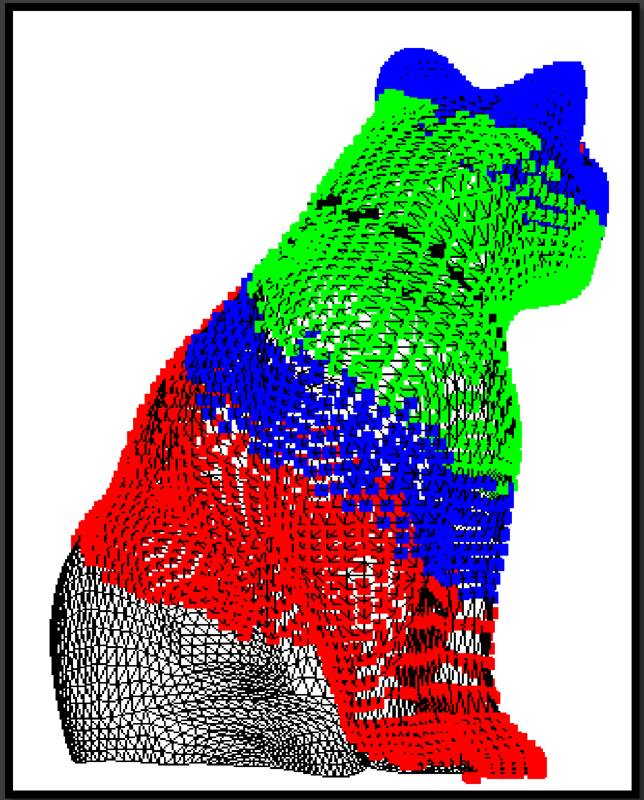
4 extensions

…are neccesary to implement some steps of the method for edition of triangulated surfaces…

3.1-Computing a geodesic neighborhood

…stop the advancing front when the geodesic distance reaches a given value

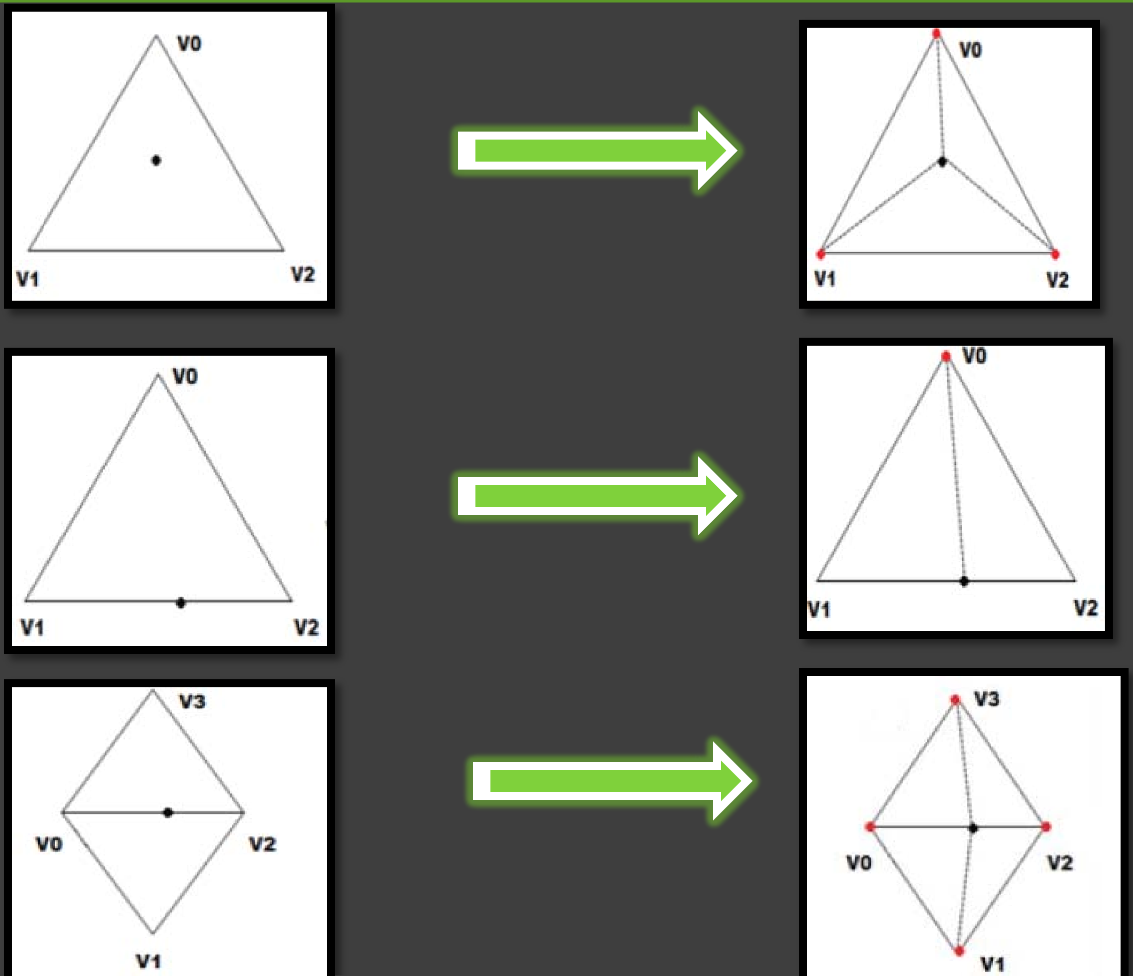




$d=0 < d_1 < d_2 < d_3$

3.2-Computing the geodesic distance for a point which is not a vertex of the triangulation

3 cases depending on the position of the point P: update the list of adjacent points for the vertices of all triangles containing P

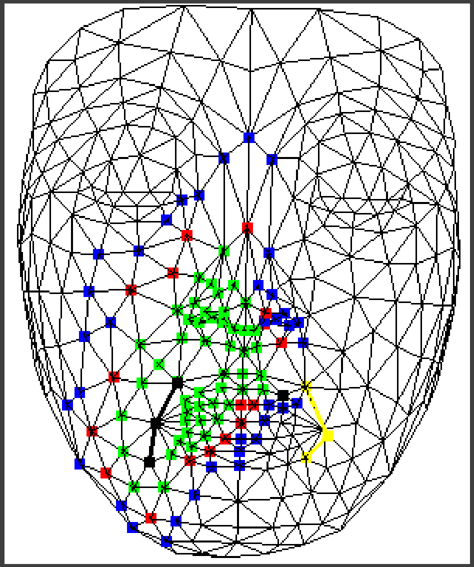


3.3-Stop the advancing front when the geodesic distance has been computed for all points in the target set

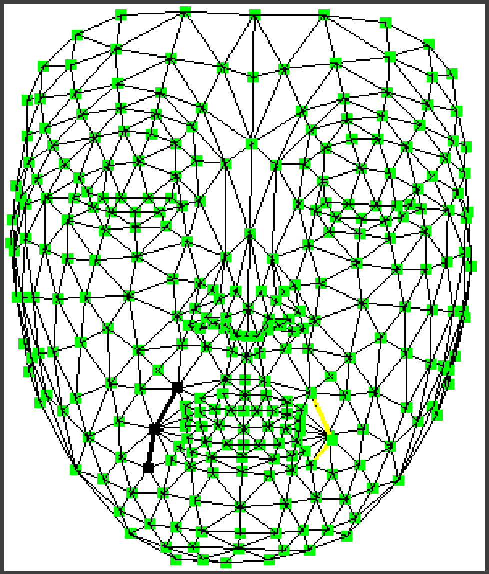
boundary $P = \{P_j\}$

$$Dg(P_j, A) = \min_i D_g(P_j, a_i)$$

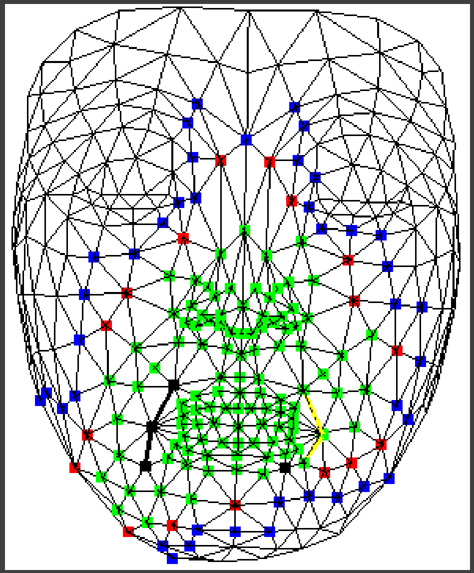
target $A = \{a_i\}$



Evolution of FMM



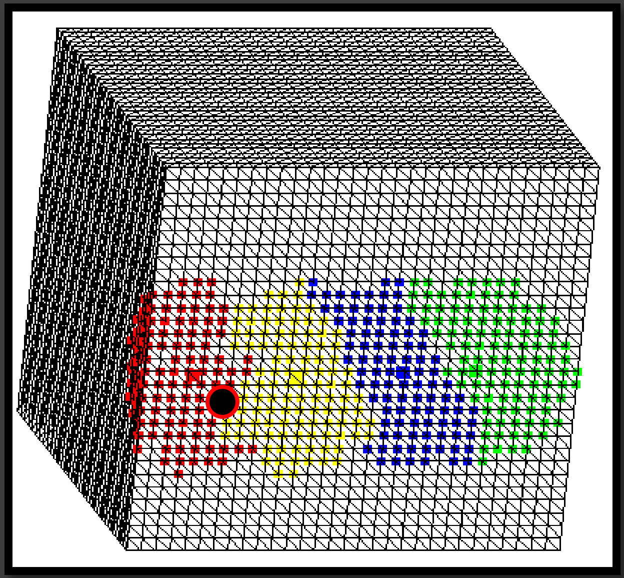
Classic FMM

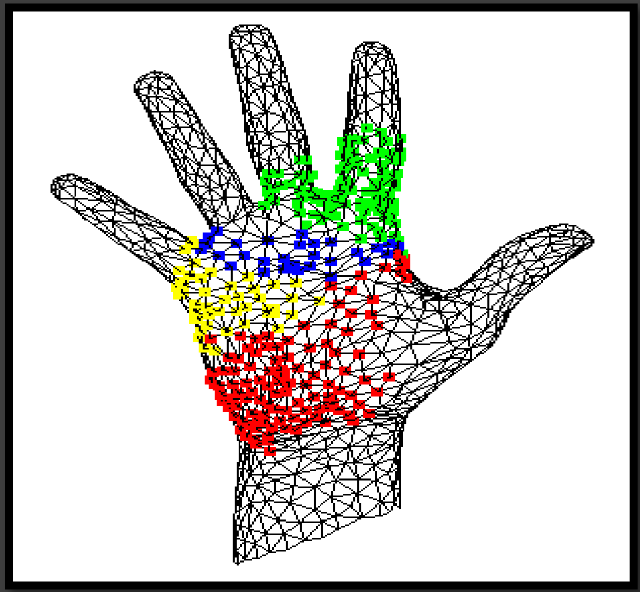


Modification of FMM

- Vertices where the geodesic distance has been computed
- Vertices in the front
- Vertices in the narrow band of the front

3.4-Computing the closest vertex





boundary
- P_0
- P_1
- P_2
- P_3

Yellow points are closer to $\mathbf{P_1}$ than to the rest of the boundary points

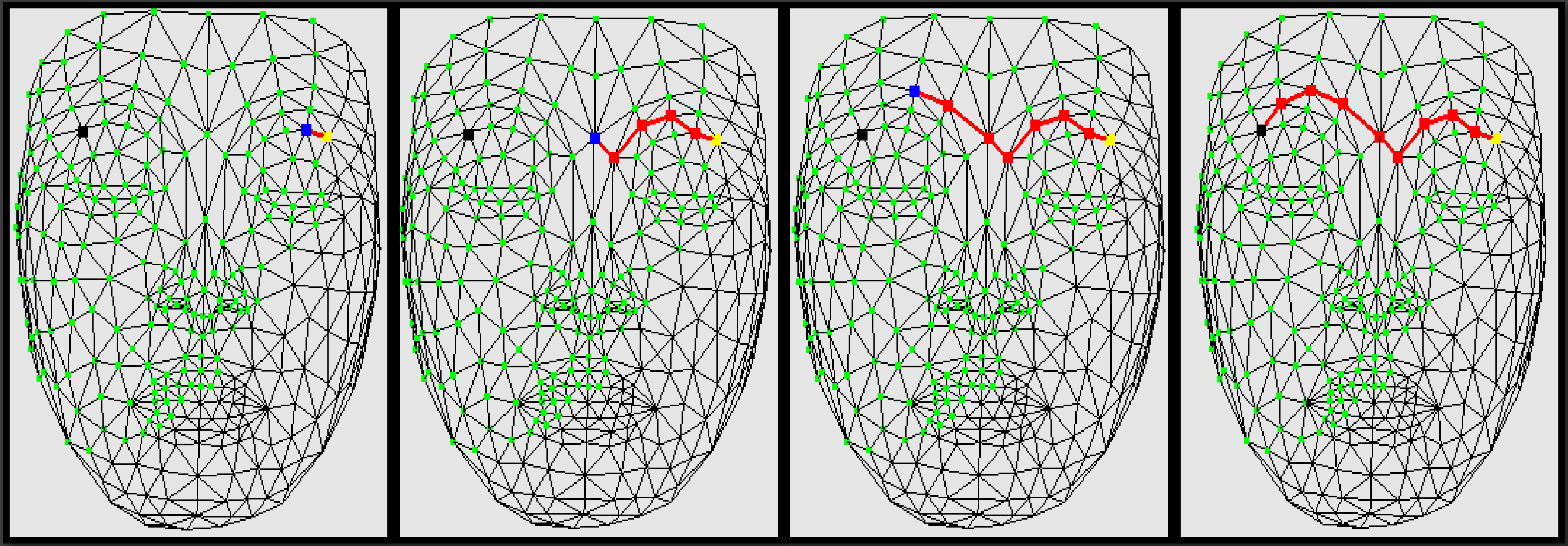
$$Dg(P\,P_0) < Dg(P\,P_1) < Dg(P\,P_2) < Dg(P\,P_3)$$

4-Computing Geodesic curves

Given two consecutive control points A,B on the triangulated surface S… D. Martínez, L. Velho , P. C. Carvalho, 2005

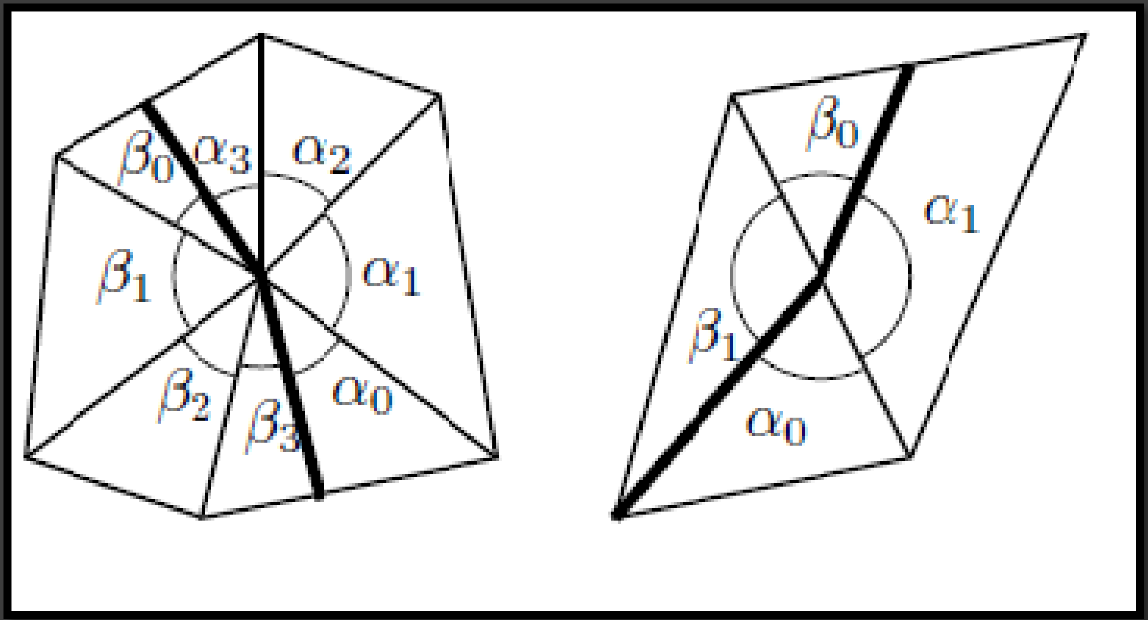
- Compute an initial approximation to the geodesic curve passing through A,B using FMM.
- Correct the initial approximation of the geodesic curve.

Computing the initial approximation to the geodesic curve



CG(AB) = CG(AC)U(C B)

Discrete geodesic curvature



$$\theta_r = \sum \alpha_i \qquad \theta_l = \sum \beta_i$$

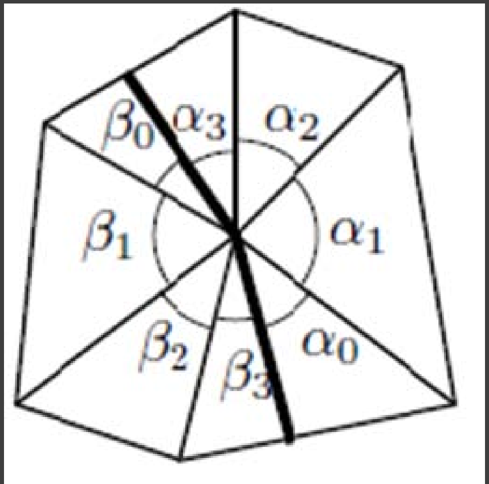
$$\kappa_g(P) = \frac{2\pi}{\theta}\left(\frac{\theta}{2} - \theta_r\right).$$

$$\theta = \theta_l + \theta_r$$

Iterative correction of the polygonal approximation to the geodesic curve

A node P of the current polygonal approximation can be **corrected** if $\theta_r < \pi$ or $\theta_l < \pi$

$$\theta_l < \pi$$

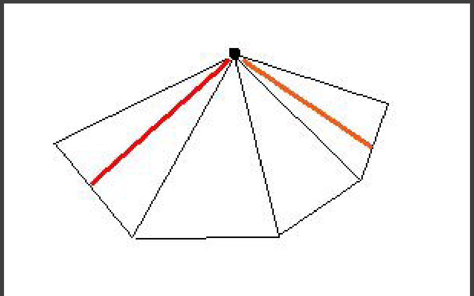


Correction

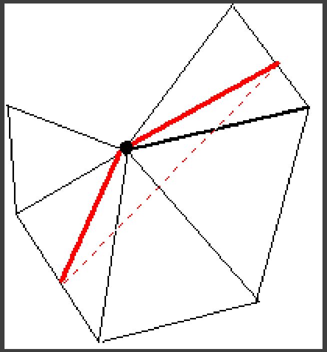
2 Cases

P is a vertex of S

3D

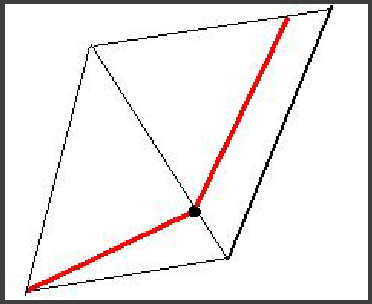


2D

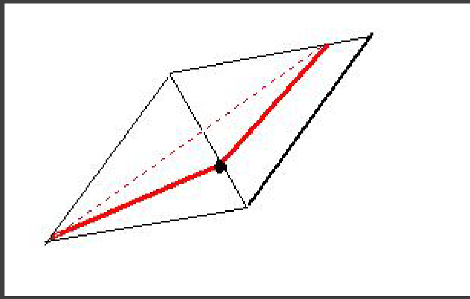


P is in the interior of an edge of S

3D

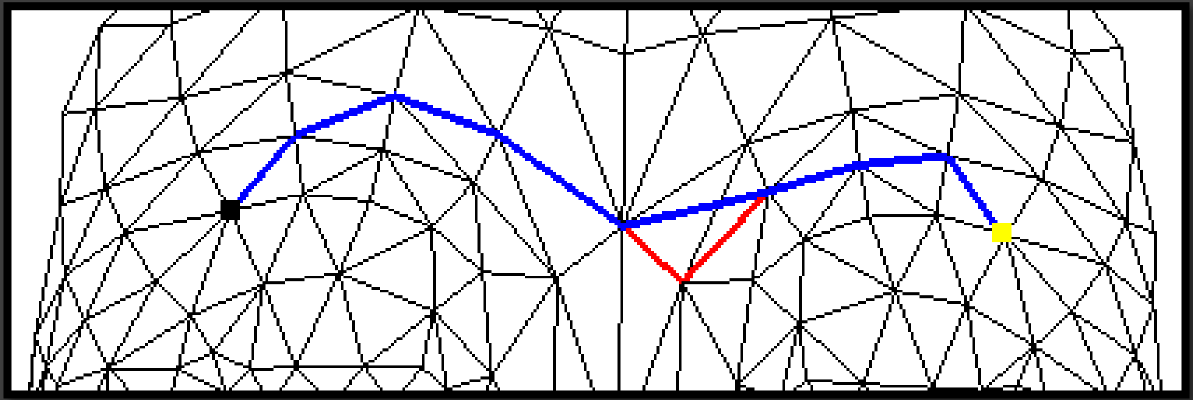


2D

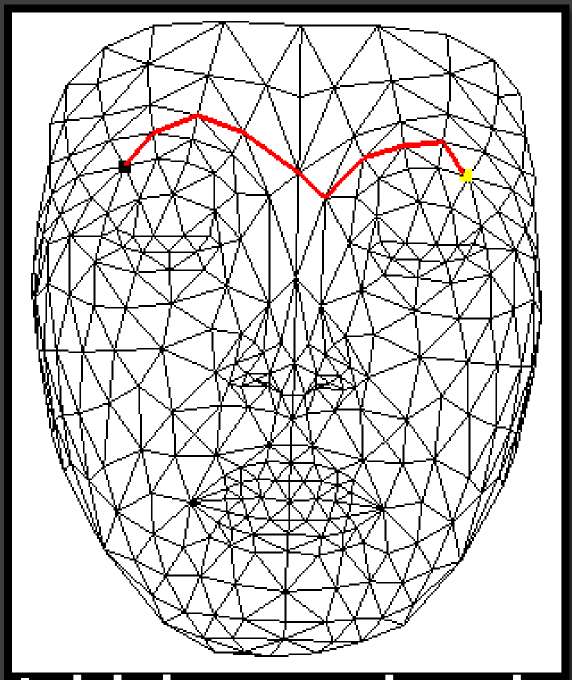


Correction of the inicial approximation

Correction

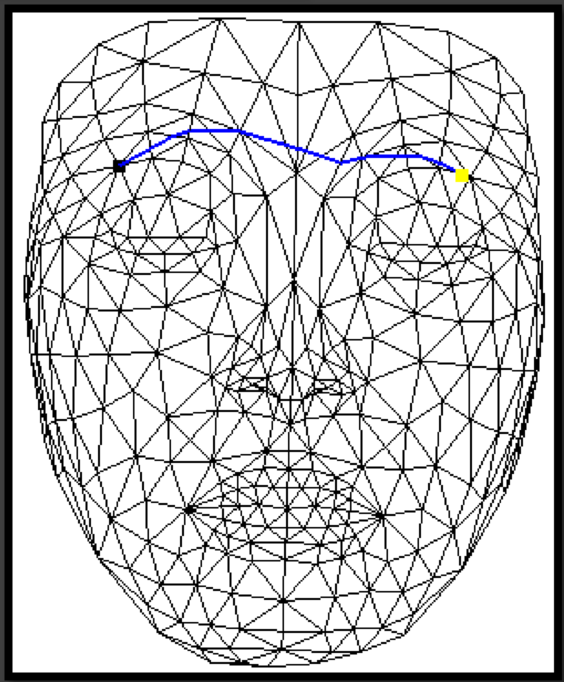


Higest geodesic curvature point



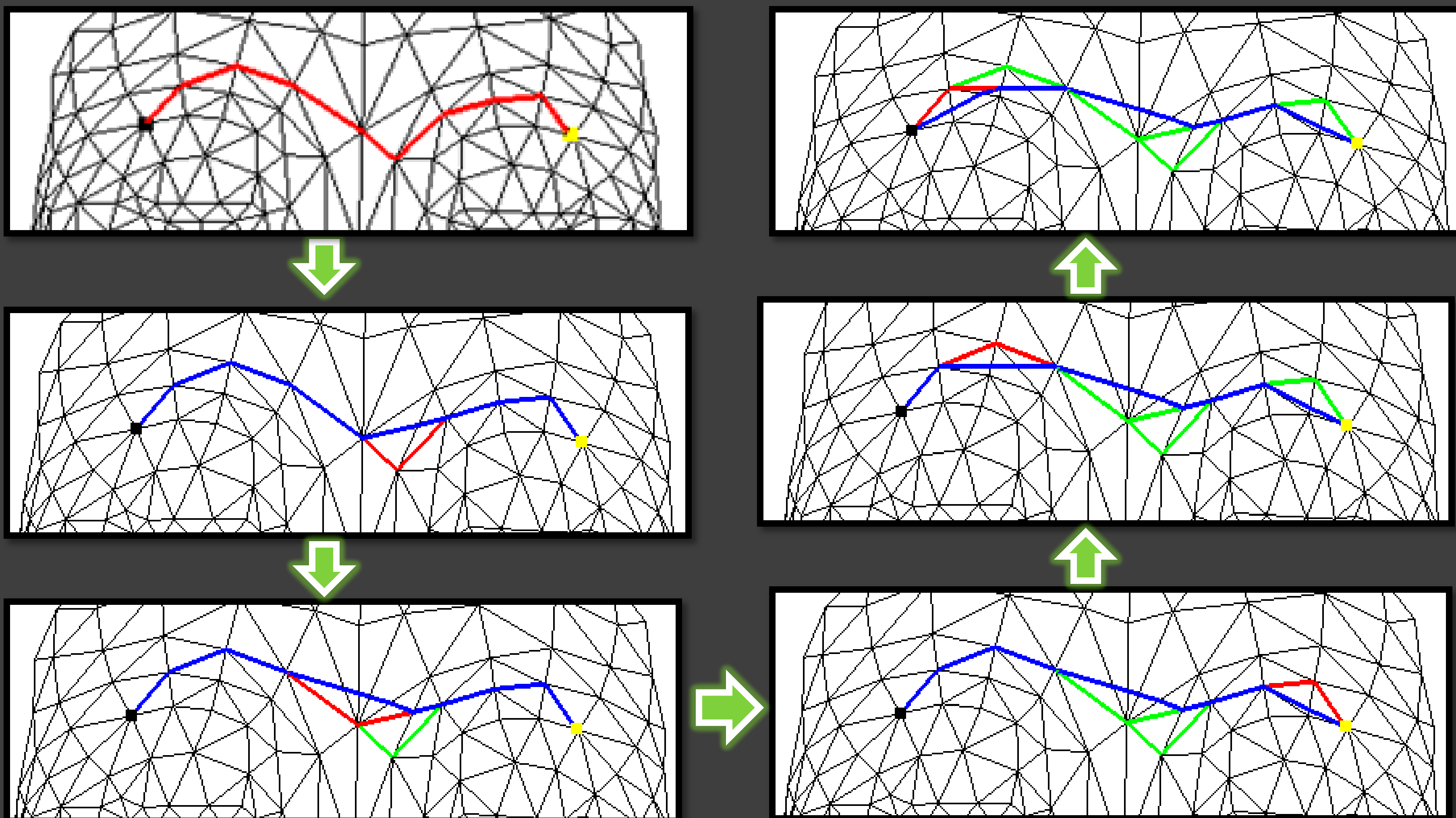
Initial approximation

Iterative process

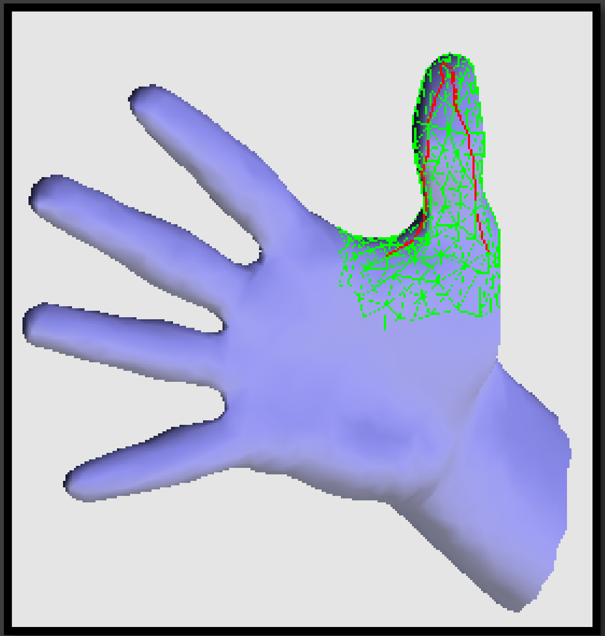


Correction

Some iterations of the correction process



5-Defining the control curve and the control region

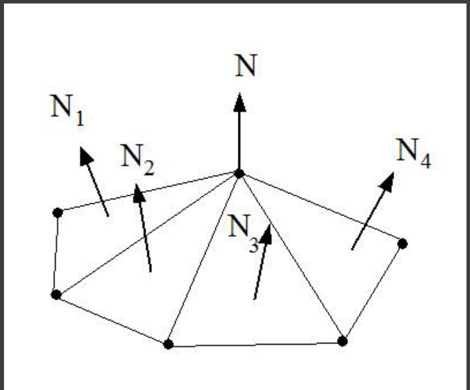


- Control curve: a geodesic polygonal curve interpolating a set of given control points
- Control region: set of vertices P on the surface such that

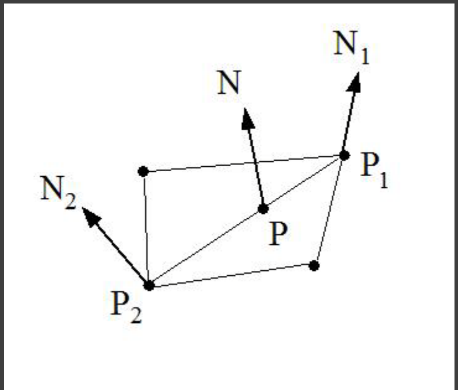
$$D_g(P) < M$$

6-Deformation of the control curve

Computing the normal vector at the vertices of the control curve

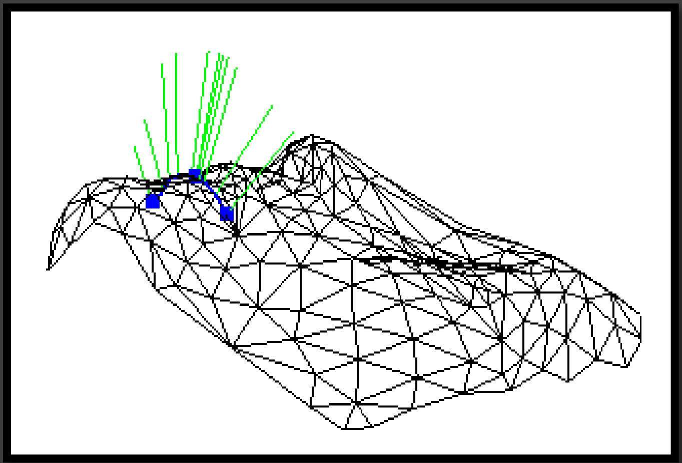


$$N = \frac{\sum_{i=1}^{n} N_i}{n}$$

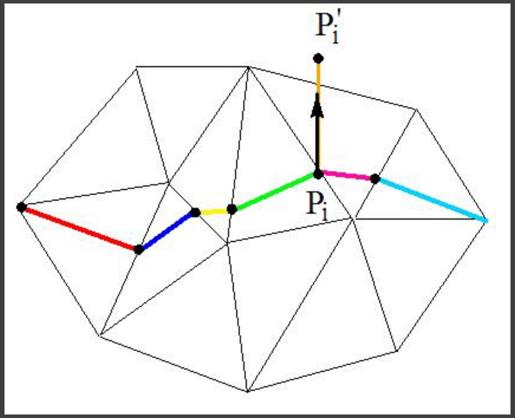


$$N = c_1 N_1 + c_2 N_2$$

$$c_i = \frac{\|P_i - P\|}{\|P_1 - P_2\|}, \quad i = 1, 2$$

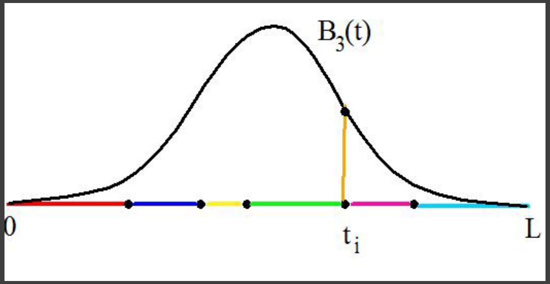


Deformation of the control curve



Parametrizing the control curve by chord length

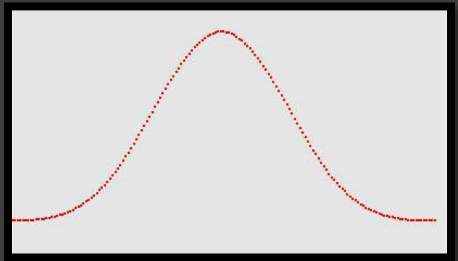
$$t_{i+1} = t_i + \|P_{i+1} - P_i\|$$



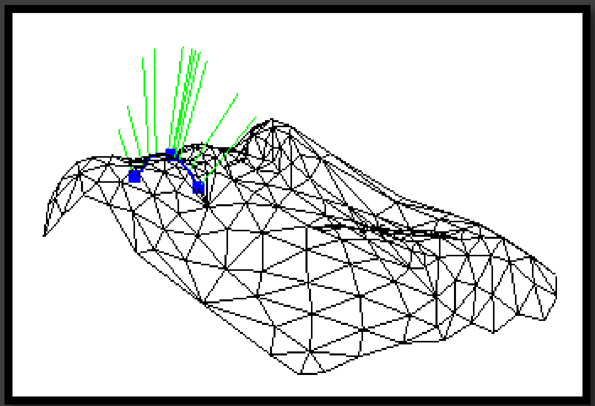
Magnitude of the displacement for a vertex in terms of a cubic B-spline.

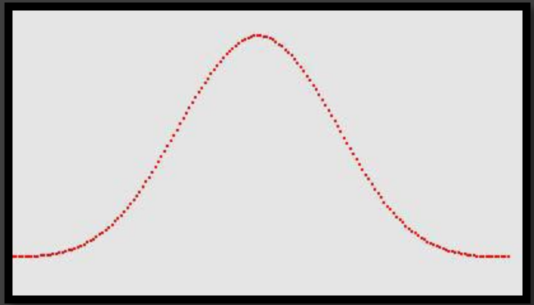
New position of P_i ⇨ $P_i' = P_i + B_3(t_i)N_i$

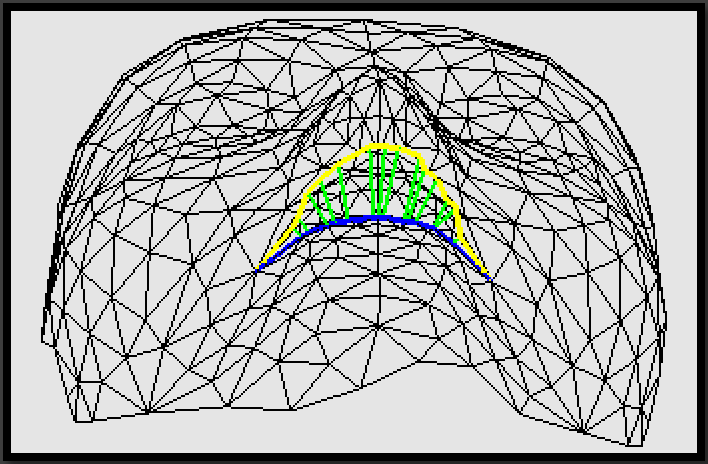
Changing the position of the knots to modify the geometry of the B-spline curve



Moving the control curve vertices





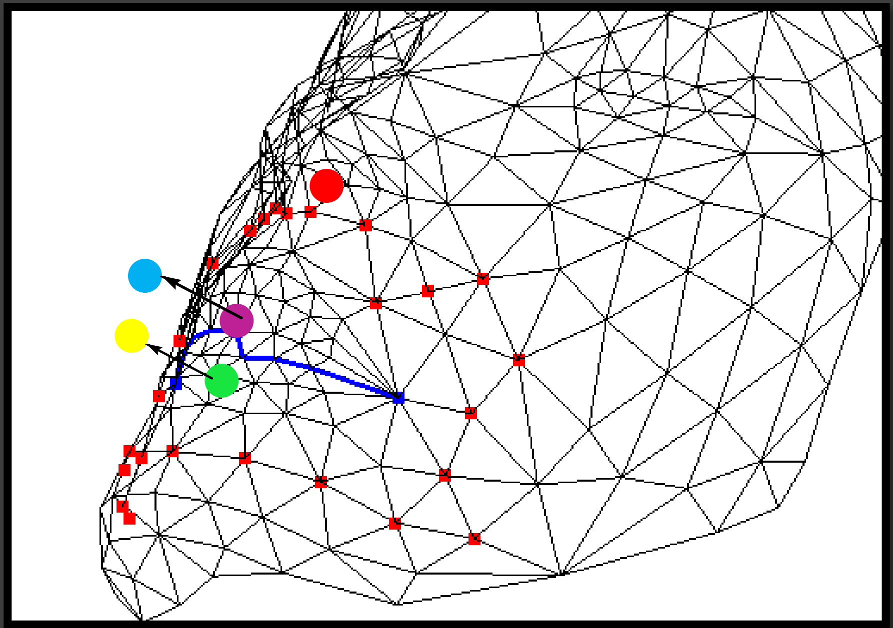


7-Edition of the control region.

Magnitude of the displacement for P in the interest region
$h(P)=(1- Dg(P,Pc)/dismax)\ h(Pc)$

Dg(P,Pc) = 0

h(P) = h(Pc)

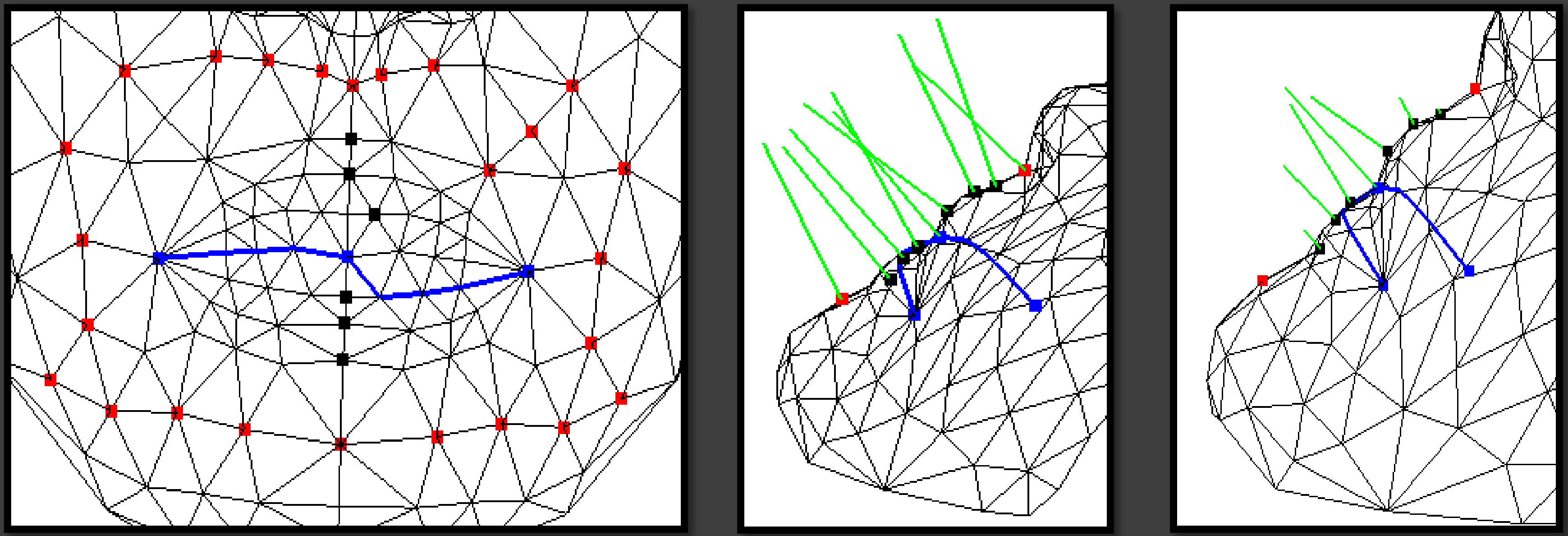


Dg(P,Pc) = distMax

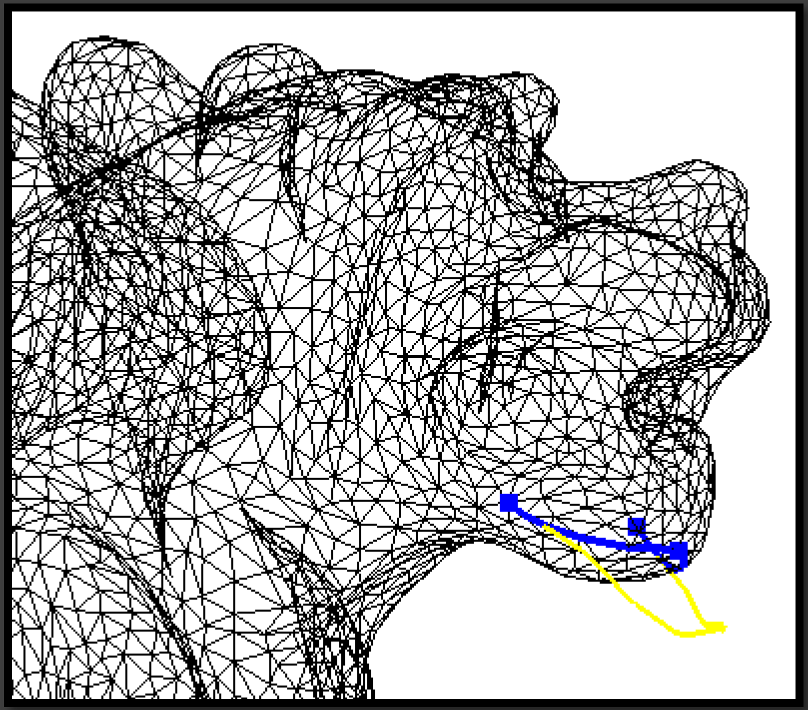
h(P) = (1-1) h(Pc) =0

Edition of the control region

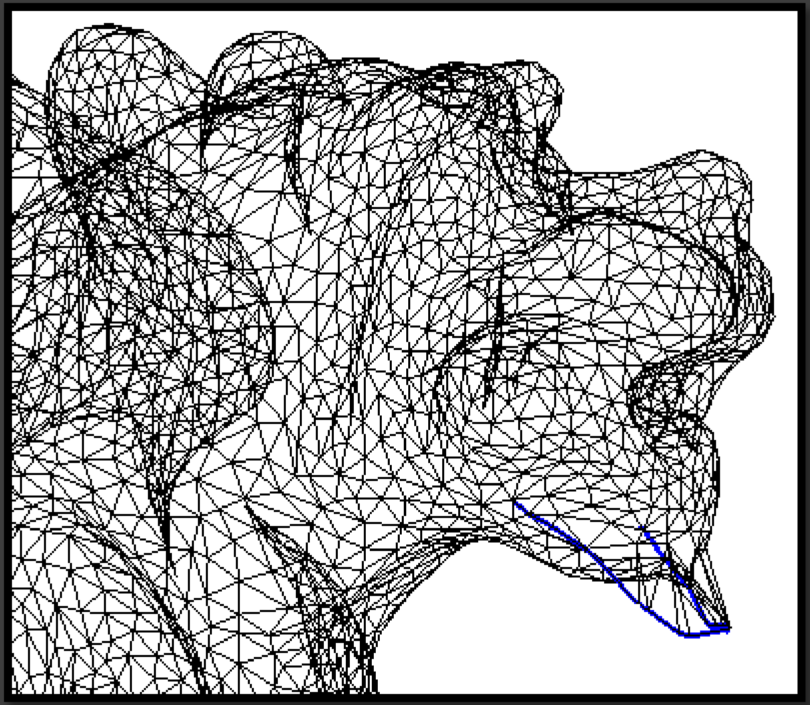
Selected points on the control region
Points on the boundary of the control region
Scaled normal vectors at selected points



Edition of the control region

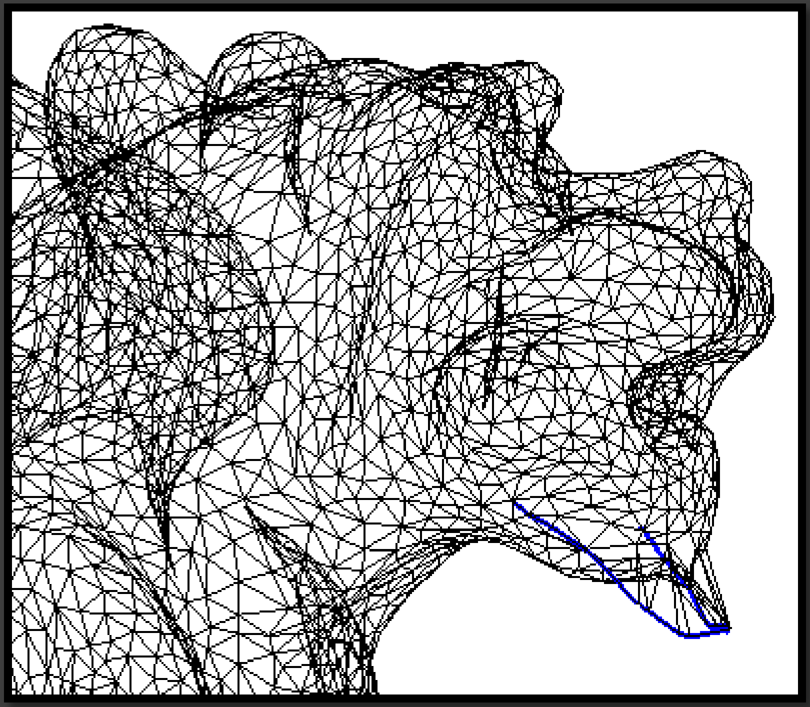


(1)

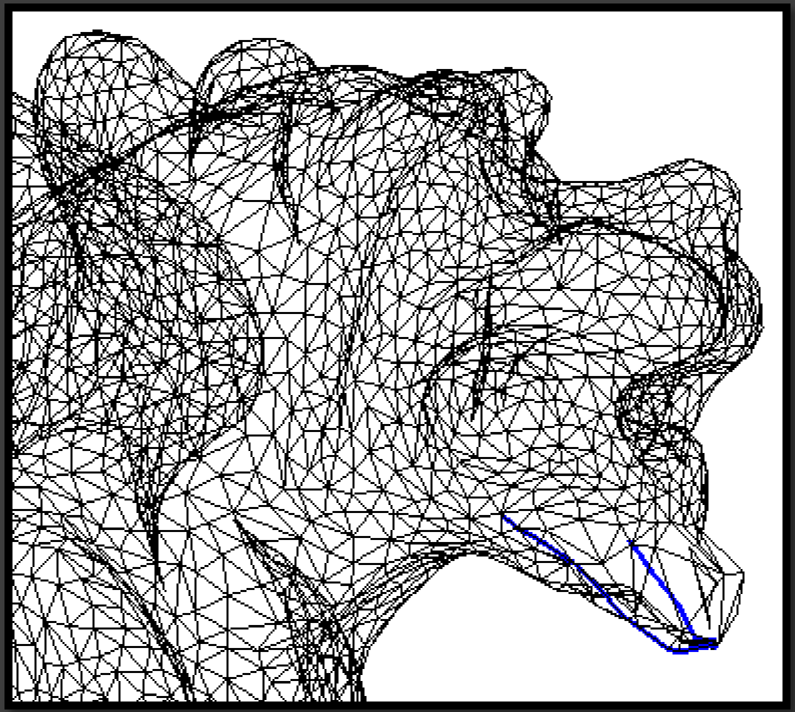


(2)

Edition of the control region

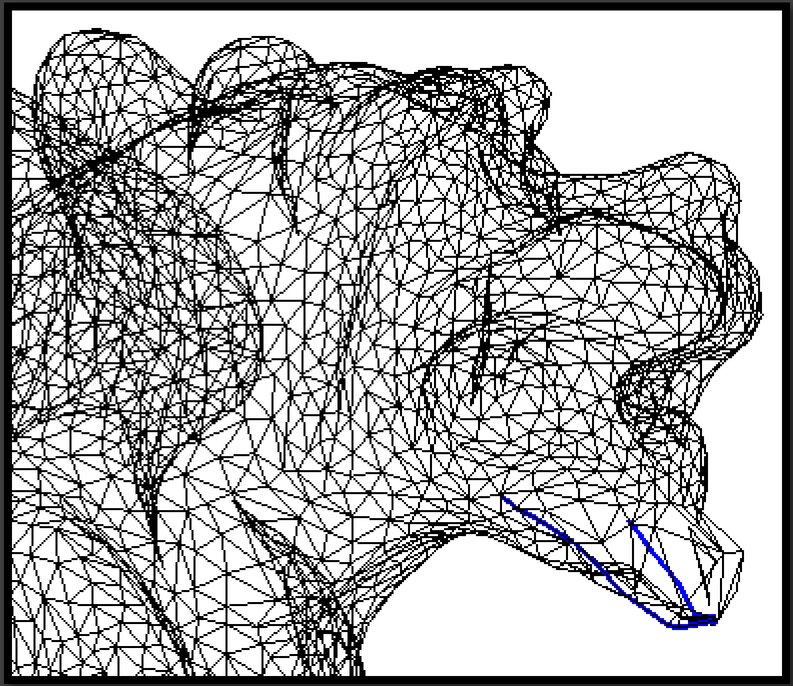


(2)

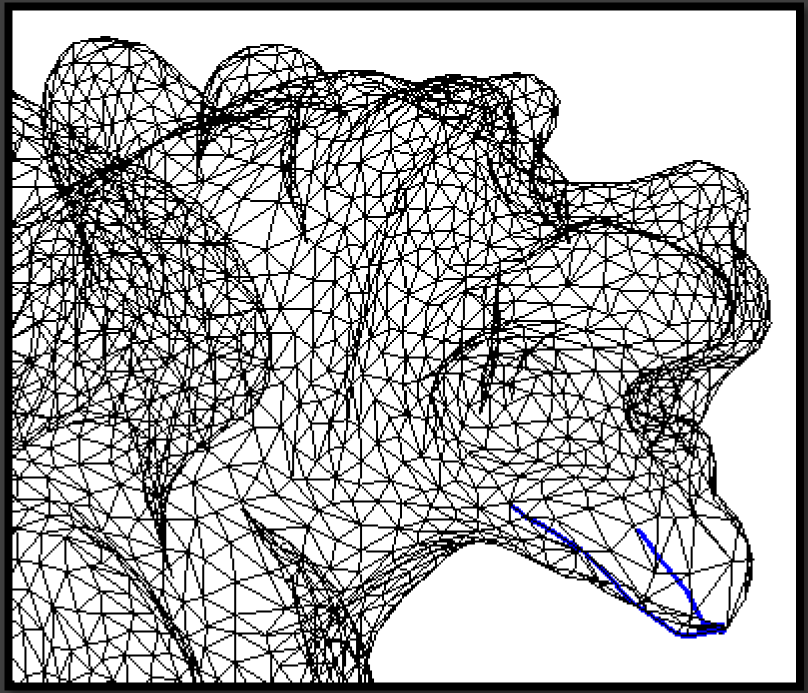


(3)

Edition of the control region



(3)



(4)

8-Implementation

Data structure:

- HalfEdge

Libraries:

- TriangMesh
- OpenGL

User Interface:

- GLUT
- GLUI

Programming Language:

- C++

Options for users

- Modify the geometry of the control curve.
- Selection of the width of the control region.
- Definition of the displacement magnitude.

Mesh Tools

Conclusions

- A new method for **editing a** triangulated surface S in an instuitive way has been proposed.

- Deformations are introduced by means of:

• **control curve**: defined as a piecewise geodesic curve on S interpolating given control points on S.
• **control region**: geodesic neighborhood on S of the control curve
• **deformations in the control curve**: introduced by means of a cubic B-spline curve
• **displacements of points in the control region**: in the direction of the normal vectors on S, with magnitude depending on the geodesic distance to the control curve.

Future Work

- To introduce closed curves as control curves.
- Study new deformation patterns for the control curve.
- To improve the computation of the initial approximation to the geodesic curve passing throught 2 prescribed points.
- To extend the method to introduce deformations preserving surface details.

References

- D. Martinez, Geodesic-based modeling on manifold triangulations, Ph.D. Thesis, IMPA, Brasil, 2006.

- R. Kimmel and J.A. Sethian, Computing geodesic paths on manifolds, In Proceedings of the National Academy of Sciences of the USA 95 (1998), no. 15, 8431-8435.

- J. A. Sethian, A fast marching level set method for monotonically advancing fronts, In Proceedings of the National Academy of Sciences of the USA 93 (1996), no. 4, 1591-1595.