Editing triangulated surfaces

Nayla López, Orlenys López, Havana Univ.
Victoria Hernández, Jorge Estrada, ICIMAF

In collaboration with:
Luiz Velho, IMPA
Dimas Martínez, Alagoas Univ.
1-Motivation

How to modify interactively a triangulated surface?
Previous Work

Laplacian Surface Editing

O. Sorkine¹, D. Cohen-Or¹, Y. Lipman¹, M. Alexa², C. Rössl³ and H.-P. Seidel³

¹School of Computer Science, Tel Aviv University
²Discrete Geometric Modeling Group, Darmstadt University of Technology
³Max-Planck Institut für Informatik, Saarbrücken

FiberMesh: Designing Freeform Surfaces with 3D Curves

Andrew Nealen
Takeo Igarashi
Olga Sorkine
Marc Alexa

TU Berlin
The University of Tokyo
TU Berlin
TU Berlin
General strategy: given a triangulated surface $S$

1. Select a set of control points on $S$

2. Construct a control curve on $S$ interpolating the control points.

3. Define a control region.
• Edit the control curve.

• Compute the deformation of the control region.
2-Geodesic curve- Geodesic distance

**Geodesic curve** \( CG(A, B) \)

\( CG(A, B) \): Shortest curve on the surface passing through A and B.

**Geodesic distance** \( Dg(S; AB) \)

\( Dg(S; AB) \): length between A and B of the geodesic curve \( CG(A,B) \).
Computing Geodesic distance

DIFERENCIABLE SURFACE

Eikonal equation: nonlinear PDE

\[ \| \nabla D_g(P) \| = 1 \]

NON DIFFERENCIABLE TRIANGULATED SURFACE

Good approximation to the gradient function restricted to a triangle

\[ \nabla D_g(P) = \left( \frac{\partial D_g}{\partial x}, \frac{\partial D_g}{\partial y} \right) \bigg|_P \]
Geodesic curve - Geodesic distance

**Theoretically**

Geodesic distance is computed as the length of the geodesic curve.

**Practically**

Solving the Eikonal equation it is possible to compute approximately the Geodesic Distance from any point on a triangulated surface to a prescribed boundary. Geodesic distances can be used to compute the geodesic curve passing through two points.
3-Fast Marching Method

Idea     Kimmel & Sethian, 1998

• To sweep the front ahead in an upwind fashion by considering a set of points in a narrow band around the existing front

- Boundary vertices
- Vertices where the geodesic distance is already known
- Vertices in the front
- Vertices in the narrow band of the front

\[ d=0 < d_1 < d_2 < d_3 \]
Fast Marching Method

Geodesic distance function restricted to a triangle

- Dg(C) is computed from all triangles ABC having C as a vertex if the geodesic distance is already known in the other vertices A, B.
- The procedure for computing Dg(C) is different for acute and obtuse triangles.

Acute triangles

Geodesic distance function in ABC is approximated by a linear function interpolating Dg(A) and Dg(B) and satisfying the Eikonal Equation. Dg(C) is computed as solution of a quadratic equation.

Obtuse triangles

Splits the obtuse angle into two acute ones, joining C with any point in the sector between the lines perpendicular to CB and CA. Extend this sector recursively unfolding the adjacent triangles, until a new vertex E is included in the sector. Compute Dg(C) as the distance between C and E on the unfolded triangles plane.
Extensions of FMM

4 extensions

…are necessary to implement some steps of the method for edition of triangulated surfaces…
3.1-Computing a geodesic neighborhood

...stop the advancing front when the geodesic distance reaches a given value

\[ d_0 < d_1 < d_2 < d_3 \]
3.2-Computing the geodesic distance for a point which is not a vertex of the triangulation

3 cases depending on the position of the point P: update the list of adjacent points for the vertices of all triangles containing P.
3.3-Stop the advancing front when the geodesic distance has been computed for all points in the target set.

$D_g(P_j, A) = \min_i D_g(P_j, a_i)$

Evolution of FMM

Classic FMM

Modification of FMM

- **Vertices where the geodesic distance has been computed**
- **Vertices in the front**
- **Vertices in the narrow band of the front**

**boundary** $P = \{P_j\}$

**target** $A = \{a_i\}$
3.4-Computing the closest vertex

Yellow points are closer to $P_1$ than to the rest of the boundary points

$$Dg(P, P_0) < Dg(P, P_1) < Dg(P, P_2) < Dg(P, P_3)$$
4-Computing Geodesic curves

Given two consecutive control points A,B on the triangulated surface S... D. Martínez, L. Velho, P. C. Carvalho, 2005

• Compute an initial approximation to the geodesic curve passing through A,B using FMM.

• Correct the initial approximation of the geodesic curve.
Computing the initial approximation to the geodesic curve

\[ \text{CG}(AB) = \text{CG}(AC) \cup (CB) \]
Discrete geodesic curvature

\[ \theta_r = \sum \alpha_i \quad \theta_l = \sum \beta_i \]

\[ \kappa_g(P) = \frac{2\pi}{\theta} \left( \frac{\theta}{2} - \theta_r \right) \]

\[ \theta = \theta_l + \theta_r \]
Iterative correction of the polygonal approximation to the geodesic curve

A node $P$ of the current polygonal approximation can be corrected if $\theta_r < \pi$ or $\theta_l < \pi$
Correction

2 Cases

P is a vertex of S

P is in the interior of an edge of S
Correction of the initial approximation

Iterative process

Initial approximation

Correction

Higest geodesic curvature point
Some iterations of the correction process
5-Defining the control curve and the control region

Control curve: a geodesic polygonal curve interpolating a set of given control points

Control region: set of vertices $P$ on the surface such that $D_a(P) < M$
6-Deformation of the control curve

Computing the normal vector at the vertices of the control curve

\[ N = \frac{\sum_{i=1}^{n} N_i}{n} \]

\[ N = c_1 N_1 + c_2 N_2 \]

\[ c_i = \frac{\|P_i - P\|}{\|P_1 - P_2\|}, \quad i = 1, 2 \]
Deformation of the control curve

Parametrizing the control curve by chord length

\[ t_{i+1} = t_i + \| P_{i+1} - P_i \| \]

Magnitude of the displacement for a vertex in terms of a cubic B-spline.

New position of \( P_i \) \( \rightarrow \) \( P_i' = P_i + B_3(t_i)N_i \)

Changing the position of the knots to modify the geometry of the B-spline curve
Moving the control curve vertices
Magnitude of the displacement for P in the interest region

\[ h(P) = (1 - \frac{Dg(P, Pc)}{\text{dismax}}) \cdot h(Pc) \]

- \( Dg(P, Pc) = 0 \)
  - \( h(P) = h(Pc) \)

- \( Dg(P, Pc) = \text{dismax} \)
  - \( h(P) = (1-1) \cdot h(Pc) = 0 \)
Edition of the control region

- Selected points on the control region
- Points on the boundary of the control region
- Scaled normal vectors at selected points
Edition of the control region
Edition of the control region

(2) (3)
Edition of the control region
8-Implementation

Data structure:
- HalfEdge

Libraries:
- TriangMesh
- OpenGL

User Interface:
- GLUT
- GLUI

Programming Language:
- C++
Options for users

- Modify the geometry of the control curve.
- Selection of the width of the control region.
- Definition of the displacement magnitude.
Conclusions

A new method for editing a triangulated surface $S$ in an intuitive way has been proposed.

Deformations are introduced by means of:

- **control curve**: defined as a piecewise geodesic curve on $S$ interpolating given control points on $S$.
- **control region**: geodesic neighborhood on $S$ of the control curve.
- **deformations in the control curve**: introduced by means of a cubic B-spline curve.
- **displacements of points in the control region**: in the direction of the normal vectors on $S$, with magnitude depending on the geodesic distance to the control curve.
Future Work

• To introduce closed curves as control curves.

• Study new deformation patterns for the control curve.

• To improve the computation of the initial approximation to the geodesic curve passing through 2 prescribed points.

• To extend the method to introduce deformations preserving surface details.
References

