QUADRICS INTERSECTION AND TANGENCY FOR REFLECTIONS ON COMPUTER GRAPHICS

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OUTLINE

- A bit of Coimbra and Vision Lab
- Non central catadioptric systems
- Forward Projection Model
- New solution to be founded
- Graphics and other applications
- Still open problems

COIMBRA AND THE VISION LAB

- Institute for Systems and Robotics
- Around 100 researchers
- Focused mainly on
  - Computer Vision
  - Medical imaging
  - Mobile Robotics
  - Autonomous vehicles
  - Energy and sustainability
- Also a magnificent weather … so you should go!

NON CENTRAL CATADIOPTRIC SYSTEMS

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Spherical mirror

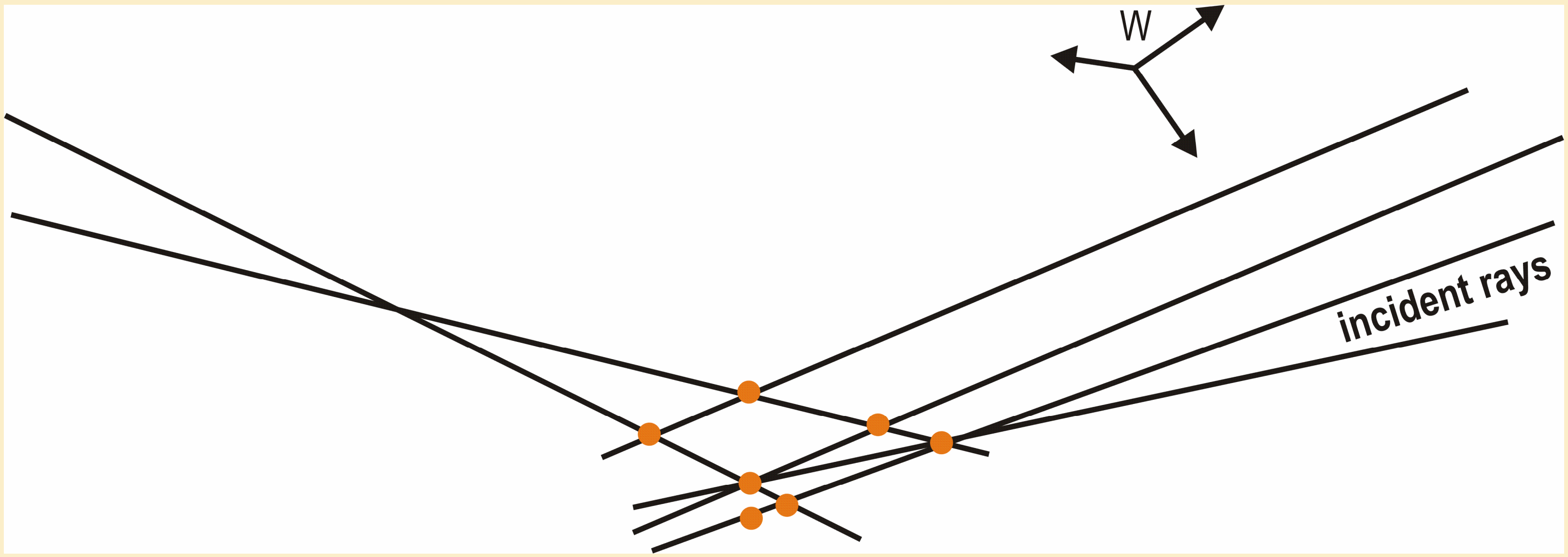
Hyperbolic mirror

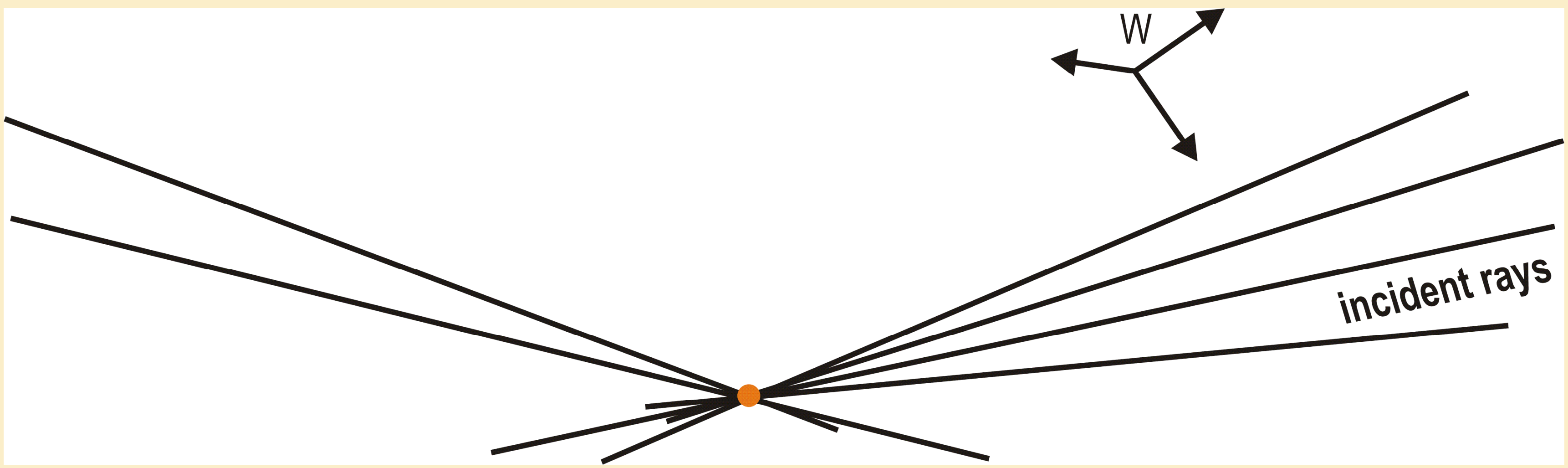
NON CENTRAL CATADIOPTRIC SYSTEMS

- The incident light rays do not intersect each other in a common single point (called viewpoint).

- When it does happen we have central projection
- Almost all methods in computer vision are designed for central projection cameras

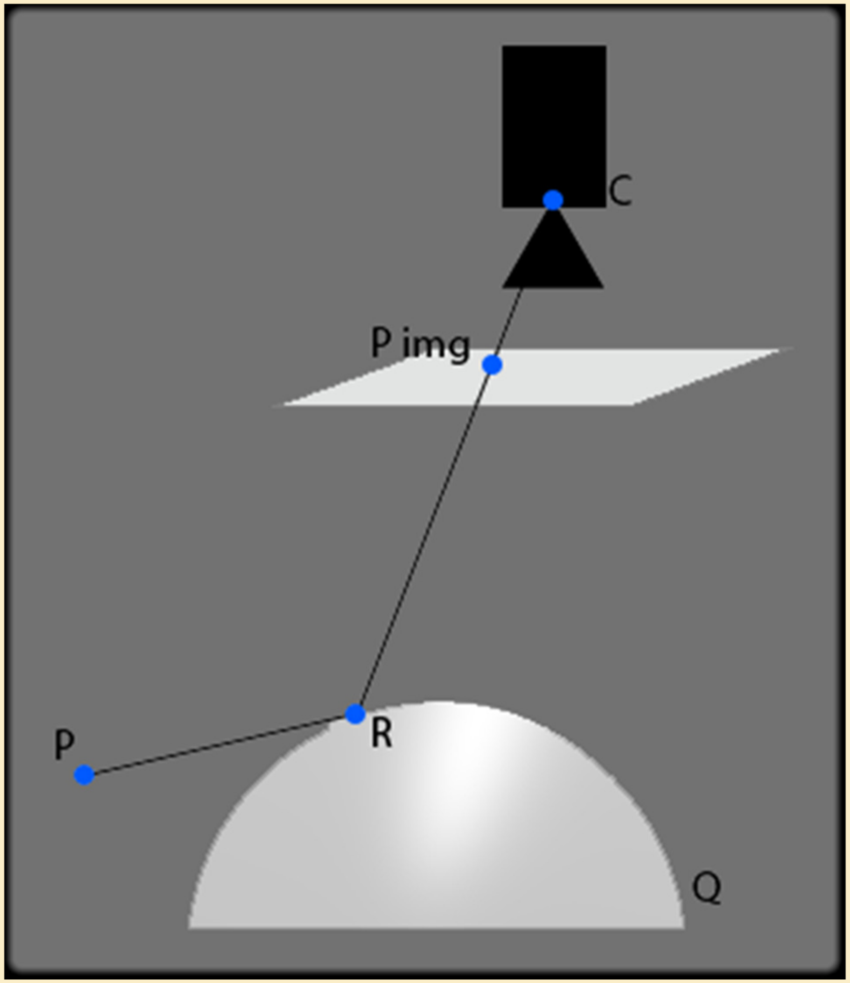
CENTRAL VS. NON CENTRAL PROJ. MODEL





FORWARD PROJECTION MODEL

- Let us consider the following vision system:



FORWARD PROJECTION MODEL

- Camera:
  - A pinhole (i.e. perspective) camera
- Mirror:
  - A quadric reflector, usually non-ruled quadrics (inertia (3,1))
    - Spheres
    - Ellipsoids
    - Hyperboloids of two sheets
    - Paraboloids

MIRROR QUADRIC SURFACE

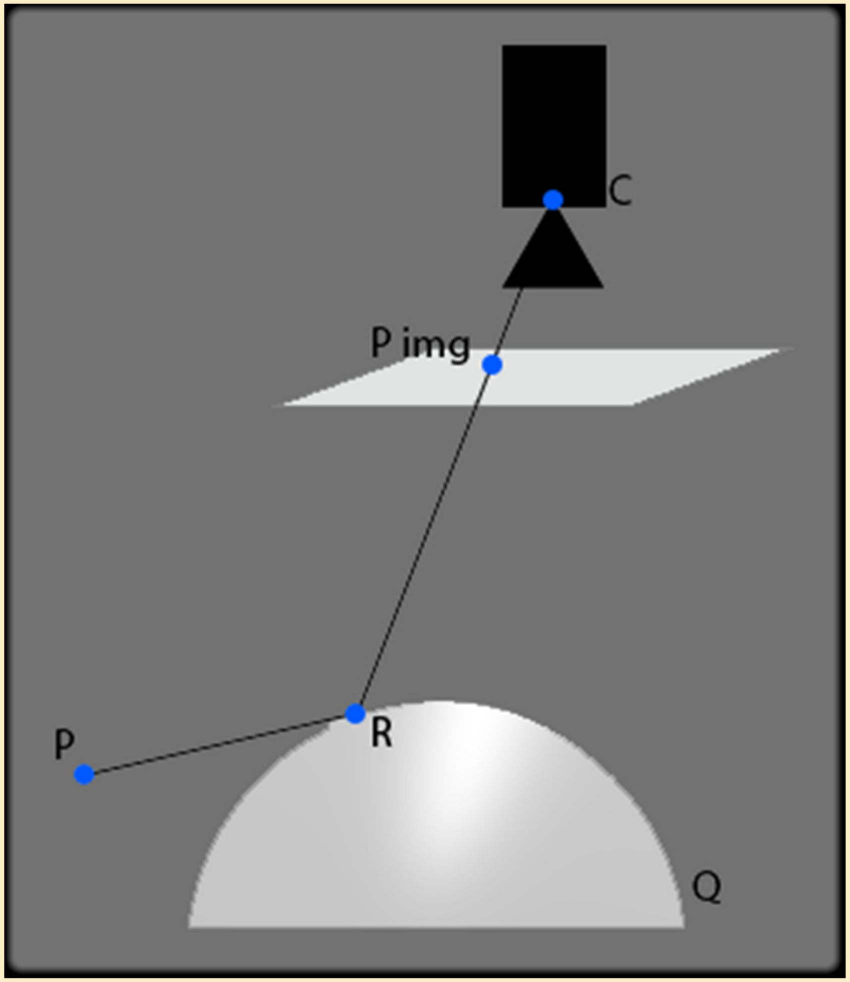
- The equation that is widely used for this type of quadrics is:

$$\boldsymbol{x^2 + y^2 + Az^2 + Bz - C = 0}$$

- They are, then, rotationally symmetric.
- Often used in robotics.

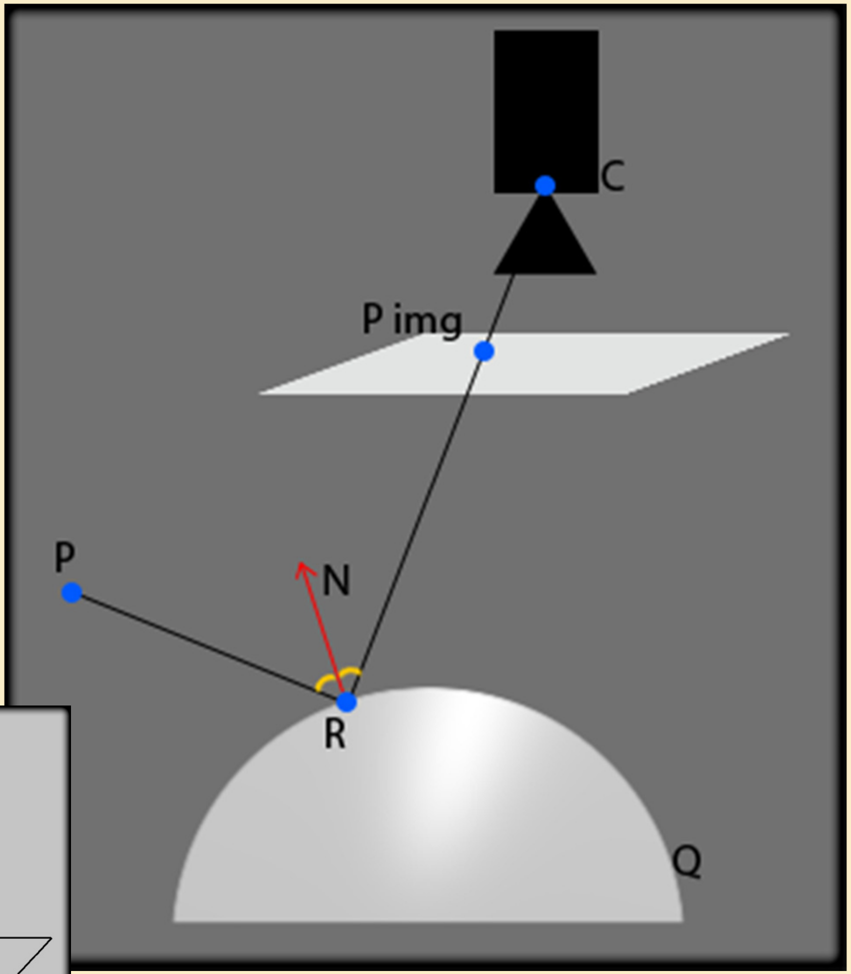
FORWARD PROJECTION MODEL

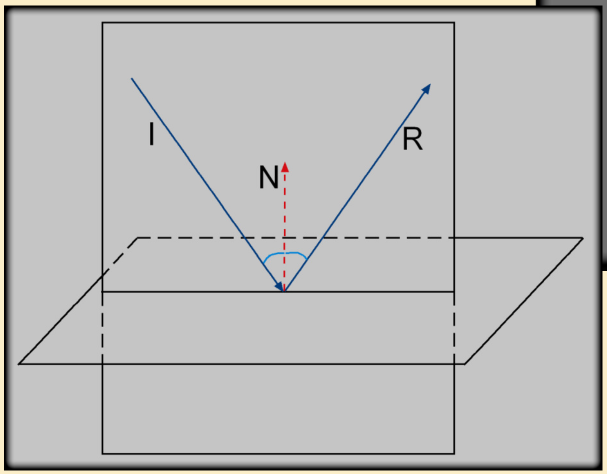
- Back to the reflection:



REFLECTION LAW

- The Law of Reflection says that:
  - The reflection is a planar phenomenon
  - The incident and reflected angles are equal



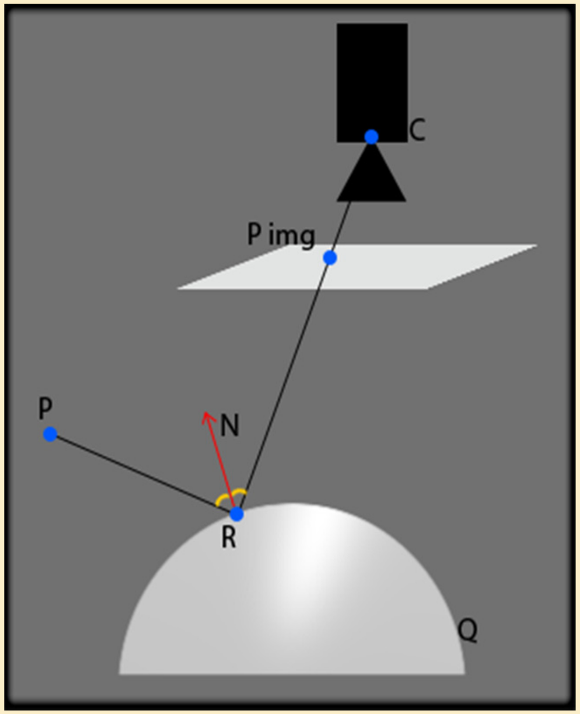


HOW TO COMPUTE THAT?

- Reflection equation:

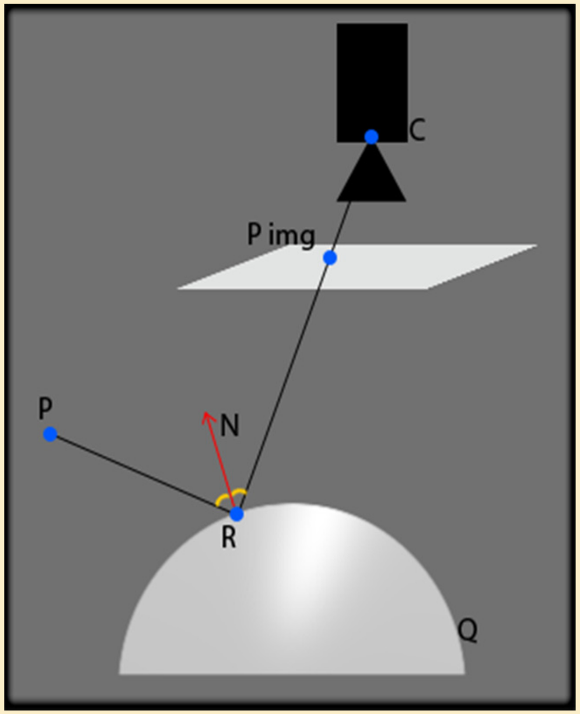
$$v_r = v_i - 2 < v_i^T n > n$$

- Reflected vector v_r:
  - $v_r = \frac{C-R}{|C-R|}$
- Incident vector v_i:
  - $v_i = \frac{R-P}{|R-P|}$



HOW TO COMPUTE THAT?

- Normal vector $\boldsymbol{n}$:
  - First compute the tangent plane to the quadric $\boldsymbol{Q}$ at the point $\boldsymbol{R}$.
  - $\boldsymbol{N} = \boldsymbol{QR}$ (homogeneous coord.)
  - Then, take the normal vector to the plane:
  - $\boldsymbol{n} = \dfrac{\boldsymbol{N}(\mathbf{1{:}3})}{|\boldsymbol{N}(\mathbf{1{:}3})|}$



HOW TO COMPUTE THAT?

- Back to the reflection equation:
  - $\boldsymbol{v_r = v_i - 2 < v_i^T n > n}$

- It is non linear and depends on three spatial coordinates: x, y and z.
- If the quadric equation is used, it can be reduced to an equation in <u>2 space coordinates</u>.

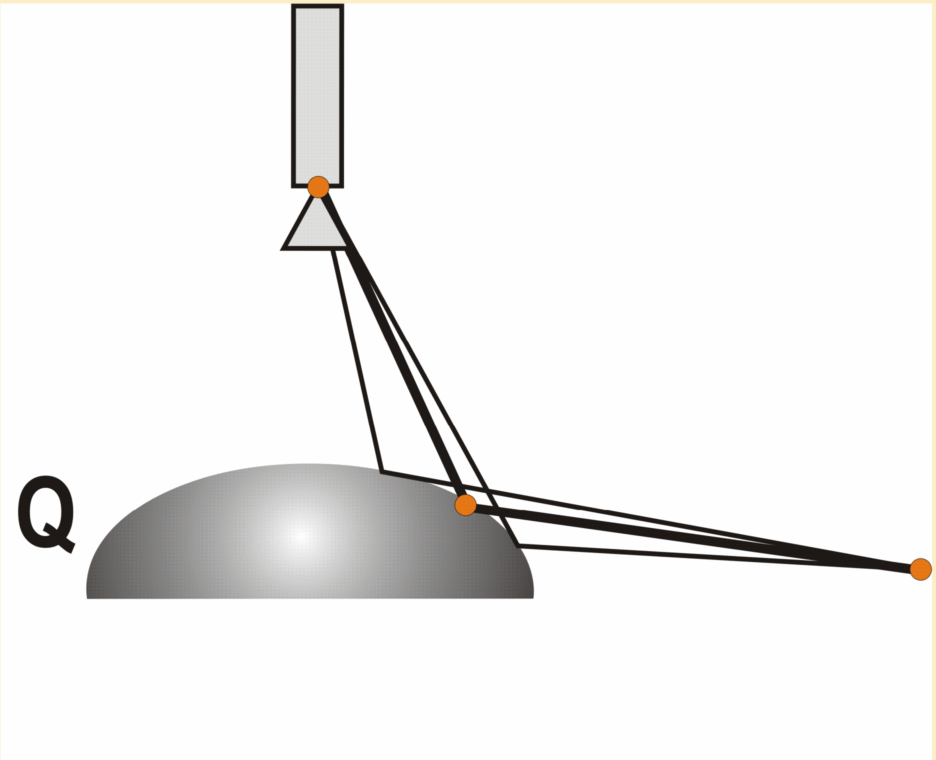
FERMAT PRINCIPLE

- The Fermat Principle says that the light always takes the **quickest path**, i.e. the shortest one in the real space.

- Light is lazy, then! Or efficient.

FERMAT PRINCIPLE

- Search for the reflection point can be made by minimizing the sum of distances from point $\boldsymbol{P}$ to the reflection point $\boldsymbol{R}$ and from this point to the eye $\boldsymbol{C}$.



HOW TO DO THAT?

- Sum of distances:

$$\boldsymbol{dist = |P - R| + |R - C|}$$

- We want to:

$$\boldsymbol{min_R\, dist}$$

- subject to:
  - $\boldsymbol{P}$, $\boldsymbol{R}$ and $\boldsymbol{C}$ defines a plane that is perpendicular to $\boldsymbol{N}$.
  - $\boldsymbol{R}$ belongs to the quadric: $\boldsymbol{R^T Q R = 0}$.

HOW TO DO THAT?

- So we've got:

$$\boldsymbol{min_R \; |P - R| + |R - C\,|}$$

- This minimization is:
  - non linear
  - dependent on at least <u>two variables</u>

SUMMARIZING

- The computation of the reflection point (forward projection model) can be computed by:
  - Law of Reflection - $\boldsymbol{v_r = v_i - 2 < v_i^T n > n}$
  - Fermat Principle - $\boldsymbol{min_R\, |P - R| + |R - C\,|}$
- The minimization is non-linear and multivariate.

SUMMARIZING

- Hence the computation of the reflection point using the classical methods is:
  - time consuming;
  - numerical unstable;
  - good accuracy is, however, generally achieved.

ALGEBRAIC SOLUTION

- Proposition: The reflection point $\boldsymbol{R}$ on a quadric mirror $\boldsymbol{Q}$, reflecting a light ray emitted by a source $\boldsymbol{P}$ to a target $\boldsymbol{C}$, is on the quadric surface $\boldsymbol{S}$, given by $\boldsymbol{S} = \boldsymbol{M}^T \boldsymbol{Q}_\infty^* \boldsymbol{Q}$. Where:
  - $\boldsymbol{M} = \boldsymbol{f}(\boldsymbol{P}, \boldsymbol{C})$
  - $\boldsymbol{Q}_\infty^*$ is the absolute dual quadric (Euclidean space)

[Gonçalves,2010] On the reflection point where light reflects to a known destination in quadric surfaces. Optics Letters 35(2), 2010.

ALGEBRAIC SOLUTION

- This means that we have the following three constraints for the reflection point $\boldsymbol{R}$:

  - $\boldsymbol{R}$ belongs to the quadric mirror $\boldsymbol{Q}$.
  - $\boldsymbol{R}$ belongs to the algebraic quadric $\boldsymbol{S}$ ($\boldsymbol{S} = \boldsymbol{M}^{\boldsymbol{T}} \boldsymbol{Q}_{\infty}^{*} \boldsymbol{Q}$).
  - $\boldsymbol{R}$ must respects the Law of Reflection and Fermat Principle (pick only one!)

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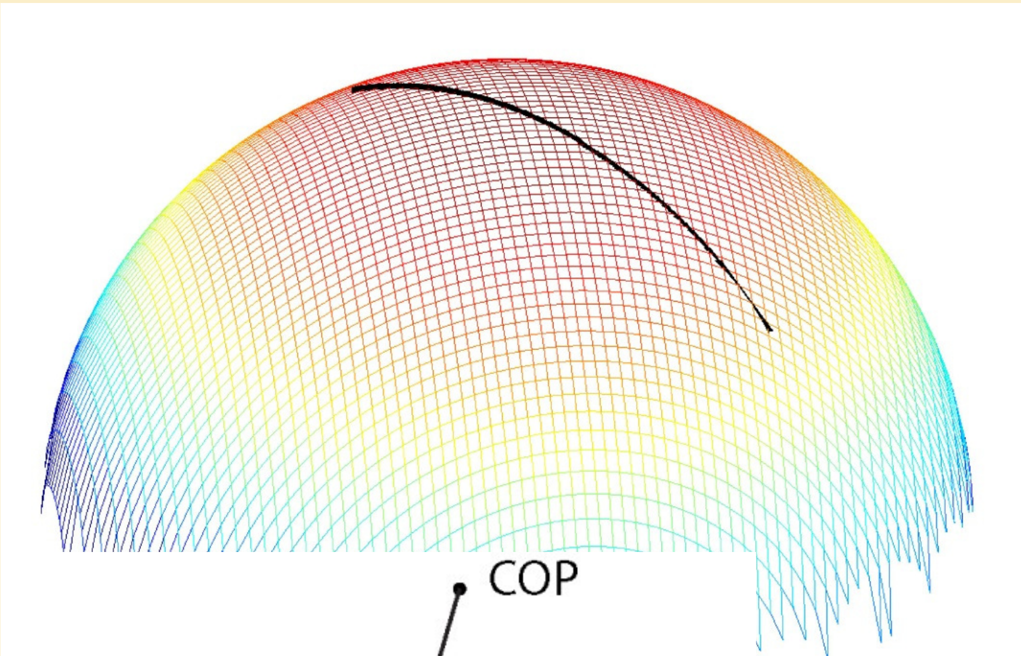
So we must intersect two quadrics.

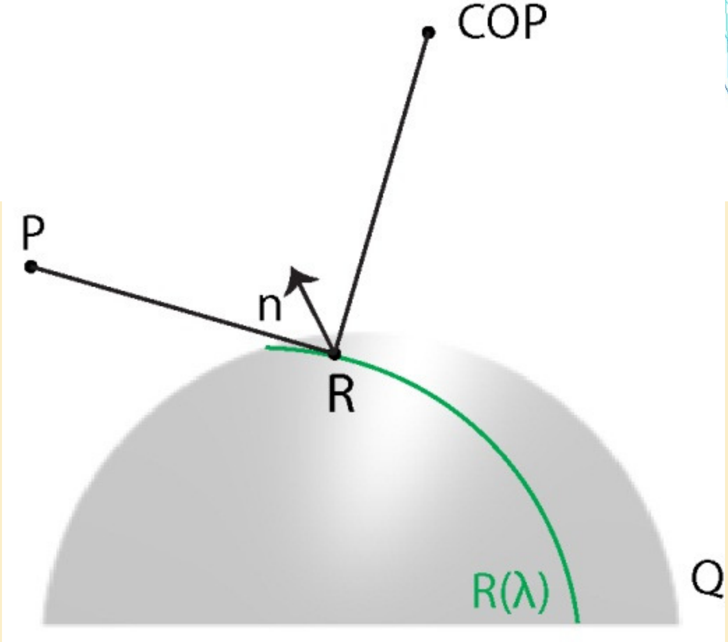
ALGEBRAIC SOLUTION

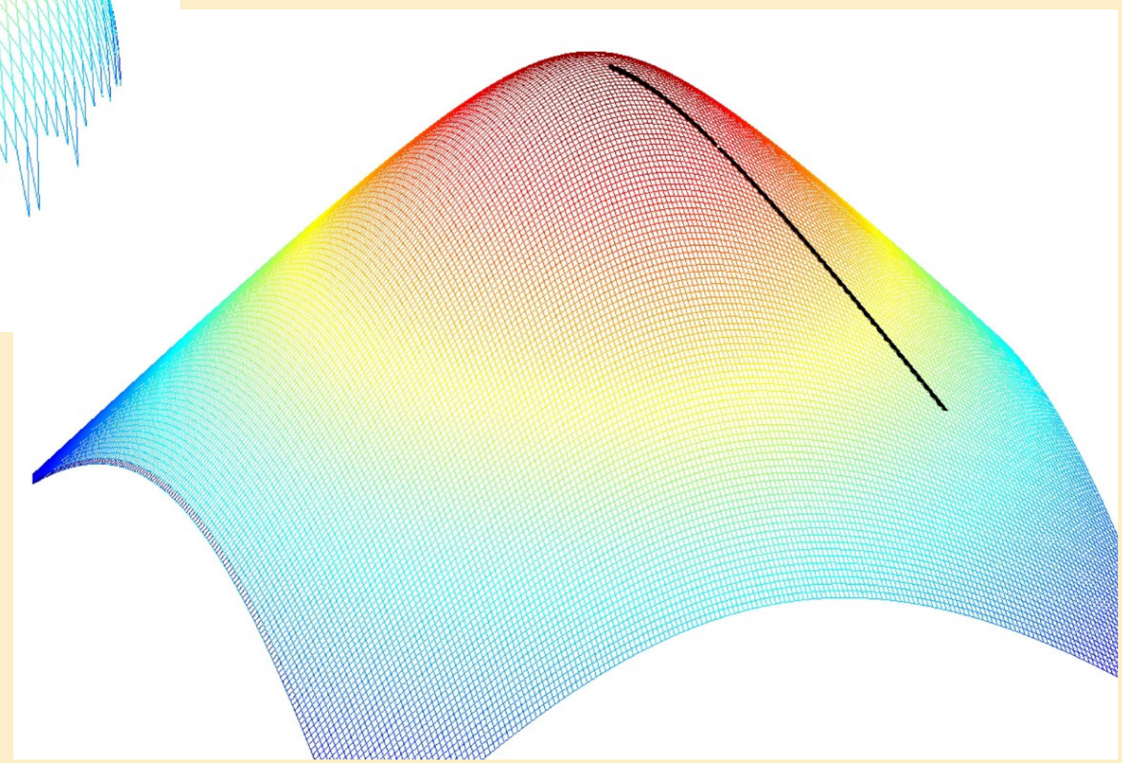
- The reflection point $\boldsymbol{R}$ is then in the curve of intersection of the two quadrics (mirror and analytically derived one).

- This curve is parameterized in only one parameter, using the method of quadrics intersection by Loria, Nancy [L. Dupont et al., 2008].

THE CURVE OF INTERSECTION







SEARCHING FOR THE REFLECTION POINT

- After intersecting the two quadrics and having a parameterization of the curve in only one parameter (Euclidean, or two in $\mathbb{P}^1$), ...

- ... the thing to do is to search on this curve for the best point that fits Law of Reflection or Fermat Principle (best way). We use any non-linear iterative minimization method.
  - Golden Section ...
  - Brent ...

GEOMETRIC INTERPRETATION FOR THE CURVE

- For spherical mirrors, the second quadric is just the plane defined by $\boldsymbol{P}$, $\boldsymbol{C}$ and the center of the mirror.
- So the curve is a just circle.

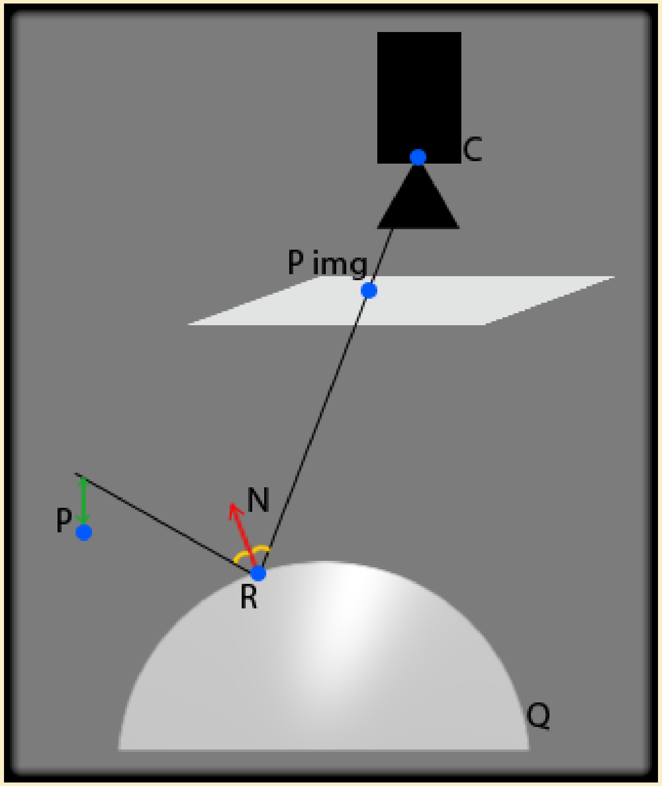
- But … for other types of mirrors (paraboloids, ellipsoids and hyperboloids of two sheets) it doesn't seem to be a geometric interpretation.

ALGEBRAIC QUADRIC S

- Quadric $\boldsymbol{S}$ has a zero diagonal.
- For rotationally symmetric mirrors (which is the general case), the quadric $\boldsymbol{S}$ has inertia (2,2), which for our case is the same to say that it is a hyperbolic paraboloid.
- For rotationally symmetric mirrors with the camera eye $\boldsymbol{C}$ in the rotation axis, the quadric $\boldsymbol{S}$ degenerates to a pair of planes.
- For spheres, it is a plane, with inertia (1,1).

HOW DOES PEOPLE COMPUTE R?

- In computer vision
  - Usually by applying Law of Reflection
  - By back projecting a pixel and iterating in the image coordinates until the incident direction passes in the point
    - They are both a non linear multivariate minimization algorithm



MITSUBISHI METHOD – POLYNOMIAL

- Recently published in CVPR 2011 [Agrawal et al. 2011] solved the problem using an 8th degree polynomial.
- Advantage: closed-form equations for the coefficients of the polynomial.
- Drawbacks:
  - It has to compute 55.000 powers, additions and multiplications.
  - Iterative approximation method to find its roots.
  - It is, hence, <u>very slow</u>.

PERFORMANCE EVALUATION

- We made some tests and concluded that:
- QI method (as we call it) is one to two orders of magnitude faster than the classical laws.
- QI method is 2 to 5 times faster than the Mitsubishi method (polynomial method) [Agrawal et al. 2011]

[Conclusions submitted to Machine Vision and Applications.]

PERFORMANCE EVALUATION

- Notice that QI method uses the method of [Dupont et al. 2008] for intersecting quadrics but not their implementation of it. Further accelerations may be expected if their exact implementation is used.

- Could we do better?

REFLECTION PROPERTY

- Some surfaces have the so called **reflection property** [Drucker 90,91].

- It states that for some surfaces in the real space the normal vector to the surface bisects, in a generic point of it, the angles made by connecting this point to two special points (not on the surface)

REFLECTION PROPERTY

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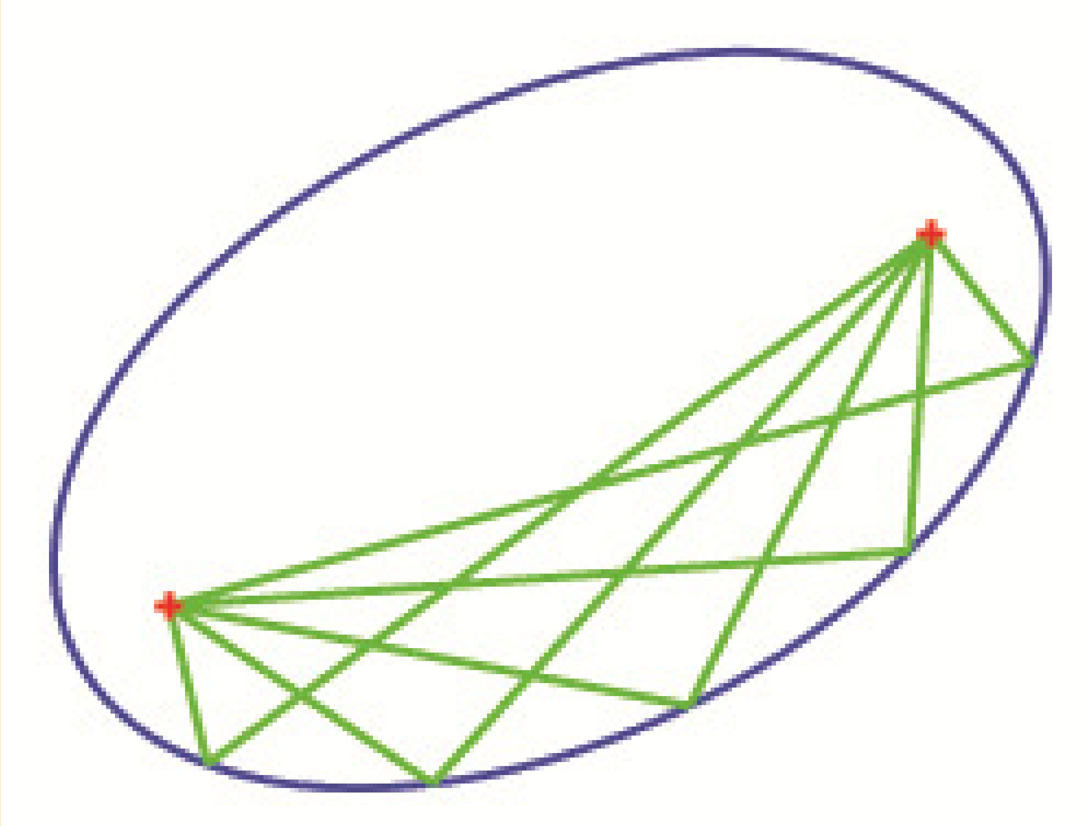
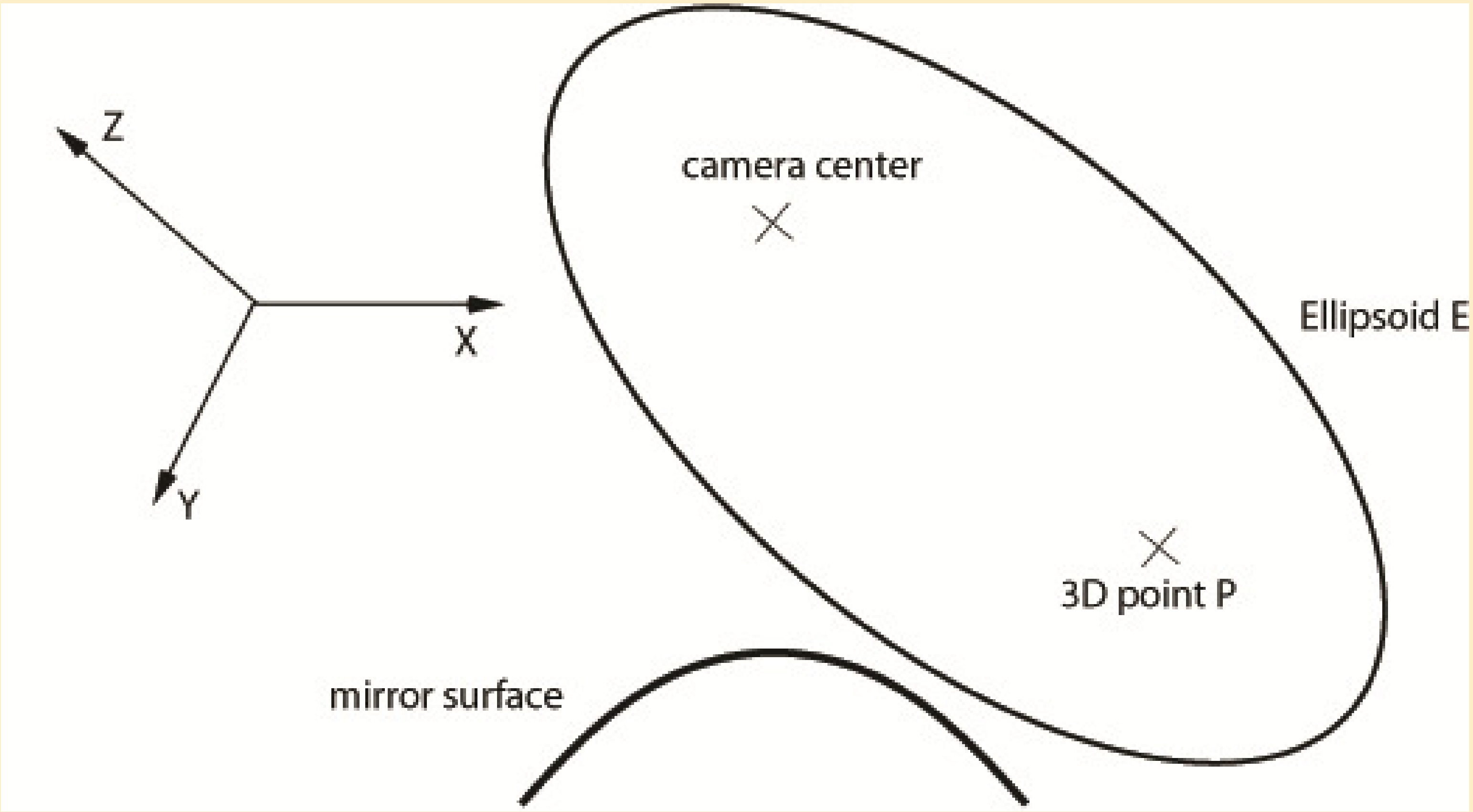


REFLECTION PROPERTY

- As shown in [Drucker 90], the smooth connected surfaces in $\mathbb{R}^3$ that have this properties are:
  - a plane
  - a sphere
  - a paraboloid of revolution
  - an ellipsoid of revolution (about the line of its foci)
  - an hyperboloid of revolution (about the line of its foci)

NEW IDEA

- This property of ellipsoids can be used to find the reflection point.



TANGENCY

- If we parameterize an ellipsoid by defining its foci (camera eye and 3D point to project) ...
- If we grow up the ellipsoid until it becomes tangent to the mirror surface ...
- Since the tangent planes to the mirror and to the growing ellipsoid, the point of intersection is the reflection point. We still have to compute the point.

ADVANTAGES OF THIS NEW IDEA

- We are working on this with the group of Prof. Sylvain Lazard and Laurent Dupont (LORIA/Nancy).

- More elegant formalism.
- Formalized by a smaller degree polynomial
- Could more easily be extended to other types of mirrors (quadrics, approximations of them or even general convex mirrors)

- Is it computationally efficient?

SUMMARIZING AGAIN

- The reflection point $\boldsymbol{R}$ can thus be found:
  - as the intersection of two quadrics: the mirror and an algebraic quadric that depends on the mirror, the camera eye and the 3D point to project (the solution is result of implicit equations and approximated using an adequate algorithm).

  or

  - growing up an ellipsoid (or hyperboloid) of revolution whose foci are the camera eye and the 3D point (work in progress).

OTHER APPLICATIONS TO COMPUTER VISION

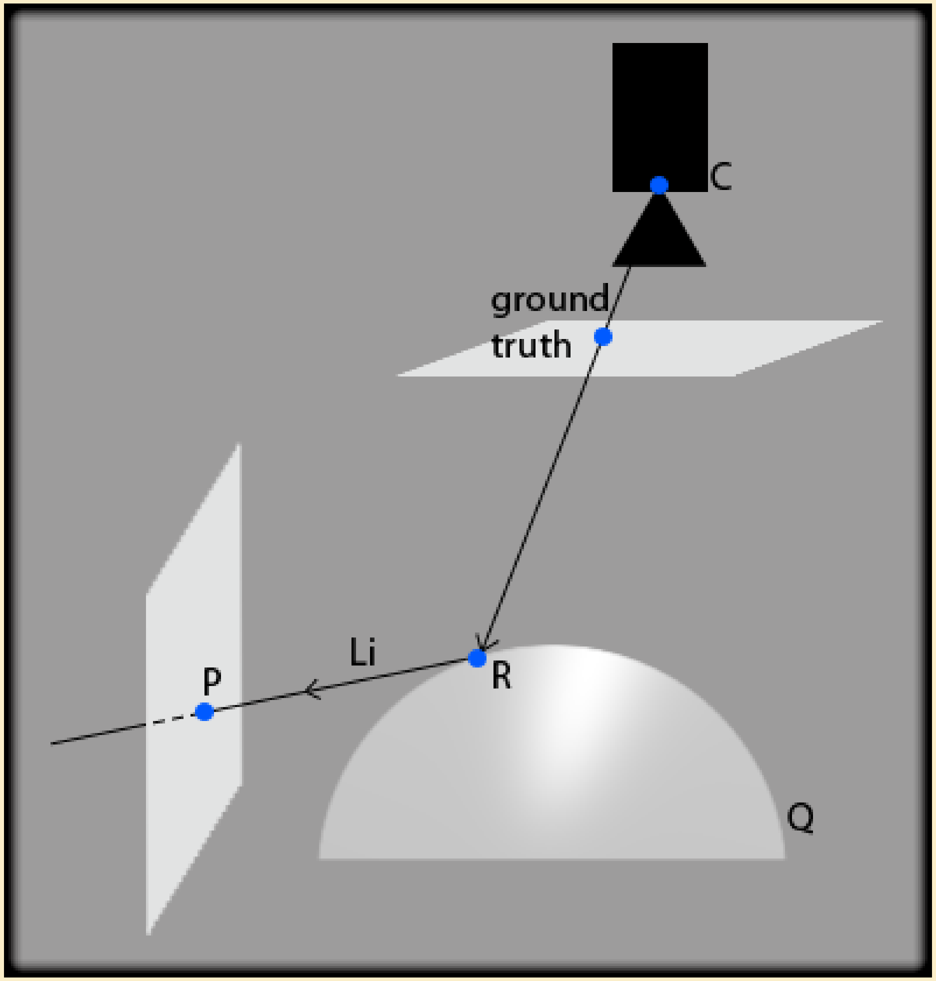
- Pose estimation
- 3D reconstruction
- Calibration of catadioptric cameras
- Structure from motion
- Epipolar geometry (2 cameras)
- Augmented reality
- …

APPLICATION TO COMPUTER GRAPHICS

- There are two ways to compute mirror reflections in computer graphics
  - ACCURATELY
  - FAST
- There is not a good trade-off between accuracy and performance.

ACCURATE SOLUTION

- Ray-tracing
  - It is the reference for all graphic motors
  - It start from the pixel and back project it through the mirror (by using Law of Reflection) and then intersect incident light rays with objects.
- It is slow! Even using Optix.



FAST SOLUTIONS

- Environment maps
- Cube maps
  - These methods pre-compute the texture of the scene and distort it and shrink it to fit to the mirror in the image
  - Suffer from parallax problem.
  - It creates many aberrations and artifacts, mainly in objects close to the mirror or to its border.
  - But they are quick and real-time.

ENVIRONMENT MAPPING

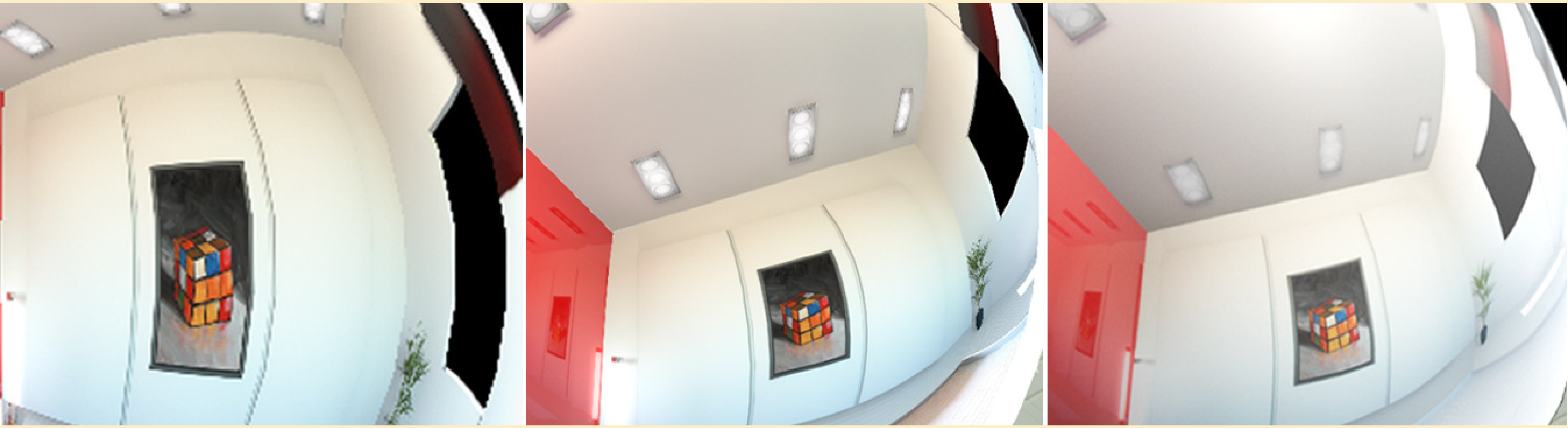
Images are from
http://www.reindelsoftware.com

REFLECTIONS ON GRAPHICS

- Even today, with all the computational power available, reflections in mirrors are not a trivial problem to solve.

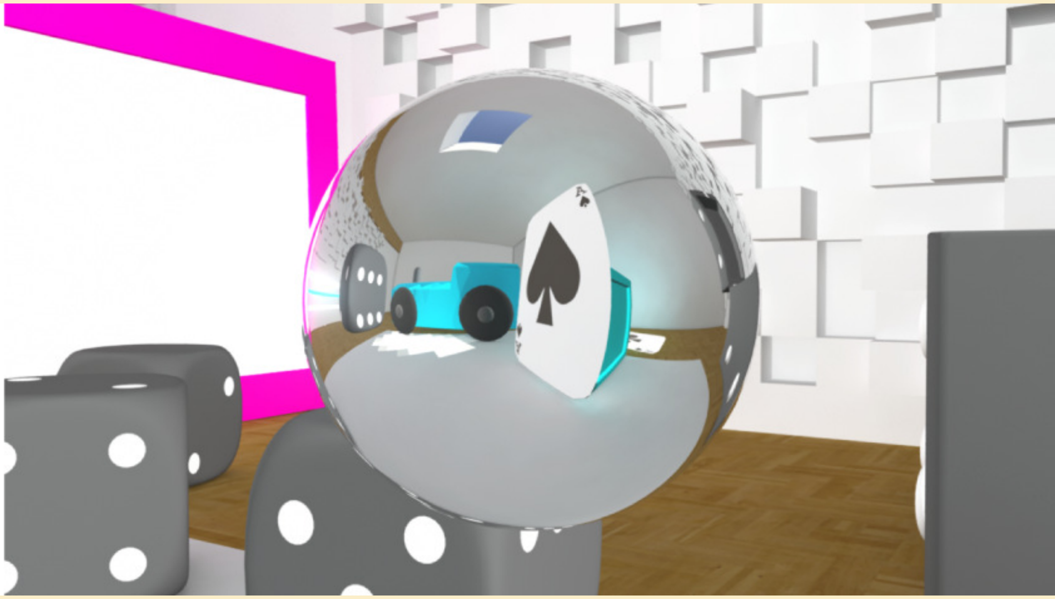
REFLECTIONS ON GRAPHICS

- It seems nice but …let us look in detail!



REFLECTIONS ON GRAPHICS

- Using the GPU we achieved real-time with very high accuracy.



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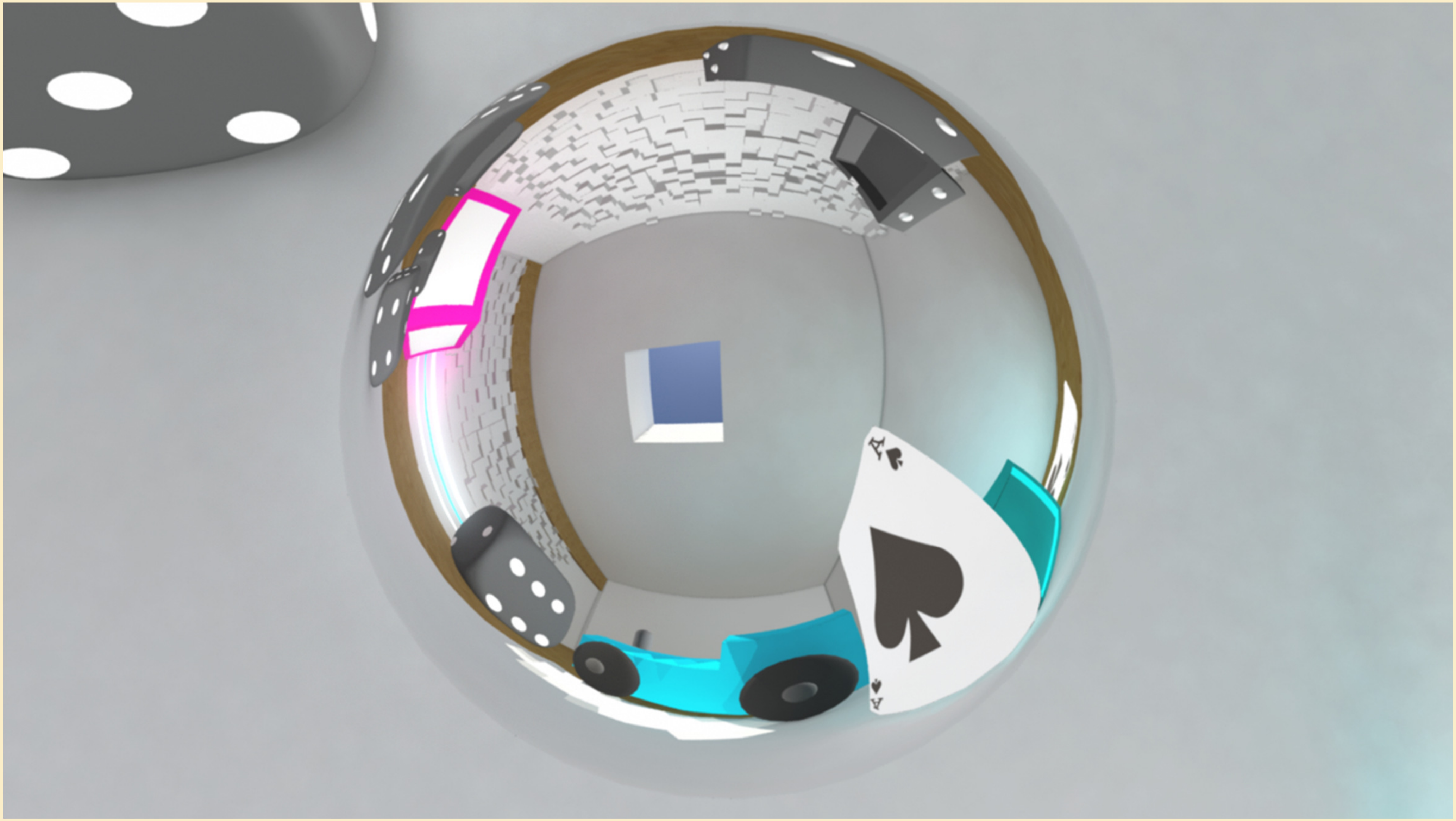

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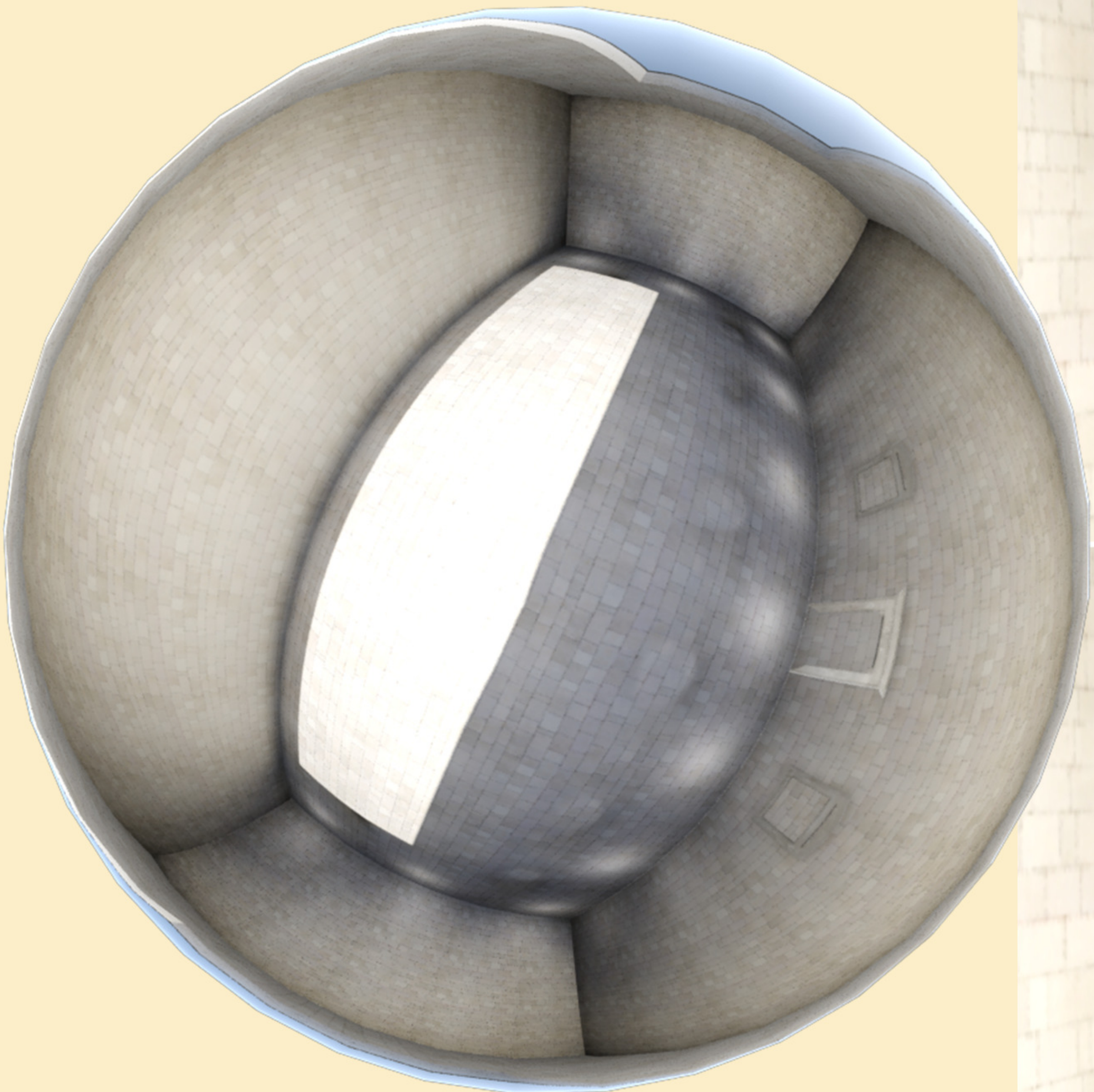
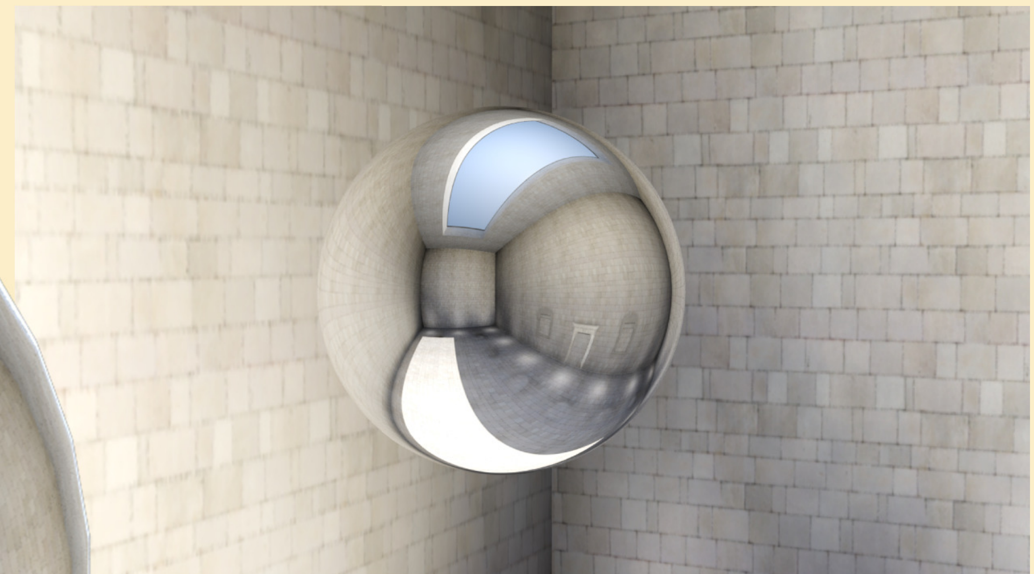
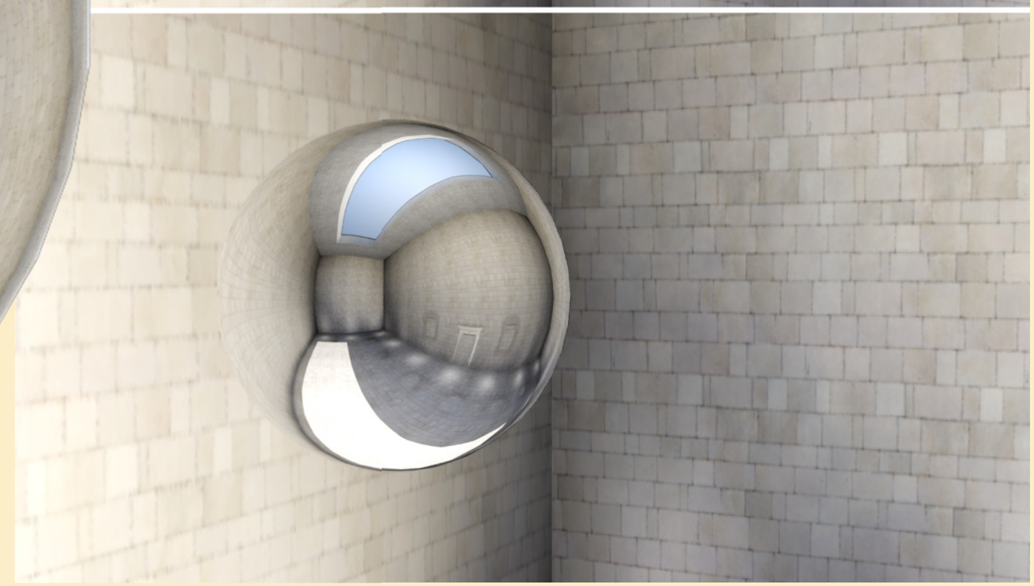






BACK TO THE EXACT SOLUTIONS

GAMES INDUSTRY

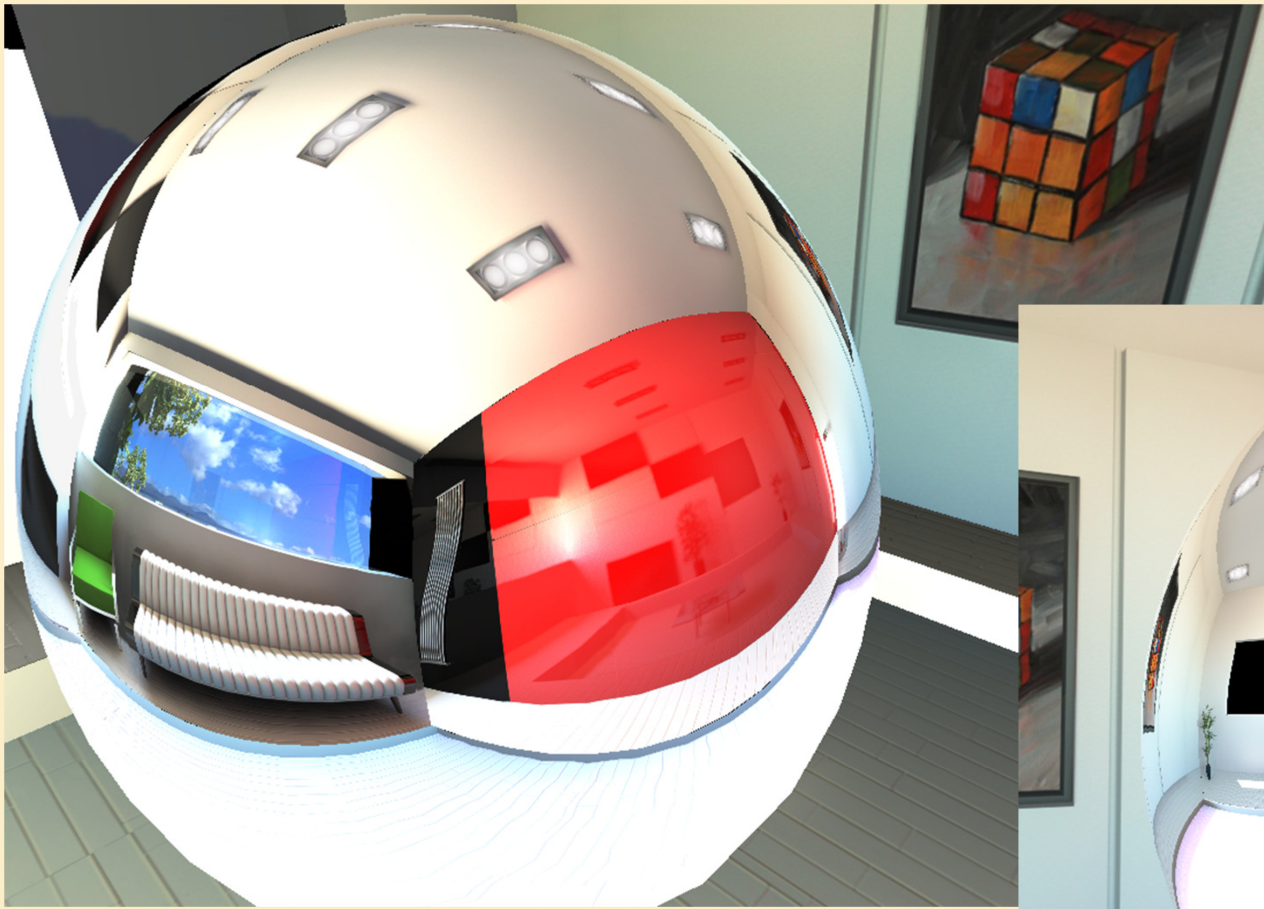
- Games industry is looking for a better method that could achieve real-time with accuracy compared with ray-tracing.
- For now, they keep using bad, since fast is better than good!

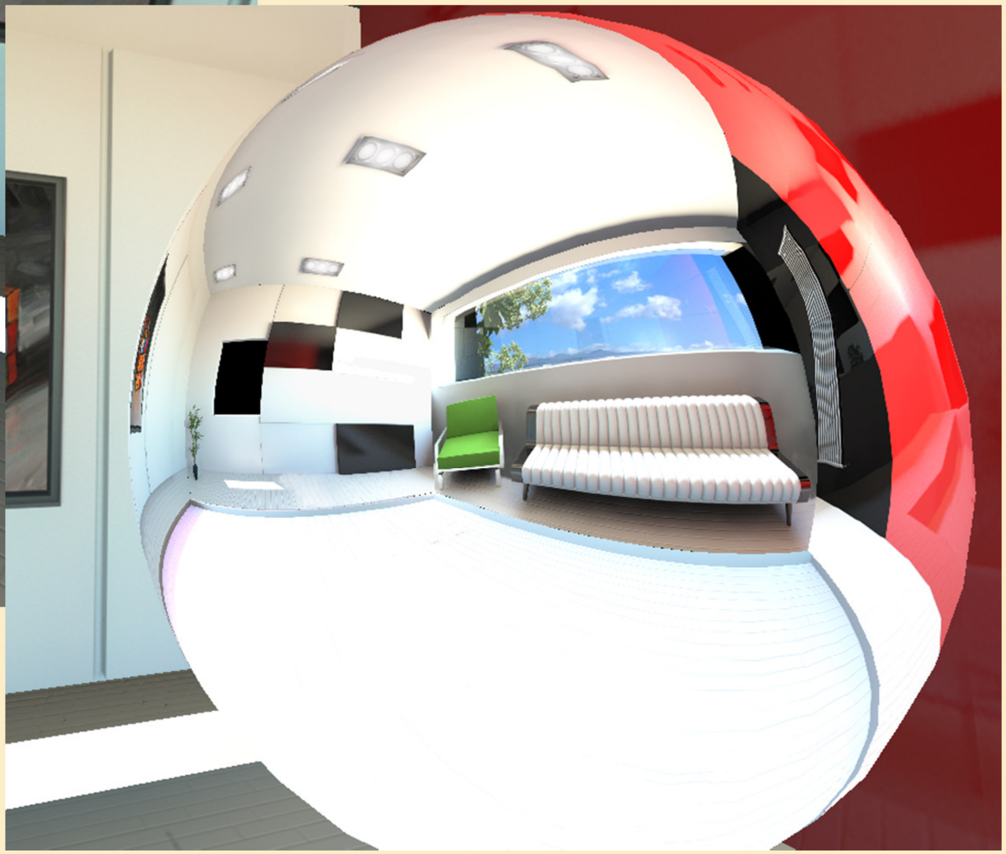
ACCURATE REFLECTION ON GRAPHICS

- Using the QI method we more than doubled the velocity when compared with Mitsubishi method and with Ray Tracing.
- For the maximum accuracy.

- We achieve average velocities of 150 fps for around 2250 points. This allows for real-time applications!

ACCURATE REFLECTIONS ON GRAPHICS



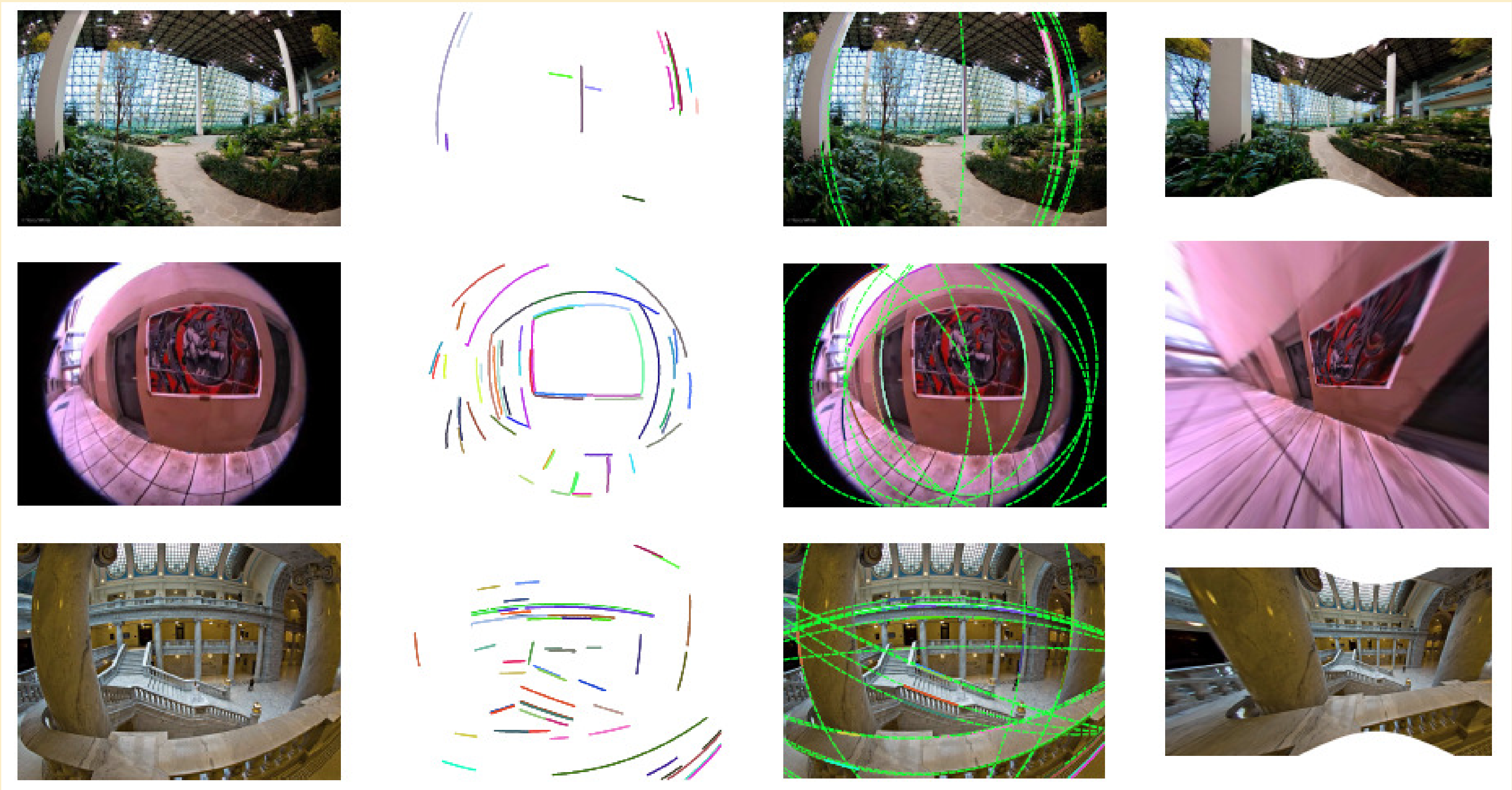


OUR FUTURE WORK

- Keep looking for an algebraic/geometrical solution better than the existing.
- Use method QI to compute reflections in general surfaces approximated (or not) by quadrics.
- Augmented Reality.

STILL OPEN PROBLEMS

- Study geometrical properties of image of lines in Non SVP cameras and using radial distortion.



(accepted to ICCV'13)

THANK YOU

- Some questions?

- Contact: nunogon@deec.uc.pt